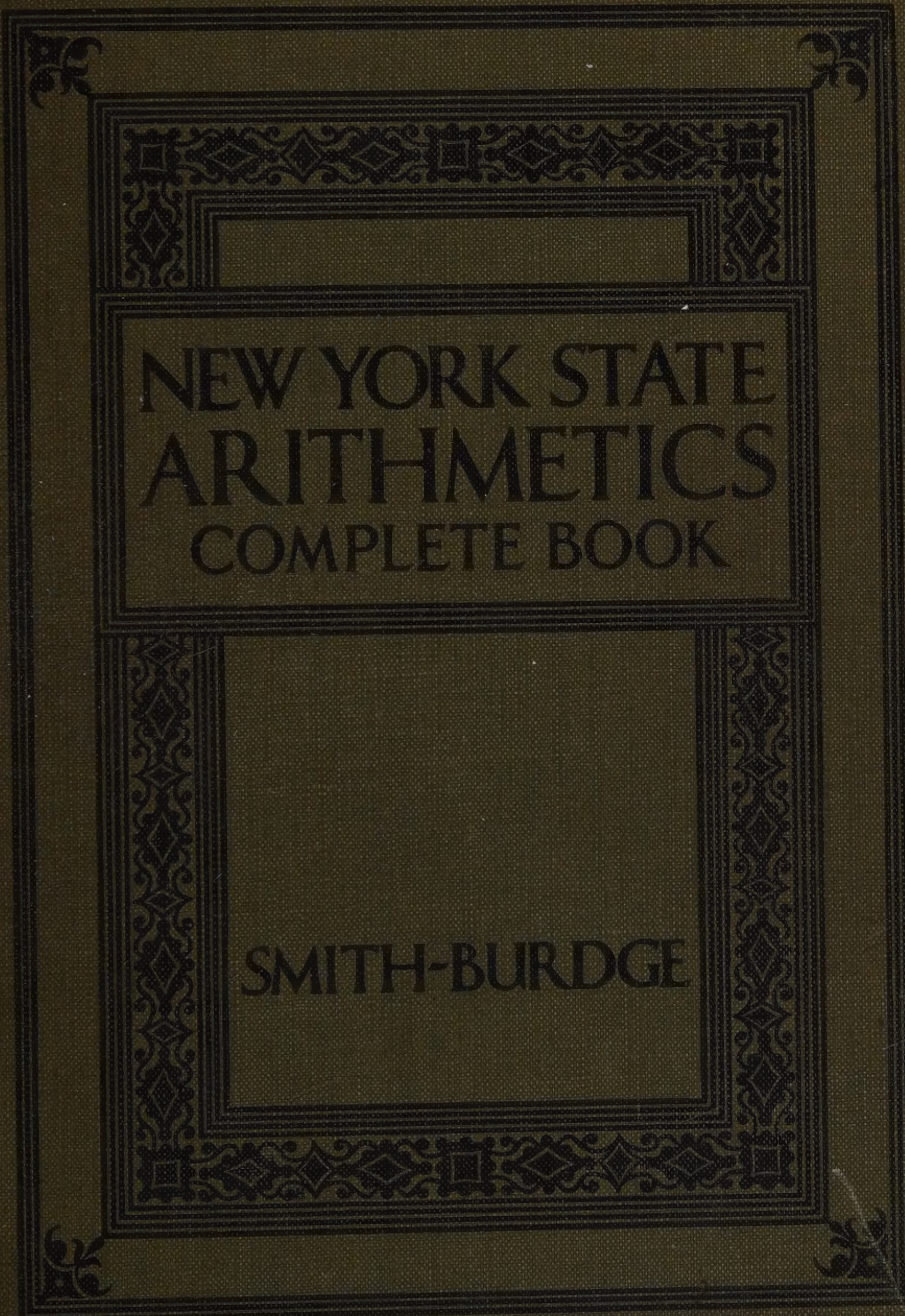

NEW YORK STATE ARITHMETICS

COMPLETE BOOK

SMITH-BURDGE

WENTWORTH-SMITH MATHEMATICAL SERIES

NEW YORK STATE ARITHMETICS

COMPLETE BOOK

BY

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PREFACE

This series of arithmetics has been prepared not only to meet the requirements of the latest official syllabus of the State of New York and to keep pace with the present progressive movement in the teaching of arithmetic, but also with the purpose of setting a new standard in the arrangement of material and in the development of the subject to meet the needs and interests of children.

The whole series has been based upon a recognition of the principle that the proper applications of arithmetic are found only in a socialized subject matter that appeals to the child's interest. The basal topics are so arranged that a pupil stays long enough with a subject at one time to acquire that feeling of mastery which is his right and privilege. Besides this, there are several other features which are noteworthy. One is the large amount of repeated drill upon the fundamental operations, which serves as the best possible preparation for modern tests; the second feature is the series of silent reading lessons by which the pupil enlarges his vocabulary without learning formal definitions that are of no interest and of but little value; the third feature is the Review and Drill section, placed near the end of each chapter, which furnishes a cumulative review of all preceding work; the fourth feature is the sets of Problems without Numbers and the Problems for Completion, each of which requires new lines of independent thought on the part of the pupils; and the fifth is the Little Examinations, a brief series of tests covering each chapter in turn.

By means of these features a teacher may be assured that the pupil's memory is kept refreshed upon those essentials of computation without which he cannot hope to succeed and that he is trained in independent thinking in arithmetic. There have also been inserted a number of elementary psychological tests relating to arithmetic, and an unusually large amount of drill material of the latest type, some or all of which should be used according to the needs of the class. Such features as the motivation of work, groups of problems relating to the interest and needs of childhood, timed tests, suggested projects, group contests, and the like will be found throughout the series.

This book, which is a combination of the Intermediate (Grades V and VI) and the Advanced (Grades VII and VIII) Books of the series, is arranged in chapters, each of which covers the work of a half-year as recommended by the syllabus. Teachers should feel free to omit as many exercises as may be necessary to carry out this plan. In general, teachers too often fail to realize the importance of omitting exercises in accordance with the needs of individual classes, and too many textbooks fail to supply a sufficient amount of material to permit a proper selection.

In the plan of the work by grades, especial recognition has been made of the fact that the first six years are, in general, devoted chiefly to the development of skill in the fundamental operations, and that, as the syllabus states, the point of view in Grades VII and VIII is somewhat changed. The pupils in these grades are beginning to realize that they are part of a great social organization in which they must prepare to take their place, and the problems must therefore have a more significant bearing on life. The school fails in its mission if it does not at this time seek to present the principles of fair business dealings and of good citizenship. All this imposes a new responsibility upon the school besides the one of teaching children to compute, which, after all, must be the chief aim of arithmetic. It should be remembered that the schools are not training experts for special technical positions, but that they should give both boys and girls as complete a view as possible of modern business procedure and civic responsibility, such as the average citizen needs in order to take his place as an intelligent member of a community.

Following the suggestions of the syllabus, the work of Grade VII relates chiefly to those applications of percentage which the pupil will be most likely to need later, — profit and loss, commission, commercial discount, insurance, taxes, and simple problems of banking. It also includes further work in measurement and in drawing simple plans and maps. Along with this work there will be found a series of systematic reviews of the fundamental operations, in recognition of the experience of all successful teachers that there is "no skill without drill." Interest in this drill is maintained by relating it to the applied work and by the proper use of tests for speed and accuracy.

In Grade VIII the pupil extends his knowledge by studying the practical uses of ratio, proportion, and roots, and by a further consideration of those important phases of arithmetic which relate to thrift, savings, investments, and general business practice. He also takes up briefly some work in the important field of intuitive geometry and is shown the practical use of simple algebraic expressions, thus obtaining some insight into the nature and uses of mathematics in the sciences and in industry.

In the Teachers' Manuals accompanying this book there will be found detailed suggestions that will assist the teacher in still further vitalizing the work of Grades V–VIII.

DAVID EUGENE SMITH
HOWARD GRIFFITH BURDGE

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NEW YORK STATE ARITHMETICS

INTERMEDIATE BOOK

CHAPTER I

I. FUNDAMENTAL OPERATIONS

A SILENT READING LESSON

Our common numerals are sometimes spoken of as Hindu, because it is believed that they were first used by the Hindus in ancient India. They are also spoken of as Arabic, because they were passed on to Europe by the Arabs.

We use also the Roman numerals, which are seen on clocks and watches. These numerals were used by most Europeans until about the time that America was discovered.

Our common numerals are more convenient than the Roman numerals, because we can write most numbers with fewer figures; for example, where we write 87 the Roman system requires LXXXVII.

If we write one after five, we have either 51 or VI. They do not have the same meaning, however, because 51 means $50 + 1$ and VI means $5 + 1$.

If we write ten, then five, and then one, we have 1051 or XVI. What is the difference in meaning?

A SILENT READING LESSON (CONTINUED)

Our numerals are better than any others, because we can easily write any number that we need to use by means of only ten different figures. The ten figures are

0 (zero), 1, 2, 3, 4, 5, 6, 7, 8, and 9.

In the number 51, 5 represents tens, because it is in tens' place; that is, in our system each figure has a *place value.*

It is easier to read a number like 3,248,630 if we separate it into *periods* of three figures each, beginning at the right. In a large number, the left-hand period may have one, two, or three figures. Periods are not commonly indicated in numbers of only four figures. We sometimes separate the periods by spaces instead of commas.

In telephoning, we read the number 4090 as "four-o-nine-o." Bookkeepers often read the number $4207.09 as "four-two-o-seven point o-nine."

A thousand million is called a *billion.* There are also names for larger numbers, which, however, are not used in practical life.

The number shown at the right is read "thirty-eight billion four hundred thirty-five million five hundred six thousand one hundred forty-seven."

$$\overbrace{38}^{\text{Billions}},\overbrace{435}^{\text{Millions}},\overbrace{506}^{\text{Thousands}},\overbrace{147}^{\text{Units}}$$

The figures 1, 2, 3, and so on to 9, and also their values, are sometimes called *digits.* Formerly, the writing of numbers was called *notation,* and the reading of numbers was called *numeration,* but as people seldom use these words today, we do not have to learn the definitions.

A LITTLE WRITING LESSON

Write answers to the following questions:

1. By looking at your fingers do you see why people came to count by tens? Why was it?

2. Some people, particularly in warmer parts of the earth where they went barefooted, used to count by twenties. Can you tell what the reason was?

3. Some people even now speak of a score, which means 20. From what ancient way of counting does this come?

4. In buying eggs do you count them by 10's? by 20's? by 12's? What name do we give to 12? Probably at one time many people usually counted by 12's.

5. What advantage has our method of writing nine (9) over the method the Romans used (IX)?

6. In writing 542, in which place is the 5? the 4? the 2?

7. In 26,623,054,689, what name has the period 689? the period 054? the period 623? the period 26?

8. What name do you give to 1000 thousand?

9. How do you write in figures the number six million twenty-five thousand fifty-nine?

10. How do you write 3,492,006 in words?

11. How do you read the telephone number 3207?

Write the following in words:

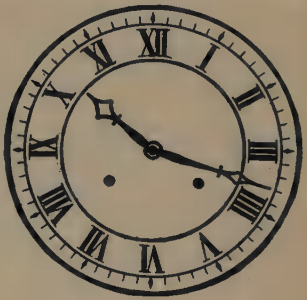
12. $6.25.
13. $0.37.
14. 45,850.
15. 84,909.
16. 230,438.
17. 700,007.
18. 6,842,759.
19. 32,329,646.

20. Write in figures the number six million six.

A SILENT READING LESSON

We do not need to learn much about Roman numerals except to be able to read them on clocks, in the chapter numbers of books, in dates on buildings, or similar cases.

The twelve numerals on a clock face are as follows:



I (1)
II (2)
III (3)
IV or **IIII** (4)
V (5)
VI (6)
VII (7)
VIII (8)
IX (9)
X (10)
XI (11)
XII (12)

The Romans usually wrote IIII for four and VIIII for nine, but this is no longer the common way.

In writing Roman numerals we combine the following letters, usually capitals, according to the rules below:

Letters,	I	V	X	L	C	D	M
Values,	1	5	10	50	100	500	1000

1. *Writing a letter of less value after a letter of greater value indicates that the values are to be added.*

Thus, using V (5) and I (1), we have VI = 5 + 1 = 6.

2. *Writing a letter of less value before a letter of greater value indicates that the first value is to be subtracted.*

Thus, using I (1) and X (10), we have IX = 10 − 1 = 9.

3. *Repeating a letter indicates that its value is repeated.*

Thus, C = 100, and so CC = 100 + 100 = 200. Usually letters are not taken more than three times, but the old forms IIII for 4 and CCCC for 400 are exceptions.

ANOTHER WRITING LESSON

Write answers to the following questions:

1. In writing a number in Roman numerals, do you begin at the left or the right? Which is the easier way?

2. What does the year 1776 mean in the history of our country? How many figures do you use in writing this date with our common numerals? with Roman numerals?

M = 1000
DCC = 700
LXX = 70
VI = 6
MDCCLXXVI = 1776

3. The Romans would have written the number 1928 as MDCCCCXXVIII. We often see 1928 written as MCMXXVIII. Write the present year and the next year in these two ways.

4. Write the tens in Roman numerals, from X to XC.

5. Write the hundreds from C to DCCCC (or CM).

6. After reading Chapter XXXIX of a book of sixty-four chapters, how many more chapters have you to read?

7. On the cornerstone of a church there is the date MDCCCLXXXVIII. When was the church built?

Write the following numbers in common numerals:

8. XIV. **10.** XXV. **12.** LXXI. **14.** LXXXVII.
9. XVI. **11.** XIX. **13.** CXIX. **15.** MDCCCCXXX.

Write the following numbers in Roman numerals:

16. 39. **17.** 48. **18.** 58. **19.** 1775. **20.** 1932.

OUR COMMON COINS AND BILLS

All work oral

1. How many cents make a nickel?

2. How many nickels make a dime? a quarter?

3. A dollar is how many cents? nickels? dimes?

4. Which would you rather have, 2 dimes, 3 nickels, and 4¢, or a quarter, a dime, and a nickel? Why?

5. Which would you rather have, a quarter, 3 dimes, 2 nickels, and 3¢, or a half dollar and a nickel? What is the difference in value?

6. Which would you rather have, a half dollar, a quarter, and a dime, or 3 quarters and 2 nickels? What is the difference in value, if any?

7. If you buy four ice-cream cones at 5¢ each, and give the clerk a quarter, how much change is due you?

8. If you are the clerk and have to give me 17¢ in change, how can you do it in the best way; that is, with the smallest number of coins?

9. If I am the clerk and have to give you 56¢ in change, how can I do it in the best way?

State the best way of giving change so as to take

10. 11¢ from 25¢.

11. 16¢ from 50¢.

12. 23¢ from 50¢.

13. 56¢ from 75¢.

14. 56¢ from $1.

15. 18¢ from 25¢.

16. 19¢ from 50¢.

17. 37¢ from 50¢.

18. 52¢ from 75¢.

19. 52¢ from $1.

A SILENT READING LESSON

Read, filling in the blanks as you read:

When we write $2 + 3 = 5$, we are adding 2 and 3, and the process is called ——.

The answer in addition may be called the ——.

The word *addends* is sometimes used for the numbers which we ——.

Taking one number from another, as in $9 - 4 = 5$, is called ——.

In subtraction, the larger of the two given numbers is sometimes called the ——, and the smaller one the ——.

The answer in subtraction may be called the ——.

One of the best checks to show that we have added correctly, is to ——.

The best check to show that we have subtracted correctly, is to —— the answer to the smaller number and to see if the result is the larger number.

In $3 + 7 + 2 + 8 + 9$, we can add more quickly by *grouping* the numbers like this: $(3 + 7) + (2 + 8) + 9$. We then see that $10 +$ —— $+ 9 =$ ——.

Instead of speaking of addends, we may simply speak of the numbers to be ——.

Instead of speaking of a minuend, we may simply speak of the number from which we ——.

Instead of speaking of the subtrahend, we may simply speak of the number to be ——.

Instead of using the words *sum* and *difference* in speaking of an addition or a ——, we may use either the word *result* or the word *answer*.

WRITTEN SPEED TEST CHART IN ADDITION

Add the numbers in the horizontal lines, grouping them conveniently; then add the numbers in the columns. Write the time taken after the last result:

	9.	10.	11.	12.	13.	14.	15.	16.	17.
1.	8	7	5	3	7	6	4	7	5
2.	2	3	5	2	8	6	4	3	7
3.	5	3	2	1	4	4	6	6	8
4.	5	7	8	9	1	4	2	8	4
5.	7	8	5	4	1	3	3	5	8
6.	3	2	5	1	9	7	3	6	6
7.	6	5	4	5	8	2	3	7	4
8.	4	5	6	5	2	8	7	3	8

	27.	28.	29.	30.	31.	32.	33.	34.	35.
18.	9	1	7	3	8	2	7	3	7
19.	2	8	7	3	5	5	3	7	6
20.	8	2	4	6	7	3	4	6	5
21.	7	7	6	7	3	8	2	4	6
22.	3	5	2	6	4	5	5	7	3
23.	2	4	4	3	7	1	9	8	2
24.	5	5	6	4	7	3	8	2	9
25.	1	9	7	3	2	8	4	6	6
26.	9	1	3	7	8	2	6	4	0

WRITTEN SPEED TEST CHART IN ADDITION

Copy the following examples and then, at a signal from the teacher, see how many columns you can add in 2 min.:

1.	4.	7.	10.	13.	16.	19.	22.	25.	28.
1	2	8	5	7	4	6	9	2	6
5	7	9	2	9	2	7	2	1	4
2	3	4	7	6	7	8	3	9	9
6	6	3	3	2	3	2	4	8	8
4	4	6	6	3	6	3	6	7	3
9	3	7	2	0	9	8	5	2	8

2.	5.	8.	11.	14.	17.	20.	23.	26.	29.
9	2	2	1	6	2	9	9	4	1
1	8	5	9	1	6	1	1	4	9
2	6	3	3	2	5	1	7	1	7
4	3	2	3	3	7	0	3	2	7
4	1	5	4	4	2	8	0	3	6
8	8	2	8	8	1	4	8	6	5

3.	6.	9.	12.	15.	18.	21.	24.	27.	30.
1	7	9	2	5	5	6	7	5	0
7	3	0	7	0	4	2	6	6	6
3	0	3	2	8	6	8	4	0	8
4	8	4	2	9	8	3	4	6	8
6	2	3	6	3	2	7	2	8	4
2	9	0	3	2	5	0	6	2	2

ORAL SPEED TEST CHART IN ADDITION

In the following additions state the results rapidly, seeing how long it takes you to do them all:

1. 24 6	**10.** 75 3	**19.** 70 30	**28.** 72 35	**37.** 85 22	**46.** 34 63
2. 34 6	**11.** 75 13	**20.** 73 30	**29.** 42 60	**38.** 25 82	**47.** 43 34
3. 36 4	**12.** 75 73	**21.** 42 65	**30.** 41 88	**39.** 51 49	**48.** 65 34
4. 67 3	**13.** 96 4	**22.** 42 63	**31.** 30 73	**40.** 51 69	**49.** 74 57
5. 77 3	**14.** 94 6	**23.** 46 63	**32.** 81 40	**41.** 61 69	**50.** 94 80
6. 88 2	**15.** 94 10	**24.** 85 15	**33.** 71 31	**42.** 52 69	**51.** 94 49
7. 88 3	**16.** 94 20	**25.** 87 15	**34.** 40 60	**43.** 59 72	**52.** 55 55
8. 48 10	**17.** 94 40	**26.** 87 5	**35.** 31 20	**44.** 30 10	**53.** 40 20
9. 49 2	**18.** 94 16	**27.** 87 13	**36.** 31 39	**45.** 35 95	**54.** 45 45

WRITTEN SPEED TEST CHART IN ADDITION

Place the edge of a sheet of paper under the first row of examples and add, writing only the answers. Then fold the answers under, and do the same for the next row, and so on. Write the results as rapidly as you can:

1.	26	42	28	36	57	63	29
	30	21	15	14	18	27	19
	14	13	10	22	16	30	24
2.	10	38	83	37	56	28	36
	14	22	9	42	49	27	48
	23	14	40	61	78	33	73
3.	21	53	75	29	29	38	85
	32	41	25	38	66	55	37
	41	66	32	67	75	16	44
	60	78	59	48	88	72	68
4.	26	33	18	67	48	54	33
	44	68	25	39	35	92	68
	25	42	32	56	29	61	79
	15	30	48	75	16	76	66
5.	12	22	32	33	40	22	12
	8	31	18	42	21	19	19
	20	6	70	20	27	38	27
	16	14	20	9	60	6	68
	24	13	15	16	19	11	23

PROBLEMS IN ADDITION

1. I bought a small radio set for $12.25, a small phonograph for $10.30, a small moving-picture machine for $10.50, and a film for $2.25. How much did I pay?

2. Find how much you would have to pay in all for a croquet set costing $1.45, a set of tenpins costing $1.20, a set of dominoes costing 95¢, a magic lantern costing $3.75, and slides costing $1.45.

3. A gas meter shows that in November a family used 3800 cu. ft. of gas; in December, 4400 cu. ft.; and in January, 4500 cu. ft. Find the total amount used.

4. On the Dixie Highway, connecting Chicago and Jacksonville, a man drove 168 mi. the first day, 157 mi. the second, 164 mi. the third, and 154 mi. the fourth. What was the total distance traveled?

Copy the following columns, add, and check by adding down instead of up. Write the time taken after the last result:

5. $2.75	**6.** $3.42	**7.** $5.20	**8.** $3.92	**9.** $5.75
3.40	2.68	2.40	4.78	2.25
.65	4.00	.80	.64	3.46
2.82	.75	6.00	.73	2.78
1.47	3.05	4.35	1.20	9.29
6.00	2.70	5.26	5.00	5.42
.95	1.55	7.88	2.73	2.80

DRILL CHART IN RAPID ADDITION

Without copying, add these columns, writing the results on the folded edge of a piece of paper placed under the examples. Write the time taken after the last result:

1.	$13.48	**5.**	$42.25	**9.**	$19.75	**13.**	$2738.75
	26.31		37.06		9.25		3138.76
	7.92		2.90		16.00		96.37
	8.74		32.66		4.37		800.46
	19.00		5.44		5.63		3517.38
2.	$37.78	**6.**	$17.36	**10.**	$29.00	**14.**	$3867.40
	62.41		7.92		9.25		2209.46
	39.29		18.00		16.37		500.73
	8.76		.69		4.29		1529.96
	29.29		32.20		15.39		4082.32
3.	$21.23	**7.**	$41.73	**11.**	$50.00	**15.**	$4887.95
	7.68		62.81		7.45		196.75
	.92		42.96		17.68		98.80
	8.00		80.00		3.92		764.80
	15.26		9.37		29.89		526.46
4.	$35.52	**8.**	$48.72	**12.**	$51.25	**16.**	$4240.35
	7.28		9.68		72.00		280.98
	16.93		18.43		6.86		3507.05
	14.36		17.67		.78		3086.76
	6.87		13.00		1.96		945.50
	25.91		39.26		19.29		3095.05

Using Subtraction. John was born in a village in which 3204 people now live. The population was only 1728 when John was born. How much has the village grown?

Just as we count John's growth by the increase in inches or pounds, so we count the growth of the village by the increase in the number of people. That is, to find the answer to the question we must subtract 1728 from 3204.

Subtracting in the way already learned, we have 1476 for the result.

That is, there are 1476 more people.

Check. The work is right because

$$1476 + 1728 = 3204.$$

$$\boxed{\begin{array}{r}3204\\ \underline{1728}\\ 1476\end{array}}$$

SUBTRACTING NUMBERS

Subtract as indicated, check each result, and write the time taken after the last result:

1. $\begin{array}{r}3980\\ \underline{1296}\end{array}$
2. $\begin{array}{r}4238\\ \underline{2879}\end{array}$
3. $\begin{array}{r}7235\\ \underline{6686}\end{array}$
4. $\begin{array}{r}8231\\ \underline{6192}\end{array}$
5. $\begin{array}{r}7802\\ \underline{973}\end{array}$
6. $\begin{array}{r}5870\\ \underline{2096}\end{array}$
7. $\begin{array}{r}6230\\ \underline{3915}\end{array}$
8. $\begin{array}{r}7202\\ \underline{6654}\end{array}$
9. $\begin{array}{r}3126\\ \underline{1792}\end{array}$
10. $\begin{array}{r}5200\\ \underline{1275}\end{array}$
11. $\begin{array}{r}7094\\ \underline{3396}\end{array}$
12. $\begin{array}{r}7230\\ \underline{2847}\end{array}$
13. $\begin{array}{r}9170\\ \underline{6694}\end{array}$
14. $\begin{array}{r}8207\\ \underline{6352}\end{array}$
15. $\begin{array}{r}6173\\ \underline{2767}\end{array}$
16. $\begin{array}{r}8060\\ \underline{3397}\end{array}$
17. $\begin{array}{r}6231\\ \underline{2806}\end{array}$
18. $\begin{array}{r}8500\\ \underline{5286}\end{array}$
19. $\begin{array}{r}3300\\ \underline{1927}\end{array}$
20. $\begin{array}{r}7080\\ \underline{796}\end{array}$
21. $\begin{array}{r}9024\\ \underline{4579}\end{array}$
22. $\begin{array}{r}9000\\ \underline{4926}\end{array}$
23. $\begin{array}{r}9101\\ \underline{6794}\end{array}$
24. $\begin{array}{r}5121\\ \underline{3278}\end{array}$
25. $\begin{array}{r}6000\\ \underline{926}\end{array}$

DRILL CHART IN RAPID SUBTRACTION

Copy and subtract, recording the time for each column:

1. 64,812
29,684

2. 51,206
36,819

3. 70,763
41,929

4. 20,728
16,439

5. 33,400
19,650

6. 72,323
48,207

7. 66,203
48,162

8. 42,902
18,921

9. 60,021
38,292

10. 72,302
68,474

11. 30,123
19,468

12. 67,329
30,538

13. 80,230
57,629

14. 20,726
9,829

15. 70,000
13,520

16. 82,121
68,237

17. 61,123
46,327

18. 73,291
47,182

19. 91,231
44,444

20. 82,230
66,627

21. 71,247
56,493

22. 91,426
48,790

23. 78,123
49,206

24. 81,111
66,666

25. 70,020
37,575

26. 60,101
44,211

27. 80,298
37,126

28. 42,070
16,720

29. 50,273
39,182

30. 61,791
48,072

31. 70,000
24,303

32. 82,573
49,696

33. 80,042
28,768

34. 50,000
6,809

35. 52,121
16,809

36. 67,123
48,070

DRILL CHART IN RAPID SUBTRACTION

Copy and subtract, recording the time for each column:

1. $950.00
652.90

2. $527.63
343.56

3. $675.62
181.98

4. $570.06
9.99

5. $802.68
469.81

6. $906.97
423.49

7. $729.81
49.93

8. $824.75
568.45

9. $402.25
387.56

10. $926.00
547.62

11. $972.93
598.98

12. $820.36
428.92

13. $829.80
167.98

14. $970.45
291.66

15. $700.00
293.76

16. $982.75
93.89

17. $408.72
329.86

18. $711.20
667.58

19. $884.75
615.28

20. $824.70
652.35

21. $504.76
362.97

22. $800.10
469.86

23. $708.07
298.78

24. $709.16
387.68

25. $508.76
235.92

26. $700.00
387.50

27. $898.05
509.79

28. 850,000
583,867

29. 990,900
666,907

30. 900,000
328,965

31. 975,438
821,795

32. 864,560
238,989

33. 347,892
9,999

34. 865,555
27,777

35. 658,250
51,175

36. 572,527
221,962

ORAL DRILL CHART IN MAKING CHANGE

1. If I buy goods costing 58¢ and hand the clerk $1, he makes change, saying, "58 and **2** are 60, and **10** are 70, and **5** are 75, and **25** makes $1." How much change does he give me?

Make change from 25¢ when you buy goods costing

2. 10¢.
3. 13¢.
4. 9¢.
5. 19¢.
6. 5¢.
7. 15¢.
8. 8¢.
9. 18¢.
10. 22¢.
11. 17¢.

Make change from 50¢ when you buy goods costing

12. 38¢.
13. 28¢.
14. 32¢.
15. 42¢.
16. 26¢.
17. 24¢.
18. 16¢.
19. 15¢.
20. 23¢.
21. 29¢.

Make change from $1 when you buy goods costing

22. 28¢.
23. 36¢.
24. 50¢.
25. 56¢.
26. 86¢.
27. 84¢.
28. 75¢.
29. 77¢.
30. 93¢.
31. 97¢.

Make change from $2 when you buy goods costing

32. $1.25.
33. $1.70.
34. $1.35.
35. $1.75.
36. $1.50.
37. $1.83.
38. $1.48.
39. $1.87.

Make change from $5 when you buy goods costing

40. $3.52.
41. $2.85.
42. $2.28.
43. $4.17.
44. $3.98.
45. $2.71.
46. $4.54.
47. $1.17.

Price List. Fred and Kate made the following price list of toys which they saw in a store window:

Air gun . . .	$1.25	Engine	$3.98
Airplane . . .	.69	Rubber ball . .	.89
Bank	.97	Taxi	.89
Basket ball . .	.98	Teddy bear . .	1.39
Box of tools . .	5.98	Train	3.45
Box of tricks .	.75	Tricycle . . .	6.98
Drum	1.19	Wheelbarrow . .	1.39

Use this list in the following problems.

BUYING TOYS

1. If you buy a train, how much change is due from $5?

2. If you buy a wheelbarrow, a box of tools, and a Teddy bear, they will cost how much less than $10?

3. Fred buys a rubber ball and a basket ball. How much change does he get from $5?

4. If you have $5 to spend, which toys should you choose from the above list so as to use nearly all the money? How much money should you have left?

5. Suppose that your class decided to send a box of toys to a children's hospital and that you had collected $15 for this purpose. Make a list of the toys that you would buy from the above list, find the total cost, and find how much you would have left from the $15.

6. Suppose that you buy all the toys in the above list, how much do you have to pay? How much change do you receive from $35?

BUYING THINGS AT A STORE

1. Suppose that you buy a fountain pen for $1.50 and a silver pencil for $1.25. How much do the two articles cost? How much change is due from a 5-dollar bill?

Add $1.50 and $1.25 to find the cost.	$1.50	$5.00
Then subtract $2.75 from $5.	1.25	2.75
The result is the change due.	$2.75	$2.25

2. If you buy a small radio set for $12.50 and a book on radio for $1.40, how much do you pay?

3. If you buy a suit for $17.10 and a hat for $2.25, how much do you pay? If you give the dealer two 10-dollar bills, how much change do you receive?

4. If you buy a camera for $5.75 and pay 75¢ for films, how much do you pay? How can you pay this amount with only three pieces of money?

5. If you spend $12.25 for a second-hand phonograph and $3.50 for records, how much change is due from $20?

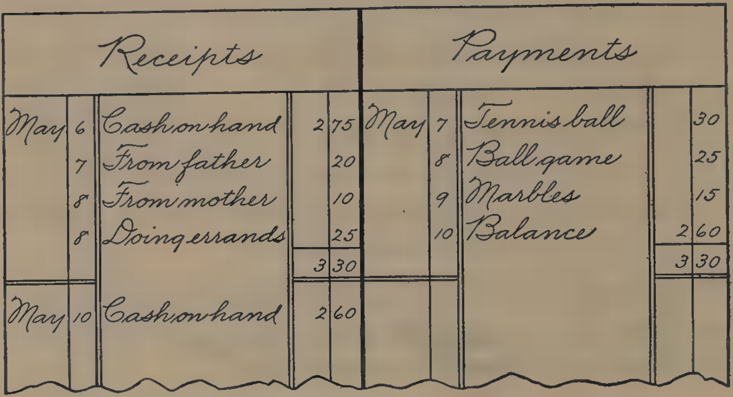
6. Suppose that you wish to buy a rubber-stamp outfit for printing names and amounts. If the price is $2.15 and you have earned $1.75, how much more do you need?

7. If your father gives you $8 and you earn $3.75, how much more do you need to buy a small typewriter costing $13.50?

8. If you save $12.75 and your uncle gives you $5, how much more do you need to buy a radio set costing $19.25?

A SILENT READING LESSON

When Robert writes up his cash account, he puts the amounts that he receives on the left-hand page and the amounts that he pays on the right-hand page, like this:



Receipts					Payments				
May	6	Cash on hand	2	75	May	7	Tennis ball		30
	7	From father		20		8	Ball game		25
	8	From mother		10		9	Marbles		15
	8	Doing errands		25		10	Balance	2	60
			3	30				3	30
May	10	Cash on hand	2	60					

In Robert's account we see that the "cash on hand" on May 6 was $2.75. With the ruled columns he does not need to write the dollar sign and the decimal point.

The three items on the left-hand page, 20¢, 10¢, and 25¢, show amounts that Robert received.

The three items, 30¢, 25¢, and 15¢, on the right-hand page show amounts that Robert paid out.

To find the *balance*, Robert first adds the receipts and finds that they amount to $3.30. He then adds the payments and finds that they amount to 70¢. He then subtracts 70¢ from $3.30, and writes the result, $2.60, as the balance on the right side. He carries this to the left-hand side as "cash on hand." The double rules across the page show that the account was *balanced* at this point.

KEEPING A CASH ACCOUNT

Rule some paper to show the left and right pages of a cash account, like the one on page 20. Then make out the accounts in Exs. 1–3, balancing each one on the last date given:

1. Receipts: May 6, cash on hand, $3.20; May 7, from father, 30¢; May 8, from mother, 25¢; May 9, errands, 30¢. Payments: May 7, movies, 20¢; May 9, banana, 4¢; May 9, pencil, 6¢.

2. Receipts: Feb. 1, cash on hand, $2.75; Feb. 2, errands, 40¢; Feb. 3, helping on cleaning car, 35¢. Payments: Feb. 2, wire for radio, 65¢; Feb. 3, radio magazine, 25¢; Feb. 4, knife, 75¢.

3. Receipts: June 3, cash on hand, $4.62; June 3, errands, 60¢; June 4, selling papers, 52¢; June 6, gardening, 45¢. Payments: June 4, book on radio, 85¢.

4. Make out a cash account with the following items, and balance it on May 31. Receipts: May 24, cash on hand, $13.20; May 25, J. L. Roe, $3.20; May 26, F. J. Case, $2.75; May 27, N. P. Doe, $1.80; May 29, A. B. Coe, $4.20. Expenses: May 29, cap, $1.35; May 30, football, $2.25.

5. Make out a cash account with the following items, and balance it on Dec. 9. Receipts: Dec. 6, cash on hand, $145.80; Dec. 6, M. P. Jones, $85.70; Dec. 7, A. B. Smith, $68.90; Dec. 8, P. H. Haynes, $38.25. Expenses: Dec. 6, rent, $85; Dec. 7, salaries, $75; Dec. 8, coal, $28.60; Dec. 8, account book, $2.50; Dec. 8, telephone, $5.80; Dec. 8, gas bill, $4.75; Dec. 8, stationery, $3.50.

Copy and balance each of the following accounts:

6.

Receipts		Payments	
1726	48	639	82
492	00	375	40
1237	42	1825	50
1050	75	88	92
128	68	*Balance*	

7.

Receipts		Payments	
1075	85	842	60
1293	42	1387	65
878	68	29	82
975	00	1000	00
320	72	*Balance*	

8.

Receipts		Payments	
1582	83	1127	20
298	30	176	39
3400	00	2173	48
1287	64	976	92
2176	92	1106	54
384	08	*Balance*	

9.

Receipts		Payments	
1720	91	1000	00
8000	00	278	92
625	43	1072	65
1426	00	2575	00
782	00	*Balance*	

10.

Receipts		Payments	
3075	00	2000	00
296	82	342	75
1921	32	1282	60
1072	00	928	34
800	00	*Balance*	

11.

Receipts		Payments	
3802	42	3015	24
2173	60	2432	30
984	38	874	60
2500	00	1687	52
3078	06	943	70
984	37	*Balance*	

USING WHAT YOU HAVE LEARNED

1. The expense account of a family showed the following items for June: Food, $48.50; rent, $60; gas for cooking, $3.35; clothing, $32.50; insurance, $17.75; church and charity, $6.50; amusements, $4.80; maid's wages, $45; incidentals, $15.80. If the family income was $250 for that month, how much could be saved?

2. If a book has 275 pages, and if, starting at page 1, you read 38 pages on Monday, 23 on Tuesday, and 36 on Wednesday, how many pages have you read? How many more pages have you to read?

3. Last month Mr. Sinclair paid $110 for rent, $137.50 for food, $72.80 for clothing, $16.20 for books and magazines, $4.50 for church and charity, and $15.20 for amusements. If his income that month was $360, how much money did he have left?

4. On an automobile trip of 641 mi. a man and his family went 139 mi. the first day, 147 mi. the second day, 98 mi. the third day, and 126 mi. the fourth day. How many more miles had they to go?

5. A farmer has 120 A. (acres) of corn in three fields. The first field contains 36 A. and the second 34 A. How many acres are there in the third field? This is how many more acres than there are in the first field?

6. In a year when a farmer's accounts showed his income to be $8940.78, he received $2286.65 from the sale of hogs and $2792.87 from the sale of cattle. What was his income from other sources besides hogs and cattle?

A SILENT READING LESSON

Read, filling in the blanks as you read:

Multiplying one number by another is called ——. Thus, if we take 3 twice, we are using ——. We may write

$$2 \times 3 = ——.$$

The number multiplied may be called the ——, which in the case at the right is ——.

$$\boxed{\begin{array}{r} 48 \\ 7 \\ \hline 336 \end{array}}$$

The number by which we multiply may be called the ——, which in this case is ——.

The result in multiplication is sometimes called the ——, which in this case is ——.

When two whole numbers are multiplied, the result is called a *multiple* of either one of the numbers. For example, 35 is a —— of 5 and also of ——.

Numbers like 2 and 5, which have no names such as "feet" or "pounds" written after them, are called *abstract numbers*. When such names are written, the numbers are called *concrete numbers*. For example, 17 and $2\frac{3}{4}$ are —— numbers, and 6 ft. and 15¢ are —— numbers.

A whole number (not including 1 or the number itself) that is exactly contained in another is called a *factor* of that number. For example, 2, 3, 4, and 6 are factors of 12. The factors of 30 are 2, ——, ——, 6, ——, and 15.

If we multiply a number by itself any number of times, the result is called a *power* of the number. For example, $2 \times 2 = 4$, and so we call 4 the *second power* (or *square*) of 2, and we write $2^2 = 2 \times 2$. In the same way, because $2 \times 2 \times 2 = 8$, we write $2^3 = 8$ and say that 8 is the *third power* (or *cube*) of 2.

ORAL DRILL CHART ON EASY COMBINATIONS

1. Two or three years ago we learned all the combinations of two numbers, of one figure each, that we shall need in multiplication. Do we know them now? Do we know one combination, like 8 × 7, as well as any other, like 2 × 1? We must do so if we are to multiply easily and quickly. Try 9 × 6 and 7 × 9.

See if you can do all these multiplications in 1 min.:

2.	1	1	0	3	2	0	4	0	2	0
	2	1	1	1	2	2	1	9	3	3
3.	1	1	2	8	8	3	5	0	0	3
	0	3	1	2	0	2	1	7	0	4
4.	9	3	2	4	1	6	1	0	5	4
	0	3	0	2	4	1	5	4	0	3
5.	7	5	2	7	5	5	3	1	6	1
	1	2	4	0	3	5	0	6	2	7
6.	1	8	4	6	0	9	0	7	6	0
	9	1	0	8	8	1	5	2	0	6
7.	2	1	2	3	2	7	4	4	5	6
	5	8	6	5	7	3	9	5	8	4
8.	9	2	2	8	3	3	9	8	9	4
	2	8	9	2	9	8	3	3	4	8

DRILL CHART ON TROUBLESOME COMBINATIONS

In these multiplications, see if you can give orally all the results in 1 min.:

1.	5	4	8	4	6	5	4	4	5	7
	4	5	3	8	4	5	6	7	8	9
2.	6	5	2	5	9	6	6	7	6	6
	3	7	9	6	3	8	5	4	9	6
3.	7	8	6	8	7	7	8	7	8	5
	6	9	7	4	5	7	6	8	5	9
4.	8	9	9	9	9	9	9	4	9	8
	7	6	9	4	8	2	5	9	7	8

See if you can give the results in these specially troublesome cases, some of which are repeated, in 1 min.:

5.	9	8	7	7	8	5	5	2	7	9
	5	7	5	8	9	5	9	9	6	9
6.	9	9	5	8	9	7	6	4	8	3
	4	3	8	7	7	4	5	9	4	9
7.	8	9	5	7	7	5	8	4	9	6
	6	6	6	7	9	7	8	7	8	8
8.	4	6	9	7	6	9	8	6	6	8
	8	6	8	9	8	9	5	7	9	8

A SILENT READING LESSON

Read, filling in the blanks as you read:

Here are three examples in multiplication. In (1) we first multiply by ——, then by —— (which is one ten), and then by —— (which is 2 ——). We then add. The final result is ——. We know where to put the decimal point because there are —— decimal places in $635.52.

(1)	$635.52	(2)	3247	(3)	3247
	215		1305		1350
	317760		16235		162350
	63552		9741		9741
	127104		3247		3247
	$136636.80		4237335		4383450

In (2) we first multiply by ——. We do not need to multiply by 0 tens because we know that the result would be ——. We next multiply by —— hundreds and write the 1 in hundreds' place. We next multiply by —— thousand and write the 7 in —— place. The final result is ——.

In (3) we first multiply by 0, which is done by placing a zero in units' place. We then multiply by 5, then by 3, and then by 1. The final result is ——.

If we wish to multiply a number by 207, we first multiply by ——. We need not multiply by —— because we know that the result would be ——. We then multiply by —— hundreds, being careful to write the right-hand figure in —— place. We then —— these two results.

FILLING MAIL ORDERS

1. Imagine that you are a clerk in the toy department of a large mail-order business house, and that you have to ship 16 toy fire engines which cost $1.19 each. How much is received for these toys?

As in Ex. 1, suppose that you ship the following toys:

2. 12 steam tractors @ $6.98. The symbol @, which means "at," refers to the price of each toy.

3. 72 small motion-picture films @ $1.98.

4. 34 trains of cars @ $4.98.

5. 128 taxis @ 89¢ and 97 garages @ 92¢.

6. 75 mouth organs @ 11¢ and 18 nickel trumpets @ 39¢.

7. 6 sets of building blocks @ 96¢, 12 sets of water color paints @ 96¢, and 24 bears on wheels @ 96¢.

8. If you have just filled an order for 8 boys' carts which amounted to $25.20, and the next order calls for 48 carts of the same kind, to how much does the second order amount? In this case 48 = 6 × 8, and so the 48 carts sell for how many times $25.20?

SPEED TEST CHART IN CARRYING

1. In multiplying 48 by 7, we think first "seven 8's are 56," and then "seven 4's are 28, and 5 are 33." That is, we have

$$7 \times 4 + 5 = 33,$$

in which we carry the 5. Try $3 \times 2 + 1$ and $3 \times 5 + 2$ for a start.

$$\begin{array}{r} 48 \\ 7 \\ \hline 336 \end{array}$$

Without repeating the statements, see how many results you can give orally in 2 min.:

2. $2 \times 1 + 1$.
3. $3 \times 1 + 1$.
4. $3 \times 5 + 1$.
5. $4 \times 1 + 1$.
6. $3 \times 2 + 2$.
7. $3 \times 6 + 1$.
8. $2 \times 3 + 1$.
9. $4 \times 1 + 3$.
10. $3 \times 7 + 2$.
11. $2 \times 6 + 1$.
12. $3 \times 4 + 1$.
13. $3 \times 8 + 2$.
14. $3 \times 1 + 2$.
15. $2 \times 2 + 1$.
16. $3 \times 4 + 2$.
17. $3 \times 9 + 2$.
18. $3 \times 3 + 1$.
19. $4 \times 2 + 1$.
20. $3 \times 6 + 3$.
21. $3 \times 9 + 1$.
22. $2 \times 4 + 1$.
23. $4 \times 2 + 3$.
24. $4 \times 3 + 2$.
25. $2 \times 7 + 1$.
26. $3 \times 2 + 1$.
27. $3 \times 6 + 2$.
28. $4 \times 1 + 2$.
29. $2 \times 5 + 1$.
30. $3 \times 7 + 1$.
31. $4 \times 3 + 1$.
32. $2 \times 8 + 1$.
33. $3 \times 3 + 2$.
34. $4 \times 2 + 2$.
35. $2 \times 9 + 1$.
36. $3 \times 8 + 1$.
37. $4 \times 3 + 3$.
38. $2 \times 0 + 1$.
39. $3 \times 0 + 2$.
40. $4 \times 0 + 3$.

SPEED TEST CHART IN CARRYING

Without repeating the statements, see how many of the results in Exs. 1–30 you can give orally in 2 min.:

1. $4 \times 5 + 3$.
2. $4 \times 8 + 2$.
3. $4 \times 4 + 2$.
4. $4 \times 7 + 2$.
5. $5 \times 2 + 4$.
6. $4 \times 7 + 3$.
7. $4 \times 9 + 2$.
8. $4 \times 6 + 3$.
9. $6 \times 2 + 1$.
10. $6 \times 2 + 5$.
11. $4 \times 4 + 1$.
12. $4 \times 8 + 1$.
13. $4 \times 5 + 2$.
14. $4 \times 9 + 1$.
15. $4 \times 4 + 3$.
16. $4 \times 6 + 2$.
17. $6 \times 2 + 2$.
18. $5 \times 2 + 1$.
19. $4 \times 9 + 3$.
20. $6 \times 2 + 3$.
21. $4 \times 8 + 3$.
22. $4 \times 6 + 1$.
23. $5 \times 2 + 2$.
24. $4 \times 5 + 1$.
25. $6 \times 1 + 5$.
26. $5 \times 2 + 3$.
27. $6 \times 1 + 4$.
28. $4 \times 7 + 1$.
29. $5 \times 1 + 4$.
30. $6 \times 2 + 4$.

See how many of these you can do in 2 min.:

31. $7 \times 6 + 4$.
32. $8 \times 6 + 7$.
33. $7 \times 7 + 5$.
34. $9 \times 6 + 6$.
35. $8 \times 8 + 6$.
36. $7 \times 7 + 6$.
37. $8 \times 8 + 7$.
38. $7 \times 9 + 6$.
39. $9 \times 9 + 6$.
40. $8 \times 9 + 6$.
41. $8 \times 6 + 6$.
42. $7 \times 6 + 5$.
43. $8 \times 7 + 6$.
44. $7 \times 8 + 6$.
45. $9 \times 6 + 7$.
46. $8 \times 9 + 7$.
47. $7 \times 8 + 5$.
48. $9 \times 9 + 7$.
49. $9 \times 8 + 8$.
50. $9 \times 9 + 8$.
51. $7 \times 7 + 4$.
52. $7 \times 8 + 4$.
53. $9 \times 6 + 8$.
54. $7 \times 6 + 6$.
55. $8 \times 7 + 7$.
56. $9 \times 8 + 6$.
57. $9 \times 6 + 4$.
58. $7 \times 9 + 5$.
59. $9 \times 6 + 5$.
60. $9 \times 7 + 8$.

USING WHAT YOU HAVE LEARNED

1. Flour is sold in different sized sacks, some of which weigh about 24 lb., others 12 lb., and others 6 lb. How many pounds are there in 125 sacks of the large size? of the medium size? of the small size?

2. If the average amount of meat eaten in a year is 194 lb. for each person, how many pounds will be eaten yearly in a village in which there are 2450 people?

3. Mr. Kane saves $6.75 out of his wages each week. How much does he save in 26 wk.? in 52 wk.? in 104 wk.?

4. Mr. Roberts put in 14 T. (tons) of coal costing $11.25 a ton. In the cellar he already had 3 T. for which he had paid $10.75 a ton. Find the cost of all the coal in the cellar.

5. Mr. Sinclair owns a house and a garage, which he rents to Mr. Robinson at $67.50 a month for the house and $7.50 a month for the garage. How much rent does he receive from Mr. Robinson in a year?

Find the total cost in each of the following cases:

6. 175 automobiles at $875.50 each.

7. 136 cows at $97.50 each.

8. 146 tires at $32.50 each, and 128 at $48.75 each.

9. 112 stoves at $78.24 each, and 69 at $87.36 each.

10. 136 horses at $157.50 each, and 96 at $178.50 each.

11. 128 chairs at $5.35 each, and 64 more at $5.35 each.

12. 1208 camp chairs at $1.08 each, 96 tents at $29.48 each, and 650 camp tables at $1.08 each.

A SILENT READING LESSON

Using the following short cuts will save time and work:

To multiply a whole number by 10, annex a zero; by 100, annex two zeros; by 1000, annex three zeros.

For example, $10 \times 27 = 270$,

and $100 \times 41 = 4100$.

To multiply a number representing United States money by 10, move the decimal point one place to the right; by 100, move the decimal point two places to the right; and so on.

For example, $100 \times \$3.17 = \$317$,

and $1000 \times \$5.26 = \$5260$.

To multiply by numbers like 20, 30, 200, 300, and so on, multiply by the 2 or 3 and annex zeros as necessary.

For example, to multiply 27 by 20, multiply by 2 and annex a zero. Similarly, to multiply $362 by 700, multiply by 7 and annex two zeros.

To multiply $25.16 by 300, multiply by 3 and move the decimal point two places to the right.

To multiply by 5, multiply by 10 and divide by 2.

For example, $5 \times 428 = 4280 \div 2 = 2140$.

To multiply by 50, multiply by 100 and divide by 2.

For example, $50 \times 344 = 34{,}400 \div 2 = 17{,}200$.

To multiply by 25, multiply by 100 and divide by 4.

For example, $25 \times 424 = 42{,}400 \div 4 = 10{,}600$,

and $25 \times \$16.80 = \$1680 \div 4 = \$420$.

DRILL CHART IN MULTIPLICATION

Multiply, in turn, by 2, 20, and 200:

1. 245.
2. $2.45.
3. 287.
4. $2.87.
5. 509.
6. $50.90.
7. 827.
8. $82.70.

Multiply, in turn, by 3, 30, and 300:

9. 682.
10. $6.82.
11. 725.
12. $72.50.
13. 933.
14. $93.30.
15. 1640.
16. $16.40.

Multiply, in turn, by 40, 400, 50, and 500:

17. 536.
18. $6.35.
19. 721.
20. $5.93.
21. 663.
22. $4.34.
23. 842.
24. $23.81.

Multiply $734.65 by

25. 9. **26.** 90. **27.** 900. **28.** 970. **29.** 907.

Multiply $875.40 by

30. 8. **31.** 80. **32.** 800. **33.** 808. **34.** 880.

Multiply $624.45 by

35. 7. **36.** 70. **37.** 700. **38.** 707. **39.** 777.

Perform the following multiplications:

40. 5280 ft.
210

41. 2932 ft.
680

42. 1460 ft.
170

43. $540.12
9000

44. $215.42
7000

45. $372.60
5000

46. $412.50
6500

47. $421.77
2900

48. $575.75
9200

USING WHAT YOU HAVE LEARNED

1. If each of the 88,756 persons in a certain county saves an average of $25 a year on the money spent for food, how much is the total saving in a year?

2. In the fall a woman bought 80 bu. of potatoes for her boarding-house. She estimated that during the winter she lost 15¢ per bushel on account of the potatoes' rotting. How much money did she lose in all?

3. If there are 2860 soldiers in a certain regiment, and the government pays 45¢ a day for food for each soldier, what is the cost of the food during the month of June?

4. When wheat advanced in price from $1.25 per bushel to $2.05 per bushel, how much more did a farmer receive from 90 A. which yielded 23 bu. of wheat per acre?

5. The average amount of meat eaten yearly by each person in this country is 194 lb. If 91 lb. per person is sufficient, what is the extra cost at 25¢ per pound?

6. Mr. Mann sold 40 barrels of apples to Mr. Smith for $3.50 a barrel at the orchard. Mr. Smith shipped them to a dealer in the city, and figured the cost of shipping and his profit at $1.17 a barrel. How much did the dealer pay Mr. Smith? How much more did the dealer pay for all the apples than Mr. Smith paid Mr. Mann?

7. A man who paid $57.50 a month for a city apartment had to furnish his own heat. In one year he used 9 T. of coal, for which he paid $12.50 a ton. The next year he moved into a heated apartment for which he paid $88.50 a month. Find the increase in his expenses for a year.

DRILL CHART IN MULTIPLICATION

Perform the following multiplications, writing the time taken for each column:

1. 798 × 798.
2. 352 × 324.
3. 682 × 726.
4. 823 × 983.
5. 435 × 481.
6. 632 × 408.
7. 563 × 481.
8. 408 × 662.
9. 621 × 826.
10. 50 × 836.
11. 80 × 906.
12. 51 × 4437.
13. 93 × 7080.
14. 69 × 7236.
15. 83 × 8192.
16. 54 × 7762.
17. 36 × 8087.
18. 49 × 9246.
19. 153 × 8129.
20. 166 × 6666.
21. 191 × 3473.
22. 222 × 4646.
23. 207 × 2007.
24. 767 × 7873.
25. 328 × 5362.
26. 50 × $68.08.
27. 60 × $42.75.

Multiply each number by each one at the right of it, and also multiply each number by each one below it, doing only those that the teacher tells you to do:

	37.	**38.**	**39.**	**40.**	**41.**	**42.**
28.	243	307	442	581	226	822
29.	376	476	208	622	642	344
30.	405	921	179	389	379	296
31.	221	330	643	427	496	473
32.	392	407	827	462	577	892
33.	470	920	906	330	640	487
34.	689	821	432	448	807	745
35.	573	139	507	229	488	307
36.	349	427	572	946	399	700

A SILENT READING LESSON

Read, filling in the blanks as you read:

In division we are given two numbers, and have to find a third. The number by which we divide is sometimes called the ——. Thus, if we divide 8 by 2, the divisor is ——.

The number divided is sometimes called the ——, and the result is sometimes called the ——. In $8 \div 2 = 4$, —— may be called the dividend, and —— the quotient.

There are several ways of indicating the division of 8 by 2; for example,

(1) A simple way often used in school is $8 \div 2 = 4$;

(2) Another way often used in school is $\begin{array}{r}4\\ 2\overline{)8}\end{array}$;

(3) A common way in practical work is $\begin{array}{r}2\overline{)8}\\ \hline 4\end{array}$;

(4) Another way is to use a fraction, as in $\frac{8}{2} = 4$.

Each of these is read "8 divided by —— equals ——."

Division is *exact* when there is no *remainder*. For example, $\frac{8}{2} = 4$, or $8 \div 2 = 4$, represents exact ——.

The case of $9 \div 2 = 4$, with 1 left over, is not one of exact division, because there is a remainder of ——.

When we divide 9 by 2, we may write —— as the whole number in the result, and then write the 1 over the 2. That is, we write the complete result as ——.

The result in addition may be called the ——; the result in subtraction, the ——; the result in multiplication, the ——; and the result in division, the ——. We may use the word *result* in any of these cases, and it is just as well to do so.

If we divide \$8 by \$4, the result must be ——; for we see that \$4 is contained in \$8 —— times.

If we divide \$8 by 2, we separate \$8 into —— equal parts, and there must be \$ —— in each part; that is, we have taken $\frac{1}{2}$ of \$8.

Thus, there are two different kinds of division; namely,

$$\$8 \div \$4 = 2,$$

and

$$\$8 \div 2 = \$4.$$

We can easily check our work in division, for we know that $8 \div 4 = 2$ is correct if $2 \times 4 =$ ——.

Here are three examples in division. The two at the left are examples of *short division*, and the one at the right is an example of —— *division*.

The practical man writes the result in short division below the number ——, because it is more convenient for him to use the result in some other operation when it is written in this way. He cannot do this in long division, and so writes the result above the number divided.

$$\begin{array}{r} 72 \\ 2\overline{)144} \end{array} \quad \text{or} \quad \begin{array}{r} 2\underline{)144} \\ 72 \end{array} \qquad \begin{array}{r} 14 \\ 13\overline{)182} \\ \underline{13} \\ 52 \\ \underline{52} \end{array}$$

In these divisions we see that the number by which we divide is ——; the number divided is ——; and the result is ——. In the case at the left we place the decimal point in the result exactly —— the —— —— in the number divided; in the case at the right, we place it ——.

$$\begin{array}{r} \$12.27 \\ 3\overline{)\$36.81} \end{array} \quad \text{or} \quad \begin{array}{r} 3\underline{)\$36.81} \\ \$12.27 \end{array}$$

SPEED TEST CHART ON EASY COMBINATIONS

In order to find out whether you have forgotten any of the combinations in division or whether you know all equally well, see if you can state these results in 2 min.:

1.	1 ÷ 1	6 ÷ 3	3 ÷ 3	8 ÷ 1	4 ÷ 2
2.	0 ÷ 4	2 ÷ 1	5 ÷ 5	2 ÷ 2	0 ÷ 8
3.	6 ÷ 2	0 ÷ 9	9 ÷ 3	0 ÷ 1	4 ÷ 4
4.	8 ÷ 4	8 ÷ 2	3 ÷ 1	0 ÷ 7	8 ÷ 8
5.	0 ÷ 2	0 ÷ 5	6 ÷ 6	10 ÷ 2	5 ÷ 1
6.	7 ÷ 7	7 ÷ 1	9 ÷ 1	12 ÷ 4	12 ÷ 6
7.	0 ÷ 6	21 ÷ 3	3 ÷ 3	4 ÷ 1	15 ÷ 3
8.	9 ÷ 9	12 ÷ 2	2 ÷ 2	0 ÷ 3	14 ÷ 2
9.	6 ÷ 1	10 ÷ 5	4 ÷ 4	12 ÷ 3	77 ÷ 7
10.	6 ÷ 3	60 ÷ 3	9 ÷ 9	30 ÷ 3	300 ÷ 3
11.	4 ÷ 2	40 ÷ 2	8 ÷ 8	20 ÷ 2	200 ÷ 2
12.	5 ÷ 5	50 ÷ 5	8 ÷ 2	40 ÷ 4	400 ÷ 4
13.	9 ÷ 3	90 ÷ 3	18 ÷ 2	90 ÷ 9	900 ÷ 9
14.	7 ÷ 7	70 ÷ 7	16 ÷ 2	80 ÷ 8	800 ÷ 8
15.	8 ÷ 4	80 ÷ 4	18 ÷ 3	80 ÷ 2	800 ÷ 2
16.	9 ÷ 1	90 ÷ 1	60 ÷ 2	00 ÷ 2	000 ÷ 2
17.	0 ÷ 2	22 ÷ 2	20 ÷ 4	25 ÷ 1	600 ÷ 3

SPEED TEST CHART ON TROUBLESOME COMBINATIONS

See if you can state all these results in $1\frac{1}{4}$ min.:

1. 27 ÷ 3   18 ÷ 6   28 ÷ 7   20 ÷ 4   48 ÷ 8
2. 24 ÷ 4   21 ÷ 7   90 ÷ 9   24 ÷ 6   36 ÷ 9
3. 14 ÷ 7   20 ÷ 2   25 ÷ 5   35 ÷ 7   27 ÷ 9
4. 56 ÷ 7   28 ÷ 4   30 ÷ 6   40 ÷ 8   42 ÷ 7
5. 16 ÷ 4   49 ÷ 7   15 ÷ 5   30 ÷ 5   64 ÷ 8
6. 36 ÷ 6   35 ÷ 5   32 ÷ 4   70 ÷ 7   32 ÷ 8
7. 20 ÷ 5   36 ÷ 4   60 ÷ 6   81 ÷ 9   18 ÷ 9
8. 40 ÷ 5   72 ÷ 9   63 ÷ 7   24 ÷ 8   16 ÷ 8
9. 24 ÷ 3   54 ÷ 6   45 ÷ 5   48 ÷ 6   40 ÷ 4
10. 45 ÷ 9   42 ÷ 6   54 ÷ 9   56 ÷ 8   63 ÷ 9

See if you can state these especially troublesome ones, some of which are repeated, in 1 min.:

11. 54 ÷ 9   36 ÷ 6   42 ÷ 7   81 ÷ 9   54 ÷ 6
12. 64 ÷ 8   32 ÷ 8   63 ÷ 7   49 ÷ 7   40 ÷ 8
13. 63 ÷ 9   45 ÷ 9   40 ÷ 8   90 ÷ 9   72 ÷ 9
14. 56 ÷ 8   56 ÷ 7   64 ÷ 8   49 ÷ 7   36 ÷ 9
15. 48 ÷ 6   45 ÷ 5   48 ÷ 8   56 ÷ 8   72 ÷ 8
16. 81 ÷ 9   56 ÷ 7   72 ÷ 9   63 ÷ 9   42 ÷ 6

DRILL CHART IN SHORT DIVISION

In the following, state the results as rapidly as you can:

1.	$2\overline{)20}$	$2\overline{)24}$	$2\overline{)42}$	$\frac{1}{2}$ of 48	$\frac{1}{2}$ of 84
2.	$3\overline{)21}$	$3\overline{)36}$	$3\overline{)63}$	$\frac{1}{3}$ of 96	$\frac{1}{3}$ of 69
3.	$4\overline{)24}$	$4\overline{)36}$	$4\overline{)48}$	$\frac{1}{4}$ of 84	$\frac{1}{4}$ of 56
4.	$5\overline{)35}$	$5\overline{)45}$	$5\overline{)55}$	$\frac{1}{5}$ of 60	$\frac{1}{5}$ of 80
5.	$6\overline{)36}$	$6\overline{)42}$	$6\overline{)54}$	$\frac{1}{6}$ of 72	$\frac{1}{6}$ of 96
6.	$7\overline{)42}$	$7\overline{)49}$	$7\overline{)63}$	$\frac{1}{7}$ of 84	$\frac{1}{7}$ of 91
7.	$8\overline{)56}$	$8\overline{)64}$	$8\overline{)88}$	$\frac{1}{8}$ of 96	$\frac{1}{8}$ of 104
8.	$9\overline{)63}$	$9\overline{)81}$	$9\overline{)27}$	$\frac{1}{9}$ of 45	$\frac{1}{9}$ of 144
9.	$7\overline{)98}$	$4\overline{)96}$	$5\overline{)85}$	$\frac{1}{3}$ of 51	$\frac{1}{4}$ of 680
10.	$4\overline{)52}$	$3\overline{)60}$	$2\overline{)94}$	$\frac{1}{4}$ of 68	$\frac{1}{6}$ of 222
11.	$4\overline{)72}$	$6\overline{)78}$	$3\overline{)57}$	$\frac{1}{5}$ of 95	$\frac{1}{8}$ of 144
12.	$6\overline{)84}$	$7\overline{)84}$	$8\overline{)96}$	$\frac{1}{9}$ of 72	$\frac{1}{8}$ of 104
13.	$5\overline{)70}$	$8\overline{)32}$	$7\overline{)91}$	$\frac{1}{7}$ of 63	$\frac{1}{9}$ of 225
14.	$3\overline{)93}$	$6\overline{)60}$	$5\overline{)65}$	$\frac{1}{4}$ of 88	$\frac{1}{8}$ of 248
15.	$7\overline{)77}$	$4\overline{)76}$	$8\overline{)48}$	$\frac{1}{6}$ of 84	$\frac{1}{10}$ of 100
16.	$5\overline{)95}$	$3\overline{)78}$	$6\overline{)78}$	$\frac{1}{9}$ of 54	$\frac{1}{10}$ of 120
17.	$3\overline{)69}$	$6\overline{)96}$	$8\overline{)72}$	$\frac{1}{5}$ of 85	$\frac{1}{12}$ of 144

DRILL CHART IN SHORT DIVISION

1. If you have 15¢ with which to buy 2-cent stamps, how many stamps can you buy, and how much money will be left? In such cases, you may say "7, and 1¢ over."

State the results, including the remainders, in the following cases as rapidly as you can:

2. 25 ÷ 6.
3. 76 ÷ 9.
4. 33 ÷ 4.
5. 59 ÷ 8.
6. 66 ÷ 7.
7. 68 ÷ 8.
8. 26 ÷ 6.
9. 35 ÷ 8.
10. 82 ÷ 9.
11. 26 ÷ 3.
12. 84 ÷ 9.
13. 28 ÷ 8.
14. 49 ÷ 5.
15. 28 ÷ 9.
16. 37 ÷ 9.
17. 32 ÷ 5.
18. 59 ÷ 7.
19. 58 ÷ 9.
20. 75 ÷ 9.
21. 66 ÷ 9.
22. 34 ÷ 5.
23. 38 ÷ 8.
24. 45 ÷ 6.
25. 26 ÷ 5.
26. 27 ÷ 7.
27. 59 ÷ 9.
28. 22 ÷ 4.
29. 74 ÷ 8.
30. 65 ÷ 9.
31. 47 ÷ 8.
32. 29 ÷ 5.
33. 45 ÷ 8.
34. 25 ÷ 3.
35. 69 ÷ 8.
36. 39 ÷ 9.
37. 68 ÷ 9.
38. 47 ÷ 5.
39. 68 ÷ 7.
40. 24 ÷ 7.
41. 39 ÷ 4.
42. 22 ÷ 7.
43. 69 ÷ 9.
44. 42 ÷ 8.
45. 48 ÷ 7.
46. 33 ÷ 8.
47. 19 ÷ 9.
48. 27 ÷ 4.
49. 58 ÷ 6.
50. 29 ÷ 9.
51. 46 ÷ 8.
52. 45 ÷ 7.

SPEED TEST CHART IN SHORT DIVISION

See how many of the following results, including the remainders, you can state in 1 min. If you make a mistake, let your neighbor take up the work:

1. 2 ÷ 7.
2. 1 ÷ 8.
3. 4 ÷ 7.
4. 12 ÷ 7.
5. 15 ÷ 8.
6. 15 ÷ 9.
7. 30 ÷ 9.
8. 41 ÷ 6.
9. 14 ÷ 9.
10. 31 ÷ 4.
11. 11 ÷ 7.
12. 26 ÷ 9.
13. 61 ÷ 8.
14. 25 ÷ 9.
15. 51 ÷ 9.
16. 33 ÷ 7.
17. 50 ÷ 9.
18. 13 ÷ 8.
19. 70 ÷ 9.
20. 23 ÷ 8.
21. 3 ÷ 9.
22. 5 ÷ 8.
23. 22 ÷ 6.
24. 44 ÷ 9.
25. 23 ÷ 6.
26. 31 ÷ 7.
27. 60 ÷ 7.
28. 32 ÷ 9.
29. 60 ÷ 9.
30. 20 ÷ 9.
31. 53 ÷ 6.
32. 31 ÷ 8.
33. 30 ÷ 7.
34. 52 ÷ 6.
35. 40 ÷ 6.
36. 35 ÷ 9.
37. 34 ÷ 9.
38. 43 ÷ 9.
39. 32 ÷ 7.
40. 54 ÷ 7.
41. 1 ÷ 6.
42. 23 ÷ 9.
43. 62 ÷ 9.
44. 52 ÷ 7.
45. 50 ÷ 8.
46. 51 ÷ 8.
47. 71 ÷ 8.
48. 62 ÷ 7.
49. 53 ÷ 8.
50. 71 ÷ 9.
51. 55 ÷ 8.
52. 40 ÷ 9.
53. 55 ÷ 7.
54. 41 ÷ 9.
55. 34 ÷ 7.
56. 52 ÷ 8.
57. 40 ÷ 7.
58. 41 ÷ 7.
59. 22 ÷ 6.
60. 21 ÷ 9.

Taking Parts of Numbers. We know that $\frac{1}{2}$ of a number is one of its two equal parts; that is, $\frac{1}{2}$ of 6 is 3, and hence 3 is one of the two equal parts of 6.

In the same way, we have the following cases:

$\frac{1}{3}$ of 6 is 6 ÷ 3, or 2

$\frac{1}{4}$ of 12 is 12 ÷ 4, or 3

$\frac{1}{5}$ of 25 is 25 ÷ 5, or 5

$\frac{1}{8}$ of 48 is 48 ÷ 8, or 6

When we study more about fractions, we shall review other parts of numbers, such as $\frac{3}{4}$ of 48.

PARTS OF NUMBERS

See how many of these parts you can state in 1 min. If you make a mistake, let your neighbor take up the work:

1. $\frac{1}{2}$ of 8.
2. $\frac{1}{4}$ of 8.
3. $\frac{1}{8}$ of 8.
4. $\frac{1}{3}$ of 9.
5. $\frac{1}{5}$ of 15.
6. $\frac{1}{4}$ of 24.
7. $\frac{1}{2}$ of 16.
8. $\frac{1}{3}$ of 18.
9. $\frac{1}{8}$ of 40.
10. $\frac{1}{4}$ of 36.
11. $\frac{1}{8}$ of 80.
12. $\frac{1}{2}$ of 10.
13. $\frac{1}{4}$ of 12.
14. $\frac{1}{8}$ of 16.
15. $\frac{1}{8}$ of 48.
16. $\frac{1}{3}$ of 21.
17. $\frac{1}{4}$ of 28.
18. $\frac{1}{2}$ of 18.
19. $\frac{1}{3}$ of 27.
20. $\frac{1}{5}$ of 25.
21. $\frac{1}{8}$ of 56.
22. $\frac{1}{4}$ of 16.
23. $\frac{1}{3}$ of 12.
24. $\frac{1}{2}$ of 12.
25. $\frac{1}{8}$ of 64.
26. $\frac{1}{8}$ of 24.
27. $\frac{1}{3}$ of 24.
28. $\frac{1}{8}$ of 72.
29. $\frac{1}{4}$ of 32.
30. $\frac{1}{5}$ of 55.
31. $\frac{1}{3}$ of 15.
32. $\frac{1}{9}$ of 27.
33. $\frac{1}{4}$ of 20.
34. $\frac{1}{7}$ of 21.
35. $\frac{1}{2}$ of 22.
36. $\frac{1}{8}$ of 32.
37. $\frac{1}{4}$ of 40.
38. $\frac{1}{9}$ of 45.
39. $\frac{1}{2}$ of 20.

A SILENT READING LESSON

We can often save work, when dividing whole numbers by 10, by powers of 10, or by multiples of these powers, if we take short cuts like the following:

To divide by 10, cut off the right-hand figure of the number divided, taking the figure cut off as the remainder.

For example, in the case of 1430 ÷ 10 we have $143\cancel{0}$, and so the result is 143. In 2637 ÷ 10 we have $263\cancel{7}$, and so the result is 263 with a remainder of 7. We may write this as $263\frac{7}{10}$.

To divide by any power of 10, cut off as many figures from the right of the number divided as there are zeros at the right of the number by which you divide, taking the figures cut off as the remainder.

For example, in 14,600 ÷ 100, we have $14{,}6\cancel{0}\cancel{0}$, and so the result is 146. In 14629 ÷ 1000, we have $14\cancel{6}\cancel{2}\cancel{9}$, and so the result is 14 with a remainder of 629, or $14\frac{629}{1000}$.

To divide by any multiple of 10, 100, and so on, proceed as shown in the following examples:

$$\begin{array}{r} 1230 \\ 3\cancel{0}\overline{)3690\cancel{0}} \end{array} \qquad \begin{array}{r} 123 \\ 3\cancel{0}\cancel{0}\overline{)369\cancel{0}\cancel{0}} \end{array} \qquad \begin{array}{r} 123\frac{13}{300} \\ 3\cancel{0}\cancel{0}\overline{)369\cancel{1}\cancel{3}} \end{array}$$

$$\begin{array}{r} 19 \\ 17\cancel{0}\overline{)323\cancel{0}} \\ \underline{17} \\ 153 \\ \underline{153} \end{array} \qquad \begin{array}{r} 19\frac{13}{1700} \\ 17\cancel{0}\cancel{0}\overline{)323\cancel{1}\cancel{3}} \\ 17 \\ 153 \\ \underline{153} \\ 13 \end{array} \qquad \begin{array}{r} 9\frac{37}{170} \\ 17\cancel{0}\overline{)156\cancel{7}} \\ \underline{153} \\ 37 \end{array}$$

PROBLEMS IN DIVISION

1. At $60 each, how many radio sets must a manufacturer sell in order to receive $1860?

2. A dealer sells some city lots at $900 a lot. If he receives $53,100 in all, how many lots does he sell?

3. A farmer spends $6880 in buying cows. At $80 apiece, how many cows can he buy for this amount?

4. An orange grower has 8460 oranges which he wishes to pack in boxes holding 100 oranges each. How many boxes can he pack, and how many oranges will be left over?

5. If a coal pocket contains 786,400 lb. of coal, how many tons (2000 lb.) does it contain, and how much over?

6. A dealer has $3850 with which to buy some village lots at $600 each. What is the greatest number that he can buy, and how much money will he have left?

7. An automobile dealer has $85,000 to invest in automobiles at $2000 each. What is the greatest number that he can buy, and how much money will he have left?

Divide as indicated:

8. 400 ÷ 20.
9. 900 ÷ 30.
10. 700 ÷ 70.
11. 37,800 ÷ 200.
12. 54,300 ÷ 300.
13. 92,400 ÷ 400.
14. 438,000 ÷ 3000.
15. 229,000 ÷ 4000.
16. 745,500 ÷ 5000.

17. In laying a floor in a corridor 80 ft. long a carpenter uses boards 16 ft. long. There are 18 boards in the width of the corridor. Allowing for the loss of one board in matching, how many boards does it take?

A SILENT READING LESSON

Read, filling in the blanks as you read:

Edward and John sell newspapers. They keep account of their sales and find that they sell 49,820 papers in a year. Allowing 52 wk. to the year, what is the average number of papers sold per week? To answer this question we see that we must —— 49,820 by 52.

To find the first figure in the result we —— 49 by 5. The result is —— with a remainder.

The figure 9 is all right, because 9 × 52 is ——, and this is less than 498. If it had been greater than 498, we should have tried ——.

$$\begin{array}{r} 958 \\ 52\overline{)49820} \\ \underline{468} \\ 302 \\ \underline{260} \\ 420 \\ \underline{416} \\ 4 \end{array}$$

We write the 9 over the figure ——, because we are first dividing 498 by 52.

We —— 468 from ——, write the remainder 30 below, and after 30 we write the figure ——, which we bring down from the number that we are dividing.

We continue to divide in the same way and find that the result is $958\frac{4}{52}$.

It would not mean much in such a case to say that the boys sell $958\frac{4}{52}$ newspapers each week. We therefore disregard the small remainder 4 and say that the average number sold per week is ——.

We check the accuracy of our work like this:

$$52 \times 958 = 49{,}816$$

and $$49{,}816 + 4 = \text{——}.$$

DRILL CHART IN LONG DIVISION

Perform the following very easy divisions:

1. 882 ÷ 21.
2. 744 ÷ 31.
3. 1558 ÷ 41.
4. 2142 ÷ 51.
5. 4686 ÷ 71.
6. 1924 ÷ 52.
7. 2976 ÷ 62.
8. 4032 ÷ 72.
9. 6724 ÷ 82.
10. 6900 ÷ 92.
11. 1419 ÷ 33.
12. 2838 ÷ 43.
13. 4851 ÷ 63.
14. 2701 ÷ 73.
15. 9207 ÷ 93.

Perform these harder divisions:

16. 957 ÷ 29.
17. 1638 ÷ 39.
18. 2597 ÷ 49.
19. 4248 ÷ 59.
20. 2553 ÷ 69.
21. 5372 ÷ 79.
22. 8722 ÷ 89.
23. 1596 ÷ 38.
24. 4466 ÷ 58.
25. 5372 ÷ 68.
26. 7744 ÷ 88.
27. 8722 ÷ 98.
28. 6396 ÷ 78.
29. 2376 ÷ 36.
30. 1406 ÷ 37.

Try these, which are still harder:

31. 2070 ÷ 45.
32. 4140 ÷ 46.
33. 3472 ÷ 56.
34. 8280 ÷ 23.
35. 2730 ÷ 78.
36. 3472 ÷ 28.
37. 3476 ÷ 44.
38. 4984 ÷ 56.
39. 7990 ÷ 85.
40. 5776 ÷ 76.
41. 7569 ÷ 87.
42. 9801 ÷ 99.

Now try these, where there are some remainders:

43. 5402 ÷ 69.
44. 7353 ÷ 68.
45. 7040 ÷ 64.
46. 4381 ÷ 95.
47. 3107 ÷ 79.
48. 6234 ÷ 87.
49. 4375 ÷ 76.
50. 8291 ÷ 78.
51. 6007 ÷ 19.

A SILENT READING LESSON

Tom's uncle is a harness manufacturer. If he sells 372 harnesses and receives \$21,375.12 in payment, at what price does he sell each harness? To answer this, we must divide \$21,375.12 by 372.

$$\begin{array}{r} \$57.46 \\ 372\overline{)\$21375.12} \\ \underline{1860} \\ 2775 \\ \underline{2604} \\ 1711 \\ \underline{1488} \\ 2232 \\ \underline{2232} \end{array}$$

Stated briefly, we have

$2137 \div 372 = 5$, with a remainder;

The 5 goes over the 7, because we are first dividing 2137. Then

$2775 \div 372 = 7$, with a remainder;
$1711 \div 372 = 4$, with a remainder;
$2232 \div 372 = 6$, with no remainder.

We place the decimal point in the result above the decimal point in the number divided.

The result is \$57.46; that is, the price at which Tom's uncle sells each harness is \$57.46.

Check. $372 \times \$57.46 = \$21,375.12.$

As another example, suppose that a mill sells 25 yd. of cloth for \$136. What price per yard does the mill receive for the cloth?

We can find the price per yard by dividing \$136 by 25. In order to do this, we first write \$136 as \$136.00 and then divide as explained above, finding that

$$\$136.00 \div 25 = \$5.44;$$

that is, the price of the cloth per yard is \$5.44.

PROBLEMS IN DIVISION

Perform the following divisions:

1. \$974.58 ÷ 111.
2. \$772.12 ÷ 199.
3. \$3867.50 ÷ 25.
4. \$296 ÷ 40.
5. \$128 ÷ 25.
6. \$385 ÷ 25.
7. \$2992.80 ÷ 27.
8. \$1059.20 ÷ 64.
9. \$6784.56 ÷ 81.

10. A class of 12 pupils bought an electric postcard projector for the schoolroom. If the projector cost \$7.80, what was the share of each pupil?

11. Three boys together buy a small electric motor for \$4.47. What is the share that each boy pays?

12. A class of 16 girls bought a slide for the school playground. If the girls paid \$13.60 for the slide and 64¢ for express, how much was the share of each?

Copy the following table, filling in the blank spaces:

	Number Divided	Divided by	Result
13.	840	——	64, remainder 8
14.	——	12	\$3.67, remainder 7¢
15.	\$17.35	144	\$——, remainder——
16.	——	79	\$13.80, remainder 10¢
17.	3867	——	143, remainder 6

18. A factory ships 60,000 cans of fruit in boxes which hold 4 doz. cans each. How many boxes are required?

19. A cubic yard contains 27 cu. ft. How many cubic yards of earth does a contractor remove in excavating 135,675 cu. ft. of earth?

DRILL CHART IN ESTIMATING ANSWERS

One of the best checks upon foolish errors, such as no one ought ever to make, is to estimate the result in advance. In each of these cases tell which estimate is probably the nearest to the result. Then do the example and see if the one you chose was the nearest:

OPERATIONS		ESTIMATES	
1. 7 × $1.43	$1001	$70	$10
2. 21 × $14.30	$28	$30	$303
3. 3 × 37,037	99,000	111,000	121,000
4. 25 × 48¢	$11.50	$12	$12.50
5. 50 × 75¢	$3.50	$35.50	$37.50
6. 100 × $2.75	$27.50	$275	$2750
7. $\frac{1}{10}$ of $375	$3.75	$37.50	$3750
8. $\frac{1}{10}$ of $2750	$27.50	$275	$2750
9. $\frac{1}{20}$ of $1500	$3000	$30,000	$75
10. $375 ÷ 25	$1500	$1.50	$15
11. $48.50 ÷ 25	$1.50	$1.90	$20
12. $151.50 ÷ 75	$202	$2	$20
13. $38.44 ÷ 62	$6.25	$0.60	$62
14. $27.01 ÷ 73	$37	$0.37	$3.75
15. $521.16 ÷ 43	$12	$121	$12,120
16. $826.50 ÷ 58	$1.50	$150	$14

DRILL CHART IN ESTIMATING ANSWERS

Write your estimated results at once; then divide rapidly, and see how close your estimates were:

1. 176 ÷ 11.
2. 192 ÷ 12.
3. 221 ÷ 13.
4. 247 ÷ 13.
5. 294 ÷ 14.
6. 330 ÷ 15.
7. 368 ÷ 16.
8. 450 ÷ 18.
9. 368 ÷ 23.
10. 408 ÷ 17.
11. 900 ÷ 18.
12. 513 ÷ 19.
13. 462 ÷ 21.
14. 700 ÷ 25.
15. 800 ÷ 32.
16. 625 ÷ 25.
17. 506 ÷ 22.
18. 544 ÷ 16.
19. 572 ÷ 22.
20. 682 ÷ 31.
21. 528 ÷ 22.
22. 572 ÷ 26.
23. 682 ÷ 11.
24. 576 ÷ 12.
25. 700 ÷ 28.
26. 825 ÷ 33.
27. 528 ÷ 24.
28. 900 ÷ 25.
29. 400 ÷ 16.
30. 850 ÷ 34.

In the following cases (only some of which are cases in exact division), write your estimate of the whole number in each result; then divide, and see how near you were:

31. 176 ÷ 16.
32. 194 ÷ 16.
33. 335 ÷ 22.
34. 234 ÷ 17.
35. 352 ÷ 22.
36. 460 ÷ 25.
37. 470 ÷ 22.
38. 130 ÷ 11.
39. 150 ÷ 24.
40. 400 ÷ 30.
41. 222 ÷ 22.
42. 322 ÷ 22.
43. 322 ÷ 16.
44. 156 ÷ 32.
45. 270 ÷ 27.
46. 270 ÷ 81.
47. 500 ÷ 50.
48. 500 ÷ 25.
49. 500 ÷ 125.
50. 750 ÷ 225.
51. 750 ÷ 250.
52. 700 ÷ 350.
53. 725 ÷ 225.
54. 775 ÷ 225.

USING WHAT YOU HAVE LEARNED

1. A builder paid 24 men, all at the same rate, $2520 for 15 da. of work. Find each man's daily wage.

2. At the same price for each set, a furniture dealer sold 4 sets of dining-room furniture each week for 3 wk. If his total receipts were $852, what was the price of each set?

3. A dealer paid $1152 for 3 doz. suits of clothes, all at the same price. How much did he pay for each suit?

4. A company has 175 men in one of its departments, all of whom receive the same wages. If in 5 da. the pay roll amounts to $5250, what are the daily wages of each man?

5. Another department in Ex. 4 has 210 men, all receiving the same wages. If in 5 da. the pay roll amounts to $7087.50, what are the daily wages of each man?

6. In a school which has 44 pupils the teacher's salary is $1500 a year. The cost of fuel is $144.20; of books and other school supplies, $172.80; of repairs and improvements, $75. Find the total cost per pupil for the year.

7. For a month in which he used 6000 cu. ft. of gas a man's gas bill was $6.60. Find the cost per 1000 cu. ft.

8. A class of 24 pupils bought these toys for a children's hospital, sharing the cost equally: an automobile for $8, a cart for $7, a building set for $9, a train for $6, a stove for $5, some blocks for $1. What was each pupil's share?

9. A merchant found that in one month he paid $30 for light, $382 for wages, $220 for rent, and $70 for other expenses. If there were 26 business days in that month, how much were the average expenses per business day?

10. A dealer pays $413.56 for 14 suits of ready-made clothes. First estimate how much he pays per suit, and then find the exact result. At the same rate, how much would he pay for 7 suits? for 28 suits?

11. If 27 A. of land are worth $2227.50, what is the average value per acre? How much are 4 A. worth?

12. A man pays $1430 for the rent of an apartment in the city for 11 mo. At this rate, how much does he pay for 1 mo.? for 5 mo.? for 7 mo.? for 3 mo.? for 8 mo.?

13. The letter M is the Roman numeral for thousand, and C for hundred. Hence 7 M means 7000, and $8.75 per C means $8.75 per hundred. Lumber is usually sold by the 1000 ft. A man bought 8 M ft. of lumber at $48.50 per M. He sold half of it at $8.75 per C, and the other half at $47.50 per M. How much did he gain?

14. At an athletic meet 860 spectators bought tickets at 45¢ each, and 1240 at 25¢ each. What was the average amount to the nearest cent paid by each spectator?

15. A builder made a profit of $865 on one house, $540 on a second, $690 on a third, and lost $275 on a fourth. What was his average profit per house?

16. At 60 lb. to the bushel, how many bushels are there in a shipment of 28,750 lb. of wheat, and how many pounds are left over?

17. A farm bulletin states that the corn raised on an acre of good land should contain 147 lb. of fat, which is one of the food elements in corn. How many such acres are needed to produce 5586 lb. of fat?

DRILL CHART IN LONG DIVISION

Perform the following divisions:

1. 85,785 ÷ 301.
2. 59,314 ÷ 631.
3. 48,594 ÷ 623.
4. 364,397 ÷ 211.
5. 976,950 ÷ 501.
6. 972,471 ÷ 111.
7. 161,784 ÷ 321.
8. 324,360 ÷ 306.
9. 140,850 ÷ 225.
10. 183,750 ÷ 175.
11. 148,555 ÷ 407.
12. 234,901 ÷ 503.
13. 208,575 ÷ 309.
14. 870,672 ÷ 204.
15. $3678.48 ÷ 786.
16. $1024.65 ÷ 495.
17. $4729.32 ÷ 604.
18. $3492.06 ÷ 407.
19. $1174.40 ÷ 320.
20. $4196.40 ÷ 538.
21. $2877.12 ÷ 216.
22. 51,716 ÷ 14.
23. 45,952 ÷ 16.
24. 85,635 ÷ 22.
25. 54,268 ÷ 37.
26. 92,357 ÷ 28.
27. 36,452 ÷ 49.
28. 88,757 ÷ 56.
29. 41,795 ÷ 56.
30. 79,145 ÷ 49.
31. 36,523 ÷ 48.
32. 67,558 ÷ 59.
33. 87,073 ÷ 66.
34. 90,601 ÷ 72.
35. 401,121 ÷ 332.
36. 210,379 ÷ 224.
37. 852,191 ÷ 315.
38. 720,830 ÷ 853.
39. 360,389 ÷ 427.
40. 692,823 ÷ 236.
41. 811,423 ÷ 2878.
42. 487,712 ÷ 4327.

DRILL CHART IN LONG DIVISION

After each of these examples are four possible results, one of which is right. Copy the examples, picking out and writing the correct result after each one:

1. 9641 ÷ 31; 101, 331, 116, or 311.
2. 29,568 ÷ 176; 108, 168, 193, or 226.
3. 12,705 ÷ 231; 505, 425, 314, or 55.
4. 17,625 ÷ 125; 125, 141, 145, or 245.
5. 42,625 ÷ 341; 145, 105, 215, or 125.
6. 47,840 ÷ 260; 199, 184, 176, or 146.
7. 51,968 ÷ 224; 232, 242, 286, or 176.
8. 77,720 ÷ 670; 126, 106, 116, or 249.

Some of the following results are right and some are wrong. Copy the statements, and divide, writing + after those which are correct and the correct result after those which are wrong:

9. 85,407 ÷ 343 = 249.
10. 76,010 ÷ 151 = 503, remainder 50.
11. 76,111 ÷ 608 = 125, remainder 111.
12. 53,165 ÷ 217 = 255; or 245, remainder 8.
13. 52,900 ÷ 116 = 456, remainder 4.
14. 85,250 ÷ 250 = 342.
15. 28,750 ÷ 1250 = 231, remainder 2.
16. 175,098 ÷ 758 = 231.
17. 162,842 ÷ 118 = 138, remainder 2.

REVIEW PROBLEMS

1. A garage owner received the following amounts from the sale of gasoline: Monday, $75.85; Tuesday, $146.20; Wednesday, $155.85; Thursday, $180.40; Friday, $190.50; Saturday, $248.60. Find the total amount for the week.

2. The owner of an automobile bought the following items: tires, $57.60; inner tubes, $16.50; oil, $1.80; tire chains, $6.50; gasoline, $5.60. What was the total amount of the bill?

3. Last week the daily sales in a grocery store were as follows: Monday, $460.20; Tuesday, $720.83; Wednesday, $652.56; Thursday, $760.54; Friday, $530.28; Saturday, $1062.49. What was the total amount of the sales for the week?

4. On the first day of last month Mr. Brown paid the following bills: gas, $4.40; electricity, $3.56; groceries, $33.65; meat, $15.40; laundry, $16.20; telephone, $4.50; milk, $12.60. What was the total amount paid?

5. Mr. Morris bought a camping outfit, which included the following articles: tent, $48.60; blankets, $23.50; air mattresses, $36.80; air pillows, $3.00; gasoline stove, $7.50; cooking utensils, $14.50; miscellaneous, $13.40. Find the total cost.

6. A town school department paid the following amounts for coal: September, $118.75; October, $106.38; November, $149.50; December, $133.60; January, $165.70; February, $162.80; March, $143.60; April, $105.13; May, $100.59. Find the total amount of the coal bill.

REVIEW PROBLEMS

1. At 56¢ each, how much will it cost to buy arithmetic books for 347 pupils?

2. How much do 13 T. of coal cost at $14.25 a ton?

3. A farmer in Steuben County sold 1750 bu. of potatoes at 65¢ a bushel. How much did he receive?

4. We bought 107 yd. of material at $4.75 a yard to make a curtain for our school auditorium. Find the cost.

5. The children in our school used 497 qt. of milk last week at a cost of 16¢ a quart. Find the cost.

6. A farmer sold 96 doz. eggs at 65¢ a dozen. How much did he receive for the eggs?

7. How much is the cost of 21,000 cu. ft. of gas at $1.25 per 1000 cu. ft.?

8. A fruit grower in Niagara County sold 785 barrels of apples at $4.50 a barrel. How much did he receive?

9. We burned 297 T. of coal last year in School No. 3. At $6.80 a ton, how much was the coal bill?

10. In School No. 7 natural gas is used for heating. Last winter 1,670,000 cu. ft. of gas were consumed. How much was the gas bill at 45¢ per 1000 cu. ft.?

11. The locomotive works at Dunkirk employed 3457 men during the first week in July. The average weekly wage paid to each man was $26.40. How much was the total pay roll for the week?

12. A school board paid 85¢ an hour for 358 hr. of labor for painting the school building. Find the total cost.

REVIEW PROBLEMS

1. In Allegany County a potato dealer loaded a freight car with 34,320 lb. of potatoes. At 60 lb. of potatoes to the bushel, how many bushels did he put in the car?

2. A high school building in a New York State village was recently destroyed by fire. The insurance company paid the village $80,000. How much more money must be raised to build a new school costing $450,000?

3. We had a school picnic last spring on the shore of Seneca Lake. We spent $101.21 for the food. How much did it cost each of the 349 persons at the picnic?

4. The Empire State Express makes a run of 441 mi. in 9 hr., including all stops. What is the average distance the train covers in an hour?

5. Last summer, on a trip from Chicago to New York and return, we drove our car 2664 mi., and used 148 gal. of gasoline. How many miles did we average per gallon?

6. We paid $32.56 for the 148 gal. of gasoline we used on the trip mentioned in Ex. 5. What was the average price per gallon?

7. In 1890 there were 297,894 pupils enrolled in the high schools of the United States. In 1920 there were 2,041,308 enrolled. What was the increase?

8. In a certain year it cost the City of Rochester $209,547.93 to run the high schools. Eleven years later, owing to the increase in attendance and to the improvement in the schools, it cost $1,020,844.23. How much was the increase in cost during this time?

II. FRACTIONS

A SILENT READING LESSON

Read, filling in the blanks as you read:

The number 3 is a whole ——, but $\frac{3}{4}$ is a ——.

In the fraction $\frac{3}{4}$, the *numerator* is ——, and the *denominator* is ——.

The *terms* of the fraction $\frac{3}{4}$ are —— and ——.

A quarter of a dollar is a fraction of a dollar, but is usually written as $0.25. This shows that fractions are sometimes written without writing the denominators.

The common way of writing three fourths is $\frac{3}{4}$. Such a fraction is called a *common fraction*, or, more often, it is called simply a *fraction*.

If the numerator of a fraction is 1, we call the fraction a *unit* ——.

A fraction like $\frac{3}{4}$, in which the numerator is less than the ——, is called a *proper fraction*.

A fraction like $\frac{2}{2}$ or $\frac{3}{2}$, in which the —— is equal to or greater than the denominator, is called an *improper* ——.

A number like $3\frac{3}{4}$ is called a *mixed number*. The numbers ——, ——, and —— are mixed numbers.

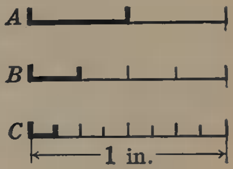
The *factors* of 6 are 2 and 3; that is, when the factors 3 and 2 are multiplied together, the result is ——.

A factor of each of two numbers is called a *common factor* of the numbers. For example, 2 is a common factor of 6 and 12, and it is also a common factor of —— and ——.

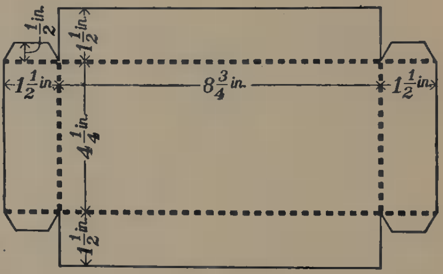
A number which has no factors except 1 and itself is called a *prime number*. For example, 7 and —— are —— numbers.

FRACTIONS IN MEASURING

1. Most rulers used in schools are divided into fractions of an inch. These fractions are often halves, fourths, and eighths. Rulers used in practical work are divided into inches, sixteenths, thirty-seconds, and sometimes sixty-fourths. Other rulers are divided into tenths of an inch. We rarely use other fractions of an inch, such as thirds, sixths, twelfths, and so on. The lines *A*, *B*, and *C* here shown are each 1 in. long. Which one is divided into fourths? into eighths? into halves?



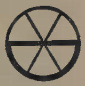
2. A set of directions for making a box told the pupils to cut a piece of cardboard $11\frac{3}{4}$ in. long and $7\frac{1}{4}$ in. wide; to draw the lines and cut out corners as shown in this figure; and then, by folding along the heavy dotted lines and pasting the four flaps, to complete the box. Make a pattern for such a box.



3. The directions for making the cover for the box were the same as those in Ex. 2, except that the measurements of the top were $4\frac{3}{8}$ in. instead of $4\frac{1}{4}$ in. and $8\frac{7}{8}$ in. instead of $8\frac{3}{4}$ in., and the width for the side pieces was $\frac{1}{2}$ in. instead of $1\frac{1}{2}$ in. Make a pattern for such a cover and show how it is to be folded.

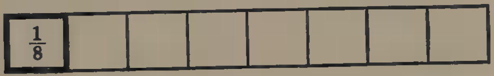
A SILENT READING LESSON

We see as we look at this circle that there are 6 sixths of a circle in the whole circle. It makes us think of the number of sixths in a whole pie.

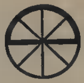


In half a pie we see that there are 3 sixths. We thus see that $\frac{1}{2}$ and $\frac{3}{6}$ are equal. When we write $\frac{3}{6}$ instead of $\frac{1}{2}$, we simply change the name of the fraction.

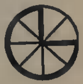
This rectangle has been divided into eight equal squares, each of which is $\frac{1}{8}$ of the rectangle. If we call a square "one," we must call the rectangle "eight"; but if we call the rectangle "one," then we must call a square "one eighth."



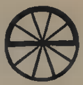
As we look at this circle we see that there are 4 eighths in half the circle. If we should cut a pie into eighths, there would be 4 eighths in half of it.



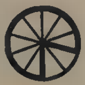
As we look at this circle we see that there are 6 eighths in $\frac{3}{4}$ of a pie. This shows us that $\frac{3}{4}$ and $\frac{6}{8}$ are equal, although one of the fractions is called fourths and the other is called eighths.



This circle shows us how many tenths there are in a half. When we write $\frac{1}{2}$ and $\frac{5}{10}$, we know that they are equal, although one of the fractions is called a half and the other is called five tenths.



This circle shows us that $\frac{4}{5}$ and $\frac{8}{10}$ are equal, although they have different names; that is, one fraction is called fifths and the other tenths.



Thus, from the examples given above, we see that

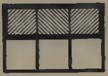
$$\frac{1}{2}=\frac{3}{6},\ \frac{1}{2}=\frac{4}{8},\ \frac{1}{2}=\frac{5}{10},\ \frac{3}{4}=\frac{6}{8},\text{ and }\frac{4}{5}=\frac{8}{10}.$$

A LITTLE WRITING LESSON

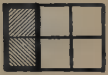
Copy the following and complete the sentences which are left unfinished, making drawings when needed:

1. Although the denominator of $\frac{1}{2}$ is less than that of $\frac{1}{4}$, the fraction $\frac{1}{2}$ is greater than $\frac{1}{4}$. These drawings (make them) show that $\frac{1}{2}$ is —— than $\frac{1}{3}$, that $\frac{1}{4}$ is —— than $\frac{1}{3}$, and that $\frac{1}{2} = \frac{2}{4}$.

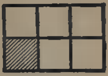
2. This diagram shows something about fractions. The heavy lines, for example, show that $\frac{3}{6} =$ ——. The diagram shows also that $\frac{1}{3}$ of the rectangle has —— small squares and that —— of the rectangle has 3 small squares; that is, —— is less than ——.



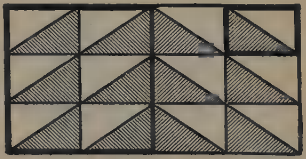
3. This diagram shows that $\frac{1}{3}$ is equal to —— sixths, that $\frac{1}{3}$ is —— than $\frac{1}{2}$, and that $\frac{4}{6}$, or $\frac{2}{3}$, is —— than $\frac{1}{2}$.



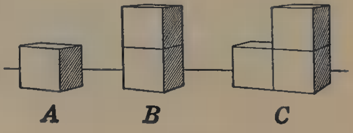
4. This diagram shows that $\frac{5}{6}$ is —— than $\frac{1}{2}$ and that $\frac{5}{6} = \frac{2}{3} +$ ——.



5. In this rectangle there are —— smaller rectangles and —— small triangles. They show that $\frac{2}{24}$ is equal to —— twelfth, and that $\frac{6}{12}$ is equal to —— half.



6. In this picture, block *A* is —— as large as block *B* and —— as large as block *C*. Similarly, block *B* is —— as large as block *C*.



A SILENT READING LESSON

From the examples on pages 61 and 62, we see that

Both terms of a fraction may be multiplied by the same number without changing the value of the fraction.

Both terms of a fraction may be divided by the same number without changing the value of the fraction.

When we change the terms of a fraction without changing the value of the fraction, we *reduce* the fraction.

For example, when we say that $\frac{1}{2} = \frac{2}{4}$, we change $\frac{1}{2}$ to fourths; or, as is commonly said, we reduce $\frac{1}{2}$ to fourths.

When both terms cannot be divided exactly by the same number, the fraction is said to be *in lowest terms*, or to be *reduced to lowest terms.*

For example, $\frac{44}{48}$ is in lowest terms when reduced to $\frac{11}{12}$.

In making a small airplane Frank found that he must use $\frac{8}{12}$ of a piece of wire. This seemed to him an awkward fraction to use. When he looked at the fraction, he saw that the largest factor of 8 and 12 is 4. He then wrote

$$\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}, \text{ in lowest terms.}$$

When the work is written as shown at the right, the factor 4 is said to be *canceled* from both terms.

$$\boxed{\frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{12}}} = \frac{2}{3}}$$

Frank then wrote this in his notebook:

To reduce a fraction to lowest terms, cancel all factors common to both terms.

We shall see on page 64 that we can tell quickly what factors to try, when numbers like 2, 3, or 5 are factors.

Divisibility of Numbers. When we speak of a number as being "divisible" by another, we mean that it can be divided without a remainder. Learn these cases:

A number is divisible by 2 if the digit in units' place is so divisible.

Thus, 54 is divisible by 2 because 4 is divisible by 2.

A whole number that is divisible by 2 is called an *even number*; one which is not is called an *odd number*.

A number is divisible by 10 if it ends in 0.

Thus, 30, 120, 800, and 5000 are divisible by 10.

A number is divisible by 5 if it ends in 0 or 5.

Thus, 180, 105, and 277,005 are divisible by 5.

Other Cases. These cases need not be memorized:

A number is divisible by 3 if the sum of its digits is so divisible, and by 6 if it is even and if the sum of its digits is divisible by 3.

Thus, 411 is divisible by 3 because $4 + 1 + 1 = 6$, and 1122, but not 1221, is divisible by 6.

A number is divisible by 4 if the number represented by its two right-hand figures is so divisible, and by 8 if the number represented by its three right-hand figures is so divisible.

Thus, we easily see that 9784 is divisible by 4 and that 175,640 is divisible by 8.

A number is divisible by 9 if the sum of its digits is so divisible.

For example, the number 70,812 is divisible by 9 because we have $7 + 0 + 8 + 1 + 2 = 18$.

Cancellation. We do not often compute with such fractions as $\frac{84}{192}$. In dividing 1620 by 192, however, we have the remainder 84, and need to reduce the fraction $\frac{84}{192}$ to lowest terms. To do this we cancel as here shown.

$$\begin{array}{r} 8 \\ 192\overline{)1620} \\ \underline{1536} \\ 84 \end{array} \qquad \frac{\overset{7}{\cancel{84}}}{\underset{16}{\cancel{192}}} = \frac{7}{16}$$

$8\frac{7}{16}$. *Ans.*

In this case we canceled 12 from 84 and 192 at once. You may, if it is easier, cancel first 4 and then 3.

In solving certain kinds of examples, we shall find it convenient to write the work in a fractional form like

$$\frac{3 \times 14 \times 15}{6 \times 35}.$$

Canceling,
$$\frac{\cancel{3} \times \overset{7}{\cancel{14}} \times \overset{3}{\cancel{15}}}{\underset{2}{\cancel{6}} \times \underset{5}{\cancel{35}}} = 3.$$

USING CANCELLATION

Reduce to lowest terms these fractions found in divisions:

1. $\frac{35}{56}$. **3.** $\frac{16}{64}$. **5.** $\frac{40}{48}$. **7.** $\frac{35}{75}$. **9.** $\frac{48}{72}$. **11.** $\frac{36}{288}$.
2. $\frac{49}{56}$. **4.** $\frac{25}{35}$. **6.** $\frac{160}{240}$. **8.** $\frac{55}{80}$. **10.** $\frac{24}{144}$. **12.** $\frac{720}{1152}$.

Simplify the following by cancellation:

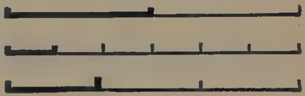
13. $\frac{9 \times 15}{10 \times 80}$. **15.** $\frac{6 \times 11 \times 35}{2 \times 33 \times 40}$. **17.** $\frac{6 \times 24 \times 35}{8 \times 15 \times 42}$.
14. $\frac{35 \times 33}{15 \times 112}$. **16.** $\frac{5 \times 27 \times 56}{8 \times 15 \times 224}$. **18.** $\frac{15 \times 24 \times 63}{18 \times 20 \times 112}$.

REDUCING FRACTIONS

Numbers 1 to 14, oral

1. In these lines how many sixths do you see in $\frac{1}{2}$? in $\frac{2}{2}$? in 1?

2. How many sixths do you see in $\frac{1}{3}$? in $\frac{2}{3}$? in $\frac{3}{3}$? in 1?



Read the following, supplying the missing terms:

3. $\frac{1}{2} = \frac{\ }{8}$. **6.** $\frac{1}{8} = \frac{4}{\ }$. **9.** $\frac{5}{8} = \frac{\ }{64}$. **12.** $\frac{3}{16} = \frac{9}{\ }$.
4. $\frac{2}{3} = \frac{\ }{9}$. **7.** $\frac{1}{8} = \frac{5}{\ }$. **10.** $\frac{7}{8} = \frac{\ }{32}$. **13.** $\frac{7}{16} = \frac{28}{\ }$.
5. $\frac{3}{4} = \frac{\ }{16}$. **8.** $\frac{3}{8} = \frac{18}{\ }$. **11.** $\frac{3}{4} = \frac{\ }{64}$. **14.** $\frac{8}{32} = \frac{1}{\ }$.

Reduce the following fractions to halves:

15. $\frac{3}{6}$. **16.** $\frac{2}{4}$. **17.** $\frac{4}{4}$. **18.** $\frac{8}{4}$. **19.** $\frac{12}{4}$. **20.** $\frac{9}{6}$.

Reduce the following fractions to fourths:

21. $\frac{1}{2}$. **22.** $\frac{6}{2}$. **23.** $\frac{18}{8}$. **24.** $\frac{24}{8}$. **25.** $\frac{28}{8}$. **26.** $\frac{32}{8}$.

Reduce the following fractions to eighths:

27. $\frac{3}{4}$. **28.** $\frac{6}{6}$. **29.** $\frac{5}{2}$. **30.** $\frac{14}{4}$. **31.** $\frac{4}{16}$. **32.** $\frac{8}{16}$.

Reduce the following fractions to twelfths:

33. $\frac{1}{3}$. **34.** $\frac{3}{4}$. **35.** $\frac{2}{3}$. **36.** $\frac{7}{6}$. **37.** $\frac{9}{6}$. **38.** $\frac{4}{24}$.

Reduce the following fractions to lowest terms:

39. $\frac{4}{8}$. **40.** $\frac{4}{6}$. **41.** $\frac{3}{9}$. **42.** $\frac{6}{10}$. **43.** $\frac{4}{12}$. **44.** $\frac{6}{15}$.

Simplify the following fractions found in divisions:

45. $\frac{48}{60}$. **47.** $\frac{60}{80}$. **49.** $\frac{72}{144}$. **51.** $\frac{75}{125}$. **53.** $\frac{65}{70}$. **55.** $\frac{55}{99}$.
46. $\frac{72}{104}$. **48.** $\frac{60}{105}$. **50.** $\frac{240}{720}$. **52.** $\frac{111}{123}$. **54.** $\frac{320}{360}$. **56.** $\frac{385}{700}$.

USING WHAT YOU HAVE LEARNED

1. Margaret needs 24 in. of cloth for some work that she is doing. Her teacher tells her that this is the same as $\frac{24}{36}$ yd. This is an awkward fraction, and she sees that it would be easier to understand it if the terms were smaller. Reduce the fraction to lowest terms.

2. In measuring an iron rod, Jack found that the diameter was $\frac{14}{16}$ in.; that is, the distance across the end was $\frac{14}{16}$ in. Express this in eighths of an inch. Is the diameter less than $\frac{1}{2}$ in.? Is it less than $\frac{3}{4}$ in.?

3. A piece of plate glass is $\frac{8}{32}$ in. thick. Express this fraction in lowest terms. Express the thickness in eighths of an inch; in sixteenths of an inch.

4. A man ordered some iron strips $\frac{1}{8}$ in. thick and some that were $\frac{5}{32}$ in. thick. Reduce $\frac{1}{8}$ to thirty-seconds and state which strips were the thicker.

5. A workman has several sizes of drills, marked $\frac{8}{16}$ in., $\frac{14}{32}$ in., $\frac{3}{8}$ in., $\frac{3}{4}$ in., and $\frac{5}{8}$ in., the fractions showing the diameters. By reducing these fractions to thirty-seconds, arrange them in order of size, beginning with the smallest.

Reduce the following fractions to sixty-fourths:

6. $\frac{7}{16}$. **7.** $\frac{9}{16}$. **8.** $\frac{9}{32}$. **9.** $\frac{15}{32}$. **10.** $\frac{16}{32}$. **11.** $\frac{12}{128}$.

Reduce the following fractions to thirty-sixths:

12. $\frac{2}{3}$. **13.** $\frac{3}{4}$. **14.** $\frac{1}{9}$. **15.** $\frac{1}{6}$. **16.** $\frac{5}{9}$. **17.** $\frac{12}{24}$.

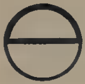
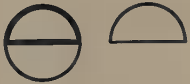
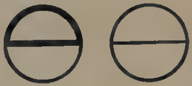
Reduce the following fractions to twenty-fourths:

18. $\frac{7}{12}$. **19.** $\frac{1}{3}$. **20.** $\frac{2}{48}$. **21.** $\frac{6}{48}$. **22.** $\frac{5}{12}$. **23.** $\frac{11}{12}$.

REDUCING TO AN IMPROPER FRACTION

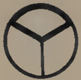
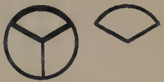
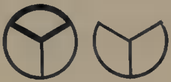
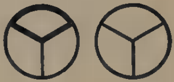
Numbers 1 to 3, oral

1. How many halves of a circle do you see in 1 circle? in $1\frac{1}{2}$ circles? Express the fraction $\frac{3}{2}$ as a mixed number.

2. How many halves of a circle do you see in 2 circles? in $2\frac{1}{2}$ circles? Express the fraction $\frac{5}{2}$ as a mixed number.

3. How many thirds do you see in 1? in $1\frac{1}{3}$? in $1\frac{2}{3}$? in 2? Express the fractions $\frac{4}{3}$ and $\frac{5}{3}$ as mixed numbers.

4. How many fourths are there in 1? in $1\frac{1}{4}$? in $1\frac{3}{4}$? in 2? Write only the results; that is, $\frac{4}{4}$, $\frac{5}{4}$, and so on.

5. How many fifths are there in 1? in $1\frac{1}{5}$? in $1\frac{2}{5}$?

6. How many sixths are there in 1? in $1\frac{1}{6}$? in $1\frac{5}{6}$? in 2? in $2\frac{1}{6}$? in $2\frac{5}{6}$? in 3?

7. How many sevenths are there in $1\frac{1}{7}$? in $2\frac{2}{7}$? in $3\frac{1}{7}$?

8. How many eighths are there in 1? in 2? in $2\frac{1}{8}$?

9. How many ninths are there in $1\frac{1}{9}$? in $2\frac{2}{9}$? in 3?

10. How many tenths are there in $3\frac{3}{10}$? in $3\frac{7}{10}$? in 4?

11. How many twelfths are there in 1? in 2? in 3? in 4? in $2\frac{1}{12}$? in $3\frac{5}{12}$? in $4\frac{7}{12}$? in $4\frac{11}{12}$?

12. How many sixteenths are there in 1? in 2? in 3? in 4? in $1\frac{1}{16}$? in $1\frac{3}{16}$? in $2\frac{1}{16}$? in $2\frac{5}{16}$? in $3\frac{7}{16}$? in $4\frac{9}{16}$?

Reducing to an Improper Fraction. The Red Cross workers wished to make bandages 1 yd. long and $\frac{1}{8}$ yd. wide. Ruth's mother had a piece of cloth $4\frac{5}{8}$ yd. long and 1 yd. wide, which she cut crosswise into pieces $\frac{1}{8}$ yd. wide and 1 yd. long. How many bandages did she make?

Ruth found the number by reducing $4\frac{5}{8}$ to eighths.

She wrote $$4 = \frac{4 \times 8}{8} = \frac{32}{8},$$

and $$4\tfrac{5}{8} = \tfrac{32}{8} + \tfrac{5}{8} = \tfrac{37}{8}.$$

After finding that $4\frac{5}{8} = \frac{37}{8}$, Ruth gave the answer. How many eighths were there? How many bandages?

REDUCING TO AN IMPROPER FRACTION

1. How many strips of cloth $\frac{1}{4}$ yd. wide can be cut from $7\frac{3}{4}$ yd. of cloth? from $8\frac{1}{4}$ yd.? from $9\frac{3}{4}$ yd.? from $5\frac{1}{2}$ yd.?

2. How many packages of pepper, each weighing $\frac{1}{8}$ lb., can be put up from $3\frac{3}{8}$ lb. of pepper? from $4\frac{7}{8}$ lb.?

Reduce the following numbers as indicated:

3. 2 to eighths.
4. 3 to thirds.
5. $1\frac{1}{2}$ to halves.
6. $3\frac{4}{5}$ to fifths.
7. $3\frac{2}{3}$ to thirds.
8. $4\frac{5}{6}$ to sixths.
9. $3\frac{1}{9}$ to ninths.
10. $2\frac{3}{5}$ to fifths.
11. $3\frac{1}{3}$ to thirds.
12. $4\frac{2}{5}$ to fifths.
13. $5\frac{4}{9}$ to ninths.
14. $6\frac{3}{5}$ to fifths.
15. $7\frac{2}{3}$ to thirds.
16. $8\frac{5}{9}$ to ninths.
17. $3\frac{2}{9}$ to ninths.
18. 32 to tenths.
19. $3\frac{1}{4}$ to fourths.
20. $6\frac{5}{8}$ to eighths.
21. $4\frac{3}{4}$ to fourths.
22. $5\frac{1}{8}$ to eighths.
23. $6\frac{7}{8}$ to eighths.
24. $9\frac{1}{6}$ to sixths.
25. 64 to halves.
26. $6\frac{3}{5}$ to fifths.

Reducing to Whole or Mixed Numbers. Edward sold tickets for an athletic meet. When he came to count his money, he found that he had 78 quarters. How many dollars were these quarters worth?

Edward saw that he must reduce $\frac{78}{4}$ to dollars; that is, to a whole number or a mixed number. He therefore wrote

$$\frac{78}{4} = 78 \div 4 = 19\tfrac{2}{4} = 19\tfrac{1}{2}.$$

That is, the 78 quarters were worth \$19½, or \$19.50.

Read, but do not memorize these statements:

A fraction is an indicated division.

To reduce an improper fraction to a whole number or a mixed number, divide the numerator by the denominator and express the result in lowest terms.

REDUCING TO WHOLE OR MIXED NUMBERS

Numbers 1 and 2, oral

1. Theresa earned 14 half dollars by taking care of a neighbor's baby. How many dollars was this?

2. In collecting for the football team, James took in 32 quarters. How many dollars did he collect?

Reduce the following fractions to whole or to mixed numbers:

3. $\frac{4}{2}$.
4. $\frac{6}{2}$.
5. $\frac{8}{2}$.
6. $\frac{7}{2}$.
7. $\frac{10}{2}$.
8. $\frac{11}{2}$.
9. $\frac{8}{4}$.
10. $\frac{9}{4}$.
11. $\frac{30}{3}$.
12. $\frac{30}{6}$.
13. $\frac{30}{5}$.
14. $\frac{30}{4}$.
15. $\frac{40}{4}$.
16. $\frac{40}{8}$.
17. $\frac{40}{3}$.
18. $\frac{40}{6}$.
19. $\frac{33}{3}$.
20. $\frac{44}{4}$.
21. $\frac{44}{5}$.
22. $\frac{44}{8}$.
23. $\frac{81}{8}$.
24. $\frac{92}{12}$.
25. $\frac{76}{32}$.
26. $\frac{100}{32}$.

27. Express as dollars: 29 quarters; 31 quarters.

28. Express as dollars: 24 half dollars; 25 half dollars.

ORAL DRILL CHART IN REDUCING FRACTIONS

Express the following fractions as whole numbers or as mixed numbers, reading each row as rapidly as you can:

1. $\frac{2}{2}$ $\frac{3}{3}$ $\frac{3}{2}$ $\frac{4}{2}$ $\frac{4}{3}$ $\frac{4}{4}$ $\frac{5}{4}$ $\frac{8}{4}$ $\frac{9}{4}$ $\frac{12}{4}$
2. $\frac{6}{2}$ $\frac{7}{2}$ $\frac{8}{2}$ $\frac{9}{2}$ $\frac{6}{3}$ $\frac{7}{3}$ $\frac{8}{3}$ $\frac{9}{3}$ $\frac{7}{4}$ $\frac{10}{4}$
3. $\frac{8}{8}$ $\frac{9}{8}$ $\frac{10}{8}$ $\frac{12}{8}$ $\frac{16}{8}$ $\frac{20}{8}$ $\frac{24}{8}$ $\frac{30}{8}$ $\frac{32}{8}$ $\frac{64}{8}$
4. $\frac{16}{16}$ $\frac{32}{16}$ $\frac{33}{16}$ $\frac{64}{16}$ $\frac{20}{10}$ $\frac{24}{12}$ $\frac{25}{12}$ $\frac{36}{12}$ $\frac{48}{12}$ $\frac{50}{12}$

Express these mixed numbers as improper fractions, reading each row as rapidly as you can:

5. $1\frac{1}{2}$ $2\frac{1}{2}$ $3\frac{1}{2}$ $4\frac{1}{2}$ $1\frac{1}{3}$ $2\frac{1}{3}$ $5\frac{1}{3}$ $7\frac{1}{3}$
6. $1\frac{1}{4}$ $1\frac{3}{4}$ $2\frac{1}{4}$ $2\frac{3}{4}$ $3\frac{3}{4}$ $1\frac{1}{8}$ $2\frac{1}{8}$ $3\frac{1}{8}$
7. $1\frac{3}{8}$ $1\frac{5}{8}$ $1\frac{7}{8}$ $2\frac{3}{8}$ $2\frac{5}{8}$ $2\frac{7}{8}$ $5\frac{1}{8}$ $4\frac{1}{8}$

Read the following, supplying the missing terms:

8. $1 = \frac{}{2}$.
9. $1 = \frac{3}{}$.
10. $1 = \frac{}{4}$.
11. $1 = \frac{}{6}$.
12. $1 = \frac{}{8}$.
13. $2 = \frac{}{2}$.
14. $2 = \frac{6}{}$.
15. $3 = \frac{}{4}$.
16. $4 = \frac{}{3}$.
17. $4 = \frac{8}{}$.
18. $5 = \frac{}{2}$.
19. $5 = \frac{15}{}$.
20. $6 = \frac{}{3}$.
21. $7 = \frac{14}{}$.
22. $8 = \frac{24}{}$.
23. $10 = \frac{20}{}$.
24. $12 = \frac{}{2}$.
25. $16 = \frac{}{2}$.
26. $20 = \frac{}{2}$.
27. $30 = \frac{60}{}$.

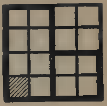
Read these fractions, reducing them to lowest terms; that is, to the form most convenient for use:

28. $\frac{2}{4}$ $\frac{3}{6}$ $\frac{2}{8}$ $\frac{2}{6}$ $\frac{3}{9}$ $\frac{2}{12}$ $\frac{3}{12}$ $\frac{4}{12}$ $\frac{8}{12}$ $\frac{6}{12}$
29. $\frac{4}{8}$ $\frac{6}{8}$ $\frac{2}{10}$ $\frac{4}{10}$ $\frac{8}{10}$ $\frac{5}{10}$ $\frac{6}{10}$ $\frac{10}{12}$ $\frac{2}{16}$ $\frac{8}{16}$
30. Which is the larger, $\frac{3}{4}$ or $\frac{5}{8}$? $\frac{1}{3}$ or $\frac{1}{2}$? $\frac{2}{3}$ or $\frac{1}{2}$?

MULTIPLYING A NUMERATOR

All work oral

1. Which is greater, \$3 or \$1? How many times as great is it?



2. In this square, $\frac{3}{4}$ of the square is how many times as great as $\frac{1}{4}$ of the square?

3. Multiplying the numerator of $\frac{1}{4}$ by 3 multiplies the fraction by what number?

4. Read and learn this statement:

Multiplying the numerator by any number multiplies the fraction by that number.

5. The shaded part of the above square is what part of the square? $\frac{12}{16}$ is how many times $\frac{1}{16}$? Multiplying the numerator of $\frac{1}{16}$ by 12 does what to the fraction?

In each of these cases state the number by which the numerator of the first fraction was multiplied to make the numerator of the second, and tell what this did to the fraction:

6. $\frac{1}{2}$, $\frac{3}{2}$.
7. $\frac{1}{4}$, $\frac{3}{4}$.
8. $\frac{1}{4}$, $\frac{5}{4}$.
9. $\frac{3}{4}$, $\frac{9}{4}$.
10. $\frac{3}{4}$, $\frac{15}{4}$.
11. $\frac{1}{8}$, $\frac{5}{8}$.
12. $\frac{1}{8}$, $\frac{7}{8}$.
13. $\frac{3}{8}$, $\frac{9}{8}$.
14. $\frac{3}{8}$, $\frac{15}{8}$.
15. $\frac{3}{8}$, $\frac{21}{8}$.
16. $\frac{5}{8}$, $\frac{15}{8}$.
17. $\frac{5}{8}$, $\frac{25}{8}$.
18. $\frac{7}{8}$, $\frac{21}{8}$.
19. $\frac{7}{8}$, $\frac{35}{8}$.
20. $\frac{7}{8}$, $\frac{49}{8}$.
21. $\frac{1}{16}$, $\frac{3}{16}$.
22. $\frac{1}{16}$, $\frac{7}{16}$.
23. $\frac{3}{16}$, $\frac{15}{16}$.
24. $\frac{5}{16}$, $\frac{45}{16}$.
25. $\frac{7}{16}$, $\frac{35}{16}$.

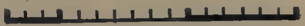
26. By what should you multiply the numerator of the fraction $\frac{3}{32}$ to multiply the fraction by 5? by 7? by 15?

27. How should you multiply the fraction $\frac{4}{5}$ by 3? by 7? by 9? by 23?

DIVIDING A NUMERATOR

All work oral

1. This line has been divided into how many equal parts? The first of the two black portions is $\frac{3}{16}$ of the line. The second is how many sixteenths of the line?



2. In Ex. 1, $\frac{3}{16}$ is what part as long as $\frac{6}{16}$? Dividing the numerator of $\frac{6}{16}$ by 2 does what to the fraction?

3. Read and learn this statement:

Dividing the numerator by any number divides the fraction by that number.

4. If you divide the numerator of the fraction $\frac{9}{16}$ by 3, what does the fraction become? By what number have you divided the fraction?

In each of these cases state the number by which the numerator of the first fraction was divided to make the numerator of the second, and tell what this did to the fraction:

5. $\frac{4}{5}$, $\frac{2}{5}$.
6. $\frac{4}{5}$, $\frac{1}{5}$.
7. $\frac{2}{3}$, $\frac{1}{3}$.
8. $\frac{3}{4}$, $\frac{1}{4}$.
9. $\frac{3}{8}$, $\frac{1}{8}$.
10. $\frac{6}{8}$, $\frac{1}{8}$.
11. $\frac{6}{8}$, $\frac{3}{8}$.
12. $\frac{6}{8}$, $\frac{2}{8}$.
13. $\frac{4}{12}$, $\frac{1}{12}$.
14. $\frac{6}{12}$, $\frac{1}{12}$.
15. $\frac{5}{8}$, $\frac{1}{8}$.
16. $\frac{5}{16}$, $\frac{1}{16}$.
17. $\frac{9}{16}$, $\frac{3}{16}$.
18. $\frac{9}{16}$, $\frac{1}{16}$.
19. $\frac{15}{16}$, $\frac{5}{16}$.
20. $\frac{15}{16}$, $\frac{3}{16}$.
21. $\frac{15}{16}$, $\frac{1}{16}$.
22. $\frac{9}{32}$, $\frac{3}{32}$.
23. $\frac{15}{32}$, $\frac{5}{32}$.
24. $\frac{21}{32}$, $\frac{3}{32}$.

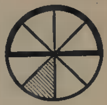
25. By what should you divide the numerator of the fraction $\frac{27}{32}$ to divide the fraction by 9? By what should you multiply the numerator of the fraction $\frac{3}{4}$ to multiply the fraction by 9?

26. How should you divide the fraction $\frac{3}{8}$ by 3?

MULTIPLYING A DENOMINATOR

All work oral

1. How many eighths of this circle are there in half of the circle? Then $\frac{1}{8}$ is what part of $\frac{1}{2}$? If we divide $\frac{1}{2}$ of the circle by 4, we have what part of the circle? What is the effect of multiplying the denominator of $\frac{1}{2}$ by 4?



2. Multiplying the denominator of any fraction by 4 does what to the fraction itself?

3. Read and learn this statement:

Multiplying the denominator by any number divides the fraction by that number.

4. Look at $\frac{1}{4}$ of the above circle. If you multiply the denominator by 2, what does the fraction become? This is what part of $\frac{1}{4}$?

In each of these cases state the number by which the denominator of the first fraction was multiplied to make the denominator of the second, and tell what this did to the fraction:

5. $\frac{1}{2}$, $\frac{1}{8}$.
6. $\frac{1}{2}$, $\frac{1}{6}$.
7. $\frac{1}{2}$, $\frac{1}{10}$.
8. $\frac{1}{2}$, $\frac{1}{12}$.
9. $\frac{1}{2}$, $\frac{1}{16}$.
10. $\frac{1}{3}$, $\frac{1}{6}$.
11. $\frac{1}{3}$, $\frac{1}{18}$.
12. $\frac{1}{3}$, $\frac{1}{15}$.
13. $\frac{2}{3}$, $\frac{2}{15}$.
14. $\frac{2}{3}$, $\frac{2}{9}$.
15. $\frac{1}{4}$, $\frac{1}{8}$.
16. $\frac{1}{4}$, $\frac{1}{16}$.
17. $\frac{1}{4}$, $\frac{1}{32}$.
18. $\frac{3}{4}$, $\frac{3}{32}$.
19. $\frac{3}{4}$, $\frac{3}{64}$.
20. $\frac{1}{8}$, $\frac{1}{16}$.
21. $\frac{1}{8}$, $\frac{1}{32}$.
22. $\frac{1}{8}$, $\frac{1}{64}$.
23. $\frac{3}{8}$, $\frac{3}{32}$.
24. $\frac{3}{8}$, $\frac{3}{16}$.

25. By what should you multiply the denominator of the fraction $\frac{7}{8}$ to divide the fraction by 5? by 4? by 6?

26. How should you divide the fraction $\frac{4}{5}$ by 3? by 9?

DIVIDING A DENOMINATOR

All work oral

1. Half of an apple is how many times $\frac{1}{4}$ of the apple? Dividing the denominator of $\frac{1}{4}$ by 2 does what to the fraction? Draw a figure on the board.

2. In the same way, dividing the denominator of the fraction $\frac{3}{8}$ by 4 gives what fraction? This is how many times the fraction $\frac{3}{8}$? Draw a figure on the board.

3. Read and learn this statement:

Dividing the denominator by any number multiplies the fraction by that number.

4. If you divide the denominator of the fraction $\frac{9}{16}$ by 4, what does the fraction become? By what number have you multiplied the fraction?

In each of these cases state the number by which the denominator of the first fraction was divided to make the denominator of the second, and tell what this did to the fraction:

5. $\frac{1}{4}$, $\frac{1}{2}$.
6. $\frac{3}{4}$, $\frac{3}{2}$.
7. $\frac{1}{8}$, $\frac{1}{2}$.
8. $\frac{1}{8}$, $\frac{1}{4}$.
9. $\frac{3}{8}$, $\frac{3}{4}$.
10. $\frac{3}{8}$, $\frac{3}{2}$.
11. $\frac{1}{16}$, $\frac{1}{8}$.
12. $\frac{1}{16}$, $\frac{1}{2}$.
13. $\frac{1}{16}$, $\frac{1}{4}$.
14. $\frac{3}{16}$, $\frac{3}{8}$.
15. $\frac{3}{16}$, $\frac{3}{4}$.
16. $\frac{3}{16}$, $\frac{3}{2}$.
17. $\frac{1}{32}$, $\frac{1}{8}$.
18. $\frac{1}{32}$, $\frac{1}{16}$.
19. $\frac{1}{32}$, $\frac{1}{2}$.
20. $\frac{3}{32}$, $\frac{3}{4}$.
21. $\frac{3}{32}$, $\frac{3}{8}$.
22. $\frac{5}{32}$, $\frac{5}{2}$.
23. $\frac{25}{32}$, $\frac{25}{16}$.
24. $\frac{25}{64}$, $\frac{25}{32}$.

25. By what should you divide the denominator of the fraction $\frac{3}{64}$ to multiply the fraction by 8? By what should you multiply the numerator of the fraction $\frac{3}{64}$ to multiply the fraction by 8?

A SILENT READING LESSON

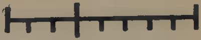
We add and subtract only like things; that is, we say

3 apples + 5 apples = 8 apples

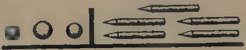
or 8 apples − 3 apples = 5 apples.

Similarly, $\frac{3}{8}+\frac{5}{8}=\frac{8}{8}$,

and $\frac{8}{8}-\frac{3}{8}=\frac{5}{8}$.



When we say 3 marbles + 5 pencils = 8 things, we give the name "thing" to each object. In adding $\frac{3}{4}$ and $\frac{5}{8}$ we must give them the same name; that is, the same denominator. We generally make this denominator as small as we can; that is, we use the *least common denominator.*



Thus, in adding $\frac{3}{4}$ and $\frac{5}{8}$, we write each fraction as eighths.

We then add as here shown.

$$\boxed{\begin{array}{rcl} \frac{3}{4} & = & \frac{6}{8} \\ \frac{5}{8} & = & \frac{5}{8} \\ \hline & & \frac{11}{8} = 1\frac{3}{8} \end{array}}$$

Since $\frac{11}{8}=1\frac{3}{8}$, the result is $1\frac{3}{8}$.

When John reduced to lowest terms three measurements which he had taken, he had $\frac{1}{4}$ ft., $\frac{3}{8}$ ft., and $\frac{5}{12}$ ft.

If he wishes to add these measurements, he must choose the same name (denominator) for the fractions.

He cannot reduce 4ths, 8ths, and 12ths to 12ths, but he can reduce them to 24ths. He therefore writes

$$\begin{array}{rcl} \frac{1}{4} & = & \frac{6}{24} \\ \frac{3}{8} & = & \frac{9}{24} \\ \frac{5}{12} & = & \frac{10}{24} \\ \hline & & \frac{25}{24} = 1\frac{1}{24}. \end{array}$$

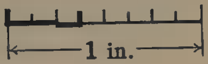
That is, the result that John finds is $1\frac{1}{24}$ ft.

Adding Fractions. Let us now have a little talk about some examples in adding fractions. You will then have no great difficulty with your work in addition.

1. Add $\frac{1}{4}$ in. and $\frac{1}{8}$ in.

Look at this picture of a 1-inch line divided into eighths.

How many eighths are there in $\frac{1}{4}$?



If to this we add $\frac{1}{8}$, how many eighths do we then have?

Copy and complete this statement: $\frac{1}{4}+\frac{1}{8}=\frac{}{8}+\frac{1}{8}=\frac{}{8}$.

2. Add $\frac{3}{4}$ in. and $\frac{5}{16}$ in.

What name (denominator) shall we use?

Write $\frac{3}{4}$ with this denominator.

Write $\frac{5}{16}$ with this denominator, if necessary.

Adding these two fractions, what is the result?

Write the result as a mixed number.

3. Add $\frac{1}{2}$ and $\frac{2}{3}$.

What name (denominator) shall we use?

Write $\frac{1}{2}$ with this denominator.

Write $\frac{2}{3}$ with this denominator.

What is the result?

4. Add $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{1}{3}$.

What is the smallest number which contains the numbers 4, 8, and 3 exactly?

Then what name (denominator) shall we use?

Then $\frac{3}{4}$ is how many twenty-fourths? $\frac{5}{8}$ is how many twenty-fourths? $\frac{1}{3}$ is how many twenty-fourths?

How many twenty-fourths do we have in all?

Is this reduced to lowest terms? What is the result?

Express the result as a mixed number.

ADDING FRACTIONS

Numbers 1 to 6, oral

1. Express $\frac{1}{4}$ as eighths. To the result add $\frac{5}{8}$.

2. If you fasten a strip of molding $\frac{1}{4}$ in. thick to a strip of wood $\frac{5}{8}$ in. thick, how thick are the two together?

3. Express $\frac{4}{5}$ as tenths. To the result add $\frac{3}{10}$.

4. If you lay a book $\frac{4}{5}$ in. thick on a notebook $\frac{3}{10}$ in. thick, how thick are the two together?

5. Express $\frac{2}{3}$ as sixths. To the result add $\frac{1}{6}$.

6. If you lay a piece of cloth $\frac{2}{3}$ yd. wide beside a piece $\frac{1}{6}$ yd. wide, how wide are the two together?

7. Some plaster $\frac{3}{8}$ in. thick is coated with a finer plaster $\frac{3}{16}$ in. thick. How thick is the plaster now?

8. A plate of brass $\frac{1}{32}$ in. thick is laid on a plate of iron $\frac{3}{16}$ in. thick. What is the total thickness?

9. An iron rod $\frac{15}{16}$ in. in diameter is plated all over with brass $\frac{1}{32}$ in. thick. What is then the diameter?

10. A piece of cardboard $\frac{1}{64}$ in. thick is laid on a book $\frac{5}{8}$ in. thick. How thick are the two together?

Add the following fractions:

11. $\frac{3}{4}$, $\frac{7}{24}$.
12. $\frac{3}{4}$, $\frac{13}{24}$.
13. $\frac{3}{8}$, $\frac{13}{48}$.
14. $\frac{7}{8}$, $\frac{23}{48}$.
15. $\frac{3}{4}$, $\frac{1}{48}$.
16. $\frac{3}{4}$, $\frac{7}{48}$.
17. $\frac{5}{16}$, $\frac{7}{24}$.
18. $\frac{3}{16}$, $\frac{11}{24}$.

19. Add $\frac{1}{2}$ in., $\frac{1}{3}$ in., $\frac{2}{3}$ in., $\frac{1}{4}$ in., $\frac{1}{6}$ in., and $\frac{1}{12}$ in.

Add the following fractions as indicated:

20. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$.
21. $\frac{1}{2} + \frac{3}{4} + \frac{1}{8}$.
22. $\frac{1}{3} + \frac{1}{4} + \frac{1}{12}$.
23. $\frac{2}{3} + \frac{3}{4} + \frac{1}{12}$.
24. $\frac{1}{3} + \frac{1}{8} + \frac{1}{12}$.
25. $\frac{2}{3} + \frac{5}{8} + \frac{1}{12}$.

Adding Mixed Numbers. 1. If Kate needs $5\frac{2}{3}$ yd. of cloth for one piece of work and $3\frac{2}{3}$ yd. for another, how much cloth does she need in all?

First, $\frac{2}{3} + \frac{2}{3} = \frac{4}{3} = 1\frac{1}{3}$.

Write $\frac{1}{3}$ under the fractions and add 1 to units.

Then $1 + 3 + 5 = 9$.

That is, the result is $9\frac{1}{3}$ yd.

$$\boxed{\begin{array}{r} 5\frac{2}{3} \\ 3\frac{2}{3} \\ \hline 9\frac{1}{3} \end{array}}$$

2. Each of three girls, Rose, May, and Edith, is making a dress. Rose uses $2\frac{1}{2}$ yd. of cloth, May $4\frac{2}{3}$ yd., and Edith $3\frac{3}{4}$ yd. How much cloth do they use in all?

Write the fractions as 12ths, because 12 is the smallest number that contains 2, 3, and 4.

Adding, the result is $9\frac{23}{12}$ yd., or, in simpler form, $10\frac{11}{12}$ yd.

$$\boxed{\begin{array}{lcl} 2\frac{1}{2} & = & 2\frac{6}{12} \\ 4\frac{2}{3} & = & 4\frac{8}{12} \\ 3\frac{3}{4} & = & 3\frac{9}{12} \\ \cline{3-3} & & 9\frac{23}{12} = 10\frac{11}{12} \end{array}}$$

ADDING MIXED NUMBERS

1. Sarah uses $\frac{7}{8}$ yd. of cloth for a dress for one of her dolls and $\frac{1}{2}$ yd. for another. She also uses $\frac{3}{8}$ yd. in making a jacket for the larger doll. How much cloth does she use in all?

2. Making no allowance for doors and windows, how many feet of picture molding will be needed to go round a classroom $22\frac{3}{4}$ ft. long and $18\frac{1}{2}$ ft. wide?

3. James has a wooden box. The cover and bottom of the box are made of $\frac{5}{8}$-inch boards, and the inside depth is $14\frac{1}{4}$ in. What is the total height of the box?

DRILL CHART IN ADDING FRACTIONS

Copy and add the following as rapidly as you can:

1. $\frac{1}{2}$ $\frac{3}{4}$	**6.** $\frac{1}{4}$ $\frac{1}{8}$	**11.** $1\frac{1}{2}$ $1\frac{7}{8}$	**16.** $3\frac{3}{4}$ $2\frac{5}{8}$	**21.** $5\frac{9}{16}$ $3\frac{5}{8}$	**26.** $1\frac{1}{3}$ $1\frac{1}{2}$
2. $\frac{1}{2}$ $\frac{3}{8}$	**7.** $\frac{3}{4}$ $\frac{1}{8}$	**12.** $2\frac{1}{2}$ $3\frac{5}{8}$	**17.** $2\frac{3}{4}$ $3\frac{7}{8}$	**22.** $2\frac{11}{16}$ $3\frac{5}{8}$	**27.** $2\frac{2}{3}$ $3\frac{1}{2}$
3. $\frac{1}{2}$ $\frac{7}{8}$	**8.** $\frac{1}{4}$ $\frac{3}{8}$	**13.** $3\frac{1}{2}$ $2\frac{3}{8}$	**18.** $2\frac{1}{16}$ $1\frac{1}{2}$	**23.** $3\frac{11}{16}$ $4\frac{7}{8}$	**28.** $2\frac{1}{3}$ $3\frac{3}{4}$
4. $\frac{1}{2}$ $\frac{5}{8}$	**9.** $\frac{1}{4}$ $\frac{5}{8}$	**14.** $4\frac{1}{2}$ $3\frac{1}{8}$	**19.** $2\frac{3}{16}$ $3\frac{5}{8}$	**24.** $5\frac{13}{16}$ $2\frac{5}{8}$	**29.** $3\frac{2}{3}$ $2\frac{3}{4}$
5. $\frac{1}{2}$ $\frac{1}{8}$	**10.** $\frac{3}{4}$ $\frac{7}{8}$	**15.** $2\frac{1}{4}$ $3\frac{5}{8}$	**20.** $3\frac{7}{16}$ $2\frac{5}{8}$	**25.** $3\frac{15}{16}$ $3\frac{7}{8}$	**30.** $3\frac{2}{3}$ $2\frac{7}{8}$

Copy and add the following:

31. $4\frac{1}{2}$ $\frac{1}{2}$	**33.** $3\frac{1}{3}$ $2\frac{2}{3}$	**35.** $12\frac{2}{3}$ $2\frac{1}{3}$	**37.** $26\frac{2}{3}$ $15\frac{2}{3}$	**39.** $18\frac{1}{3}$ $8\frac{3}{8}$	**41.** $32\frac{7}{8}$ $5\frac{9}{16}$
32. 3 $3\frac{1}{2}$	**34.** $4\frac{1}{2}$ $3\frac{3}{4}$	**36.** $12\frac{1}{2}$ $36\frac{1}{4}$	**38.** $29\frac{1}{4}$ $4\frac{1}{6}$	**40.** $22\frac{7}{8}$ $5\frac{1}{2}$	**42.** $15\frac{3}{8}$ $3\frac{5}{16}$

Copy and add the following:

43. $2\frac{1}{2}+\frac{3}{4}+\frac{5}{8}$. **46.** $3\frac{1}{2}+3\frac{3}{4}+2\frac{5}{8}$. **49.** $2\frac{3}{4}+3\frac{3}{8}+3\frac{9}{16}$.

44. $3\frac{1}{3}+\frac{5}{6}+\frac{1}{2}$. **47.** $3\frac{2}{3}+1\frac{5}{6}+\frac{1}{2}$. **50.** $3\frac{3}{4}+2\frac{7}{8}+7\frac{9}{16}$.

45. $2\frac{7}{8}+\frac{2}{3}+\frac{3}{4}$. **48.** $2\frac{3}{8}+3\frac{3}{4}+\frac{5}{16}$. **51.** $2\frac{5}{8}+3\frac{1}{2}+4\frac{15}{16}$.

Subtracting Fractions. 1. Edith and Emily walk home together after school. Edith's home is $\frac{1}{2}$ mi. from the school, and Emily's is $\frac{3}{10}$ mi. from it. How far is it between their homes?

To take $\frac{3}{10}$ mi. from $\frac{1}{2}$ mi., use the denominator 10.

The result is $\frac{2}{10}$, or $\frac{1}{5}$; that is, it is $\frac{1}{5}$ mi. between their homes.

$$\boxed{\begin{array}{rcll} \frac{1}{2} & = & \frac{5}{10} & \\ \frac{3}{10} & = & \underline{\frac{3}{10}} & \\ & & \frac{2}{10} & = \frac{1}{5} \end{array}}$$

2. Walter saws a strip $4\frac{3}{8}$ in. wide from a board 14 in. wide. How wide is the part of the board that is left?

First, think of 14 as $13\frac{8}{8}$.

Then subtract $4\frac{3}{8}$ from $13\frac{8}{8}$.

The result is $9\frac{5}{8}$; that is, the board that is left is $9\frac{5}{8}$ in. wide.

$$\boxed{\begin{array}{rcr} 14 & = & 13\frac{8}{8} \\ 4\frac{3}{8} & = & \underline{4\frac{3}{8}} \\ & & 9\frac{5}{8} \end{array}}$$

SUBTRACTING FRACTIONS

1. If Jenny has $\frac{5}{8}$ yd. of ribbon and uses $\frac{1}{2}$ yd. in trimming some sleeves, how much ribbon has she left?

2. If you live $\frac{3}{4}$ mi. from school, how far have you still to go when you have walked $\frac{3}{8}$ mi. toward home?

3. In making a bird house, Fred saws a piece $4\frac{7}{8}$ in. long from a board 16 in. long. How much is left?

4. Gladys has a piece of ribbon 34 in. long and uses $18\frac{1}{2}$ in. in trimming a bag. How much ribbon has she left?

5. On an automobile trip of 54 mi., Jack stops to fix a tire after going $9\frac{3}{4}$ mi. How much farther has he still to go?

6. How much is left after taking $4\frac{7}{8}$ gal. from 10 gal.?

Subtracting from a Mixed Number. From $23\frac{1}{8}$ in. of wire, James uses $8\frac{3}{4}$ in. for a toy airplane. How much wire has he left?

First, think of $23\frac{1}{8}$ as $22\frac{9}{8}$.

Then think of $8\frac{3}{4}$ as $8\frac{6}{8}$, and subtract it from $22\frac{9}{8}$ in the usual way.

The result is $14\frac{3}{8}$; that is, James has $14\frac{3}{8}$ in. of wire left.

$$\boxed{\begin{array}{rcr} 23\frac{1}{8} & = & 22\frac{9}{8} \\ 8\frac{3}{4} & = & \underline{8\frac{6}{8}} \\ & & 14\frac{3}{8} \end{array}}$$

SUBTRACTING FROM A MIXED NUMBER

1. If Minnie has $32\frac{1}{4}$ in. of cloth and uses $9\frac{3}{4}$ in., how much cloth has she left?

2. If one book is $8\frac{1}{4}$ in. long, and another is $5\frac{13}{16}$ in. long, what is the difference between their lengths?

3. If Frank has a piece of board $16\frac{1}{8}$ in. long and needs a piece only $13\frac{3}{4}$ in. long, how much must he cut off?

Subtract as follows:

4. $2\frac{5}{8}$ $\underline{\frac{3}{8}}$	**8.** $8\frac{3}{8}$ $\underline{\frac{3}{4}}$	**12.** $9\frac{2}{5}$ $\underline{8\frac{3}{8}}$	**16.** $9\frac{7}{12}$ $\underline{8\frac{1}{2}}$	**20.** $6\frac{1}{2}$ $\underline{4\frac{3}{4}}$	**24.** $5\frac{5}{12}$ $\underline{\frac{5}{6}}$
5. $6\frac{3}{8}$ $\underline{\frac{3}{4}}$	**9.** $9\frac{1}{4}$ $\underline{3\frac{5}{8}}$	**13.** $9\frac{5}{6}$ $\underline{8}$	**17.** $9\frac{2}{3}$ $\underline{2\frac{4}{5}}$	**21.** $6\frac{3}{4}$ $\underline{4\frac{1}{2}}$	**25.** $8\frac{5}{12}$ $\underline{\frac{7}{8}}$
6. $9\frac{1}{2}$ $\underline{4\frac{1}{3}}$	**10.** $8\frac{3}{4}$ $\underline{7}$	**14.** $6\frac{1}{4}$ $\underline{3\frac{5}{8}}$	**18.** $8\frac{1}{5}$ $\underline{6\frac{5}{8}}$	**22.** $7\frac{1}{8}$ $\underline{2\frac{1}{2}}$	**26.** $9\frac{15}{16}$ $\underline{1\frac{1}{8}}$
7. $2\frac{7}{8}$ $\underline{1\frac{1}{8}}$	**11.** $6\frac{5}{8}$ $\underline{3\frac{1}{4}}$	**15.** $6\frac{1}{3}$ $\underline{4\frac{3}{4}}$	**19.** $5\frac{1}{2}$ $\underline{2\frac{1}{4}}$	**23.** $7\frac{1}{2}$ $\underline{2\frac{1}{8}}$	**27.** $9\frac{1}{8}$ $\underline{1\frac{15}{16}}$

DRILL CHART IN FRACTIONS

Copy, add, and check each result:

1.	2.	3.	4.	5.	6.
$2\frac{1}{4}$	$2\frac{2}{3}$	$3\frac{1}{2}$	$2\frac{2}{5}$	$1\frac{3}{8}$	$3\frac{2}{9}$
$2\frac{2}{3}$	$3\frac{3}{4}$	$2\frac{1}{3}$	$4\frac{1}{3}$	$1\frac{5}{16}$	$3\frac{1}{3}$
$3\frac{1}{2}$	$4\frac{1}{3}$	$2\frac{2}{3}$	$3\frac{1}{6}$	$2\frac{3}{4}$	$7\frac{5}{6}$
$4\frac{1}{12}$	$5\frac{7}{12}$	$3\frac{5}{6}$	$5\frac{4}{5}$	$7\frac{1}{2}$	$5\frac{1}{2}$

7. From $14\frac{7}{8} + 23\frac{3}{4}$ take $9\frac{1}{2} + 7\frac{11}{16}$.

Copy, add, and check each result:

8.	9.	10.	11.	12.	13.
$3\frac{1}{2}$	$5\frac{2}{3}$	$3\frac{1}{8}$	$1\frac{1}{5}$	$2\frac{3}{4}$	$5\frac{1}{5}$
$4\frac{2}{3}$	$4\frac{1}{6}$	$4\frac{3}{4}$	$8\frac{3}{4}$	$5\frac{7}{8}$	$2\frac{2}{3}$
$2\frac{3}{4}$	$7\frac{3}{4}$	$9\frac{1}{4}$	$6\frac{5}{8}$	$3\frac{1}{2}$	$3\frac{5}{6}$
$5\frac{3}{4}$	$5\frac{7}{8}$	$6\frac{7}{8}$	$9\frac{1}{2}$	$4\frac{3}{8}$	$4\frac{3}{5}$
$7\frac{1}{12}$	$9\frac{5}{6}$	$4\frac{1}{2}$	$4\frac{9}{10}$	$9\frac{1}{4}$	$2\frac{1}{2}$

Subtract as follows:

14.	18.	22.	26.	30.	34.
$\frac{7}{8}$	$\frac{3}{4}$	$\frac{3}{5}$	$3\frac{1}{2}$	$5\frac{1}{2}$	$12\frac{1}{2}$
$\frac{1}{4}$	$\frac{5}{12}$	$\frac{1}{2}$	$2\frac{1}{3}$	$4\frac{1}{4}$	$\frac{3}{8}$

15.	19.	23.	27.	31.	35.
$\frac{5}{8}$	$\frac{5}{12}$	$\frac{7}{8}$	$5\frac{1}{4}$	$5\frac{1}{4}$	$12\frac{3}{8}$
$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{4}$	$2\frac{3}{8}$	$4\frac{1}{2}$	$9\frac{1}{2}$

16.	20.	24.	28.	32.	36.
$\frac{1}{2}$	$\frac{5}{6}$	$\frac{7}{8}$	8	$6\frac{7}{8}$	$15\frac{2}{3}$
$\frac{1}{12}$	$\frac{2}{3}$	$\frac{2}{3}$	$6\frac{2}{5}$	$5\frac{3}{4}$	$5\frac{1}{2}$

17.	21.	25.	29.	33.	37.
$\frac{7}{8}$	$\frac{1}{5}$	$\frac{2}{3}$	9	$6\frac{3}{4}$	$25\frac{1}{2}$
$\frac{1}{24}$	$\frac{1}{6}$	$\frac{1}{2}$	$7\frac{5}{8}$	$5\frac{7}{8}$	$6\frac{2}{3}$

SPEED TEST CHART IN FRACTIONS

In Exs. 1–30, see how many you can copy, subtract, and check in 10 min. or less:

1. $\begin{array}{r}8\frac{3}{4}\\ \underline{2\frac{1}{2}}\end{array}$

2. $\begin{array}{r}9\frac{5}{8}\\ \underline{3\frac{1}{2}}\end{array}$

3. $\begin{array}{r}2\\ \underline{\frac{1}{4}}\end{array}$

4. $\begin{array}{r}10\\ \underline{\frac{7}{8}}\end{array}$

5. $\begin{array}{r}12\\ \underline{\frac{7}{16}}\end{array}$

6. $\begin{array}{r}30\\ \underline{\frac{11}{12}}\end{array}$

7. $\begin{array}{r}7\frac{1}{2}\\ \underline{3\frac{3}{8}}\end{array}$

8. $\begin{array}{r}8\frac{5}{8}\\ \underline{3\frac{1}{4}}\end{array}$

9. $\begin{array}{r}4\frac{1}{2}\\ \underline{1\frac{1}{4}}\end{array}$

10. $\begin{array}{r}6\frac{1}{2}\\ \underline{2\frac{5}{8}}\end{array}$

11. $\begin{array}{r}7\frac{1}{2}\\ \underline{4\frac{3}{16}}\end{array}$

12. $\begin{array}{r}7\frac{1}{4}\\ \underline{5\frac{11}{16}}\end{array}$

13. $\begin{array}{r}5\frac{7}{8}\\ \underline{2\frac{1}{4}}\end{array}$

14. $\begin{array}{r}9\frac{2}{3}\\ \underline{4\frac{1}{6}}\end{array}$

15. $\begin{array}{r}\frac{19}{24}\\ \underline{\frac{2}{3}}\end{array}$

16. $\begin{array}{r}\frac{7}{10}\\ \underline{\frac{1}{3}}\end{array}$

17. $\begin{array}{r}2\frac{7}{8}\\ \underline{\frac{1}{16}}\end{array}$

18. $\begin{array}{r}3\frac{7}{8}\\ \underline{\frac{13}{16}}\end{array}$

19. $\begin{array}{r}9\frac{2}{3}\\ \underline{5\frac{3}{4}}\end{array}$

20. $\begin{array}{r}7\frac{1}{4}\\ \underline{3\frac{2}{3}}\end{array}$

21. $\begin{array}{r}2\frac{7}{8}\\ \underline{\frac{17}{32}}\end{array}$

22. $\begin{array}{r}3\frac{1}{8}\\ \underline{\frac{5}{32}}\end{array}$

23. $\begin{array}{r}2\frac{5}{6}\\ \underline{\frac{2}{3}}\end{array}$

24. $\begin{array}{r}7\frac{1}{6}\\ \underline{\frac{2}{3}}\end{array}$

25. $\begin{array}{r}22\frac{1}{3}\\ \underline{2\frac{1}{5}}\end{array}$

26. $\begin{array}{r}34\frac{2}{3}\\ \underline{6\frac{4}{5}}\end{array}$

27. $\begin{array}{r}12\frac{3}{4}\\ \underline{7\frac{7}{8}}\end{array}$

28. $\begin{array}{r}16\frac{3}{8}\\ \underline{2\frac{3}{4}}\end{array}$

29. $\begin{array}{r}19\frac{3}{16}\\ \underline{14\frac{3}{4}}\end{array}$

30. $\begin{array}{r}32\frac{11}{16}\\ \underline{18\frac{5}{8}}\end{array}$

31. Add $14\frac{1}{2}$ in., $23\frac{3}{4}$ in., $6\frac{1}{4}$ in., $22\frac{7}{8}$ in., and $14\frac{3}{8}$ in.

Copy, add, and check in less than 4 min.:

32. $\begin{array}{r}3\frac{3}{4}\\ 1\frac{1}{2}\\ 2\frac{11}{12}\\ 5\frac{1}{4}\\ \underline{6\frac{5}{12}}\end{array}$

33. $\begin{array}{r}8\frac{2}{3}\\ 9\frac{1}{6}\\ 8\frac{1}{3}\\ 4\frac{5}{6}\\ \underline{3\frac{1}{3}}\end{array}$

34. $\begin{array}{r}6\frac{1}{4}\\ 3\frac{1}{2}\\ 2\frac{1}{8}\\ 6\frac{3}{8}\\ \underline{7\frac{5}{8}}\end{array}$

35. $\begin{array}{r}9\frac{1}{2}\\ 2\frac{1}{3}\\ 3\frac{3}{4}\\ 8\frac{7}{8}\\ \underline{1\frac{5}{6}}\end{array}$

36. $\begin{array}{r}18\frac{1}{2}\\ 2\frac{9}{16}\\ 23\frac{7}{8}\\ 29\frac{3}{4}\\ \underline{38\frac{5}{16}}\end{array}$

Multiplying by a Whole Number, without Canceling. If Tom has a quarter of a dollar and Jane has three times as much, how much has Jane?

Just as 3 times 1 in. is 3 in.,

so 3 times 1 quarter is 3 quarters,

or $3 \times \frac{1}{4} = \frac{3}{4}$.

That is, Jane has 3 quarters of a dollar, or $\$\frac{3}{4}$.

To multiply a fraction by a whole number, multiply the numerator by the whole number.

MULTIPLYING BY A WHOLE NUMBER

Multiply rapidly as follows, reducing each result to a mixed number when possible:

1. $3 \times \frac{1}{2}$.
2. $5 \times \frac{1}{2}$.
3. $7 \times \frac{1}{2}$.
4. $9 \times \frac{1}{2}$.
5. $11 \times \frac{1}{2}$.
6. $15 \times \frac{1}{2}$.
7. $10 \times \frac{1}{3}$.
8. $11 \times \frac{1}{3}$.
9. $13 \times \frac{1}{3}$.
10. $14 \times \frac{1}{3}$.
11. $26 \times \frac{1}{3}$.
12. $20 \times \frac{1}{3}$.
13. $22 \times \frac{2}{3}$.
14. $14 \times \frac{2}{3}$.
15. $5 \times \frac{2}{3}$.
16. $7 \times \frac{2}{3}$.
17. $8 \times \frac{2}{3}$.
18. $5 \times \frac{1}{4}$.
19. $7 \times \frac{1}{4}$.
20. $9 \times \frac{1}{4}$.
21. $11 \times \frac{1}{4}$.
22. $13 \times \frac{1}{4}$.
23. $15 \times \frac{1}{4}$.
24. $17 \times \frac{1}{4}$.
25. $11 \times \frac{3}{4}$.
26. $13 \times \frac{3}{4}$.
27. $15 \times \frac{3}{4}$.
28. $21 \times \frac{3}{4}$.
29. $3 \times \frac{1}{8}$.
30. $3 \times \frac{3}{8}$.
31. $2 \times \frac{3}{4}$.
32. $3 \times \frac{3}{4}$.
33. $3 \times \frac{5}{8}$.
34. $3 \times \frac{7}{8}$.
35. $5 \times \frac{1}{8}$.
36. $5 \times \frac{3}{8}$.
37. $5 \times \frac{5}{8}$.
38. $5 \times \frac{7}{8}$.
39. $11 \times \frac{1}{8}$.
40. $11 \times \frac{3}{8}$.
41. $11 \times \frac{5}{8}$.
42. $11 \times \frac{7}{8}$.

Multiplying by a Whole Number, with Canceling. If you have 4 quarters, how many dollars have you?

In such a simple case we easily see that

$$4 \times \frac{1}{4} = \frac{\overset{1}{\cancel{4}}}{\cancel{4}} = 1.$$

That is, you have one dollar, or \$1.

Before actually multiplying, indicate the multiplication and, if possible, cancel common factors.

That is, in a case in which you have to multiply $\frac{7}{8}$ by 6, first indicate the multiplication by writing the work in this form:

$$\frac{6 \times 7}{8}.$$

Canceling, the result is $\frac{21}{4}$, or, in simpler form, $5\frac{1}{4}$.

$$\boxed{\frac{\overset{3}{\cancel{6}} \times 7}{\underset{4}{\cancel{8}}} = \frac{21}{4} = 5\frac{1}{4}}$$

MULTIPLYING BY A WHOLE NUMBER

Numbers 1 to 28, oral

Multiply rapidly as follows:

1. $2 \times \frac{1}{2}$.
2. $2 \times \frac{1}{4}$.
3. $4 \times \frac{1}{4}$.
4. $4 \times \frac{3}{4}$.
5. $2 \times \frac{3}{4}$.
6. $2 \times \frac{1}{8}$.
7. $4 \times \frac{1}{8}$.
8. $4 \times \frac{3}{8}$.
9. $8 \times \frac{5}{8}$.
10. $4 \times \frac{7}{8}$.
11. $3 \times \frac{1}{3}$.
12. $3 \times \frac{2}{3}$.
13. $6 \times \frac{1}{3}$.
14. $6 \times \frac{2}{3}$.
15. $9 \times \frac{1}{3}$.
16. $9 \times \frac{2}{3}$.
17. $4 \times \frac{1}{2}$.
18. $8 \times \frac{1}{2}$.
19. $8 \times \frac{1}{4}$.
20. $8 \times \frac{3}{4}$.
21. $6 \times \frac{1}{6}$.
22. $5 \times \frac{1}{5}$.
23. $5 \times \frac{2}{5}$.
24. $9 \times \frac{1}{3}$.
25. $4 \times \frac{5}{8}$.
26. $5 \times \frac{1}{10}$.
27. $5 \times \frac{3}{10}$.
28. $6 \times \frac{5}{6}$.

29. For some work which she is doing, Harriet needs 8 pieces of cloth, each $\frac{5}{8}$ yd. long. How many yards does she need in all?

30. In doing up some boxes of candy for a sale it was found that $\frac{1}{8}$ yd. of ribbon was needed for each box. If there were 24 boxes, how much ribbon was needed?

31. A class of 18 pupils made some ribbon badges, each 6 in. long. The length of each badge is what part of a yard? How many yards did they need for the 18 badges?

32. For a school sale the girls filled 48 boxes with salted almonds, each box containing 6 oz. of nuts. How many pounds of almonds did they use?

Multiply as follows, using cancellation when possible:

33. $6 \times \frac{2}{3}$.
34. $6 \times \frac{3}{4}$.
35. $6 \times \frac{7}{8}$.
36. $8 \times \frac{1}{2}$.
37. $8 \times \frac{2}{3}$.
38. $8 \times \frac{3}{4}$.
39. $9 \times \frac{1}{2}$.
40. $9 \times \frac{1}{3}$.
41. $9 \times \frac{2}{3}$.
42. $9 \times \frac{3}{4}$.
43. $3 \times \frac{7}{8}$.
44. $5 \times \frac{3}{5}$.
45. $5 \times \frac{3}{8}$.
46. $5 \times \frac{5}{8}$.
47. $5 \times \frac{5}{6}$.
48. $6 \times \frac{5}{8}$.
49. $5 \times \frac{7}{8}$.
50. $7 \times \frac{7}{8}$.
51. $8 \times \frac{7}{8}$.
52. $9 \times \frac{7}{8}$.
53. $4 \times \frac{7}{8}$.
54. $2 \times \frac{7}{8}$.
55. $12 \times \frac{3}{4}$.
56. $12 \times \frac{2}{3}$.
57. $12 \times \frac{3}{5}$.
58. $12 \times \frac{3}{8}$.
59. $16 \times \frac{3}{8}$.
60. $16 \times \frac{3}{4}$.
61. $16 \times \frac{3}{5}$.
62. $18 \times \frac{2}{3}$.
63. $18 \times \frac{5}{6}$.
64. $18 \times \frac{5}{8}$.
65. $18 \times \frac{7}{8}$.
66. $24 \times \frac{1}{2}$.
67. $24 \times \frac{1}{3}$.
68. $24 \times \frac{2}{3}$.
69. $24 \times \frac{3}{4}$.
70. $24 \times \frac{3}{8}$.
71. $36 \times \frac{7}{8}$.
72. $24 \times \frac{5}{16}$.
73. $24 \times \frac{5}{12}$.
74. $36 \times \frac{7}{16}$.
75. $42 \times \frac{5}{16}$.
76. $48 \times \frac{7}{16}$.
77. $48 \times \frac{15}{16}$.
78. $32 \times \frac{9}{16}$.
79. $32 \times \frac{11}{16}$.
80. $32 \times \frac{11}{32}$.
81. $64 \times \frac{1}{32}$.
82. $64 \times \frac{11}{32}$.
83. $64 \times \frac{15}{32}$.
84. $96 \times \frac{25}{32}$.

Multiplying a Mixed Number by a Whole Number. A grocer sells washing-soda in boxes that hold $1\frac{3}{4}$ lb. each. How many pounds of soda shall I get if I buy 6 boxes?

First, multiply $\frac{3}{4}$ by 6.

Then multiply 1 by 6.

Now add these two results.

The result, as shown in the work at the right, is $10\frac{1}{2}$; that is, I shall buy $10\frac{1}{2}$ lb. of soda.

$$\boxed{\begin{array}{lcr} & & 1\frac{3}{4} \\ & & \underline{6} \\ 6 \times \frac{3}{4} & = & 4\frac{1}{2} \\ 6 \times 1 & = & \underline{6} \\ 6 \times 1\frac{3}{4} & = & 10\frac{1}{2} \end{array}}$$

To multiply a mixed number by a whole number, first multiply the fractional part, then multiply the whole number, and add the two results.

MULTIPLYING A MIXED NUMBER

Multiply as follows:

1. $2 \times 1\frac{1}{2}$.
2. $2 \times 2\frac{1}{2}$.
3. $4 \times 1\frac{1}{2}$.
4. $3 \times 1\frac{1}{3}$.
5. $6 \times 3\frac{1}{3}$.
6. $8 \times 3\frac{3}{4}$.
7. $8 \times 2\frac{5}{8}$.
8. $9 \times 3\frac{7}{8}$.
9. $8 \times 2\frac{1}{2}$.
10. $8 \times 1\frac{1}{2}$.
11. $8 \times 5\frac{1}{2}$.
12. $9 \times 5\frac{1}{3}$.
13. $20 \times 1\frac{1}{2}$.
14. $30 \times 2\frac{1}{3}$.
15. $36 \times 4\frac{2}{3}$.
16. $24 \times 1\frac{3}{4}$.

17. At $27\frac{3}{4}$ mi. an hour, how far will an automobile go in 2 hr.? in 3 hr.? in 5 hr.?

Multiply as follows:

18. $24 \times 3\frac{1}{2}$.
19. $36 \times 3\frac{2}{3}$.
20. $48 \times 6\frac{3}{4}$.
21. $40 \times 5\frac{7}{8}$.
22. $96 \times 2\frac{1}{16}$.
23. $48 \times 3\frac{1}{12}$.
24. $48 \times 5\frac{5}{12}$.
25. $60 \times 5\frac{7}{12}$.
26. $72 \times 6\frac{11}{12}$.
27. $56 \times 5\frac{5}{8}$.
28. $64 \times 7\frac{7}{8}$.
29. $72 \times 7\frac{7}{8}$.

Fractional Part of a Whole Number. If there are 240 pupils in a certain school, and $\frac{3}{4}$ of them are in the primary grades, how many pupils are there in the primary grades?

Write the work in the form for canceling.

$$\frac{3}{4} \text{ of } 240 = \frac{3 \times \overset{60}{\cancel{240}}}{\cancel{4}} = 180$$

The result is 180; that is, there are 180 pupils in the primary grades.

To find a fractional part of a whole number, multiply the number by the fraction, canceling whenever possible.

FRACTIONAL PARTS

State rapidly the fractional parts indicated:

1. $\frac{1}{2}$ of 8.
2. $\frac{1}{2}$ of 16.
3. $\frac{1}{2}$ of 18.
4. $\frac{1}{2}$ of 22.
5. $\frac{1}{3}$ of 21.
6. $\frac{1}{3}$ of 27.
7. $\frac{1}{3}$ of 30.
8. $\frac{1}{4}$ of 36.
9. $\frac{1}{4}$ of 40.
10. $\frac{1}{5}$ of 40.
11. $\frac{1}{6}$ of 42.
12. $\frac{1}{7}$ of 42.
13. $\frac{1}{8}$ of 48.
14. $\frac{1}{3}$ of 24.
15. $\frac{2}{3}$ of 24.
16. $\frac{1}{3}$ of 33.
17. $\frac{2}{3}$ of 33.
18. $\frac{1}{3}$ of 36.
19. $\frac{2}{3}$ of 36.
20. $\frac{1}{3}$ of 39.
21. $\frac{2}{3}$ of 39.
22. $\frac{1}{3}$ of 60.
23. $\frac{2}{3}$ of 60.
24. $\frac{1}{3}$ of 66.
25. $\frac{2}{3}$ of 66.
26. $\frac{2}{3}$ of 90.
27. $\frac{1}{4}$ of 20.
28. $\frac{3}{4}$ of 20.
29. $\frac{1}{4}$ of 24.
30. $\frac{3}{4}$ of 24.
31. $\frac{1}{4}$ of 32.
32. $\frac{3}{4}$ of 32.
33. $\frac{1}{4}$ of 44.
34. $\frac{3}{4}$ of 40.
35. $\frac{1}{4}$ of 48.
36. $\frac{3}{4}$ of 48.
37. $\frac{1}{4}$ of 80.
38. $\frac{3}{4}$ of 80.
39. $\frac{3}{4}$ of 88.
40. $\frac{1}{5}$ of 15.
41. $\frac{2}{5}$ of 15.
42. $\frac{3}{5}$ of 15.
43. $\frac{4}{5}$ of 15.
44. $\frac{4}{5}$ of 25.
45. $\frac{1}{8}$ of 16.
46. $\frac{3}{8}$ of 16.
47. $\frac{5}{8}$ of 16.
48. $\frac{7}{8}$ of 16.
49. $\frac{3}{8}$ of 32.
50. $\frac{5}{8}$ of 32.
51. $\frac{7}{8}$ of 32.
52. $\frac{7}{8}$ of 80.

FINDING FRACTIONAL PARTS

1. If you need $\frac{3}{4}$ yd. of cloth to make a book cover, you need $\frac{3}{4}$ of how many inches? This is how many inches?

2. If you need $\frac{1}{8}$ yd. of lace for some trimming, how many inches do you need?

3. Helen asks the clerk in a store for $\frac{2}{3}$ yd. of velvet ribbon. How many inches does she wish?

4. If 1 yd. of velvet costs $4, how much does $\frac{3}{8}$ yd. cost?

5. At 80¢ a yard, how much will $\frac{1}{2}$ yd. of gingham cost? Find the cost of $\frac{1}{4}$ yd.; of $\frac{3}{4}$ yd.; of $\frac{1}{8}$ yd.; of $\frac{5}{8}$ yd.

6. Some toweling costing 72¢ a yard is cut into lengths of $\frac{3}{4}$ yd. each. Find the cost of the material in each towel.

7. Solve Ex. 6, supposing the lengths to be $\frac{5}{8}$ yd.

8. Solve Ex. 6, supposing the lengths to be $\frac{7}{8}$ yd.

9. If you buy $\frac{3}{4}$ lb. of butter at 48¢ a pound, and $\frac{1}{2}$ lb. of tea at 80¢ a pound, how much must you pay?

10. On an automobile trip of 168 mi. Frank's father said, as he looked at the speedometer, "We have gone three quarters of the way." How many miles had they gone? How many miles had they still to go?

Find the following fractional parts:

11. $\frac{5}{8}$ of 768.
12. $\frac{1}{6}$ of 840.
13. $\frac{5}{6}$ of 774.
14. $\frac{7}{8}$ of 904.
15. $\frac{7}{8}$ of 984.
16. $\frac{5}{8}$ of 3376.
17. $\frac{5}{6}$ of 1758.
18. $\frac{3}{8}$ of 4976.
19. $\frac{3}{8}$ of 5656.
20. $\frac{5}{8}$ of 6408.
21. $\frac{3}{8}$ of 1464.
22. $\frac{7}{8}$ of 3272.
23. $\frac{5}{6}$ of 7764.
24. $\frac{1}{10}$ of 7200.
25. $\frac{3}{10}$ of 7200.

Multiplying a Whole Number by a Mixed Number. If washing-soda costs 12¢ a pound, how much shall I have to pay when I buy $7\frac{1}{2}$ lb.?

To find the answer you must multiply 12 by $7\frac{1}{2}$.

First, take $\frac{1}{2}$ of 12.

Then multiply 12 by 7.

Now add these two results.

The result is 90; that is, I shall have to pay 90¢.

$$\begin{array}{rr} & 12 \\ & \underline{7\frac{1}{2}} \\ \frac{1}{2} \text{ of } 12 = & 6 \\ 7 \times 12 = & \underline{84} \\ 7\frac{1}{2} \times 12 = & 90 \end{array}$$

To multiply a whole number by a mixed number, first multiply by the fractional part, then multiply by the whole number, and add the two results.

MULTIPLYING BY A MIXED NUMBER

1. If eggs cost 60¢ a dozen, how much must I pay for $1\frac{1}{2}$ doz.? for $2\frac{1}{2}$ doz.? for $3\frac{1}{2}$ doz.?

2. If you ride for $3\frac{3}{4}$ hr. in an automobile, going at a steady rate of 18 mi. an hour, how many miles do you go?

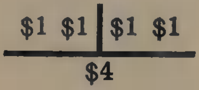
Multiply the following:

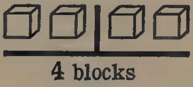
3. $3\frac{1}{2} \times 7$.
4. $5\frac{1}{3} \times 8$.
5. $2\frac{2}{3} \times 9$.
6. $4\frac{2}{3} \times 50$.
7. $7\frac{1}{6} \times 90$.
8. $8\frac{5}{8} \times 24$.
9. $3\frac{1}{4} \times 16$.
10. $4\frac{3}{4} \times 16$.
11. $5\frac{3}{4} \times 28$.
12. $3\frac{1}{8} \times 32$.
13. $3\frac{3}{8} \times 32$.
14. $4\frac{5}{8} \times 32$.
15. $5\frac{7}{8} \times 64$.
16. $7\frac{7}{8} \times 72$.
17. $2\frac{1}{12} \times 36$.
18. $2\frac{7}{12} \times 36$.
19. $5\frac{7}{10} \times 60$.
20. $9\frac{9}{10} \times 90$.
21. $7\frac{3}{16} \times 32$.
22. $8\frac{9}{16} \times 64$.
23. $9\frac{3}{32} \times 64$.

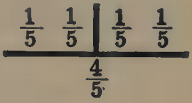
FRACTIONAL PART OF A FRACTION

All work oral

1. How much is $\frac{1}{2}$ of \$4? $\frac{1}{2}$ of 4 blocks? $\frac{1}{2}$ of $\frac{4}{5}$?

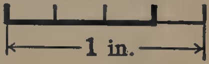


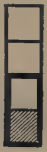




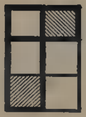
2. How do you find $\frac{1}{2}$ of \$4? $\frac{1}{2}$ of 4¢? $\frac{1}{2}$ of $\frac{4}{5}$?

3. Here is a line, 1 in. long, on which $\frac{3}{4}$ in. has been marked off. If you take $\frac{1}{3}$ of $\frac{3}{4}$ in., what part of an inch do you have? How can you find $\frac{1}{3}$ of $\frac{3}{4}$?





A



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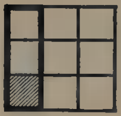
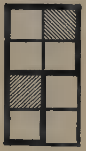


C



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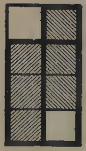
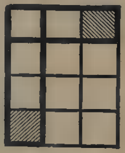


E



F

4. What part of the rectangle *A* is $\frac{1}{2}$ of $\frac{1}{2}$ of *A*?

5. Show that $\frac{1}{2}$ of $\frac{1}{3}$ of *B* is the same as $\frac{1}{3}$ of $\frac{1}{2}$ of *B*.

6. What part of *B* is $\frac{1}{3}$ of $\frac{1}{2}$ of *B*?

7. What part of *C* is $\frac{1}{3}$ of $\frac{1}{3}$ of *C*?

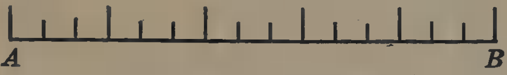
8. What part of *D* is $\frac{1}{2}$ of $\frac{1}{4}$ of *D*? $\frac{1}{4}$ of $\frac{1}{2}$ of *D*?

9. Point to $\frac{1}{2}$ of $\frac{3}{4}$ of *E*; to $\frac{3}{4}$ of $\frac{1}{2}$ of *E*. What part of *E* is $\frac{3}{4}$ of $\frac{1}{2}$ of *E*?

10. Point to $\frac{1}{3}$ of *F*; to $\frac{1}{4}$ of $\frac{1}{3}$ of *F*. Point also to $\frac{1}{4}$ of *F* and to $\frac{1}{3}$ of $\frac{1}{4}$ of *F*. What part of *F* is $\frac{1}{4}$ of $\frac{1}{3}$ of *F*? $\frac{1}{3}$ of $\frac{1}{4}$ of *F*?

A SILENT READING LESSON

The line *AB* shown below has been divided into five equal parts, and each of these, in turn, into three equal parts. Into how many equal parts is *AB* divided?



Let us consider the following questions:

$\frac{1}{3}$ of $\frac{1}{5}$ of the line is how many fifteenths of the line?
$\frac{1}{3}$ of $\frac{4}{5}$ is how many times as long as $\frac{1}{3}$ of $\frac{1}{5}$?
$\frac{1}{3}$ of $\frac{4}{5}$ is how many fifteenths of the line?
$\frac{2}{3}$ of $\frac{4}{5}$ is how many times as long as $\frac{1}{3}$ of $\frac{4}{5}$?
$\frac{2}{3}$ of $\frac{4}{5}$ is how many fifteenths of the line?

We may write $\frac{2}{3}$ of $\frac{4}{5}$ as $\frac{2}{3} \times \frac{4}{5}$, which we may read as either "$\frac{2}{3}$ of $\frac{4}{5}$" or "$\frac{2}{3}$ times $\frac{4}{5}$."

To multiply two fractions, multiply the numerators together for the new numerator and multiply the denominators together for the new denominator. Cancel whenever possible.

Thus, in finding $\frac{2}{3}$ of $\frac{3}{4}$, we proceed as follows:

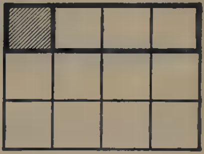
$$\frac{\cancel{2} \times \cancel{3}}{\cancel{3} \times \underset{2}{\cancel{4}}} = \frac{1}{2}.$$

Similarly, $$\frac{3}{4} \text{ of } \frac{4}{5} = \frac{3 \times \cancel{4}}{\cancel{4} \times 5} = \frac{3}{5},$$

and $$\frac{2}{3} \times \frac{3}{4} \times \frac{7}{8} = \frac{\cancel{2} \times \cancel{3} \times 7}{\cancel{3} \times \underset{2}{\cancel{4}} \times 8} = \frac{7}{16}.$$

FRACTION OF A FRACTION

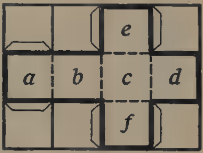
1. Draw a rectangle like the one here shown, making it 4 in. long and 3 in. wide. Divide it into four equal strips running one way and into three equal strips running the other way.



2. The shaded square is what part of a longer strip? of a shorter strip? of the whole rectangle?

3. How many twelfths of the rectangle is $\frac{1}{4}$ of $\frac{1}{3}$ of it?

4. In making cubes for studying fractions, the class cut out cardboard rectangles, each 4 in. long and 3 in. wide. Each pupil marked off the cardboard into four equal parts in length and three equal parts in width, and then cut and folded it as suggested by the figure. Show that each side of the cube, which is indicated by the heavy lines, is $\frac{1}{3}$ of $\frac{1}{4}$ of the rectangle.



5. How many twelfths of the rectangle is $\frac{1}{3}$ of $\frac{1}{4}$ of it?

6. The squares *a*, *b*, *c*, and *d* together are how many twelfths of the rectangle? This is how many thirds?

Multiply as follows:

7. $\frac{1}{2} \times \frac{1}{3}$.
8. $\frac{1}{3} \times \frac{1}{2}$.
9. $\frac{1}{2} \times \frac{2}{3}$.
10. $\frac{2}{3} \times \frac{1}{2}$.
11. $\frac{1}{2} \times \frac{1}{4}$.
12. $\frac{1}{2} \times \frac{3}{4}$.
13. $\frac{1}{3} \times \frac{3}{4}$.
14. $\frac{2}{3} \times \frac{3}{4}$.
15. $\frac{1}{2} \times \frac{1}{8}$.
16. $\frac{1}{2} \times \frac{3}{8}$.
17. $\frac{5}{8} \times \frac{1}{2}$.
18. $\frac{1}{3} \times \frac{3}{8}$.
19. $\frac{2}{3} \times \frac{3}{8}$.
20. $\frac{2}{3} \times \frac{5}{8}$.
21. $\frac{2}{3} \times \frac{7}{8}$.
22. $\frac{1}{4} \times \frac{4}{5}$.
23. $\frac{1}{5} \times \frac{5}{8}$.
24. $\frac{1}{5} \times \frac{5}{16}$.
25. $\frac{1}{4} \times \frac{1}{8}$.
26. $\frac{1}{4} \times \frac{3}{8}$.
27. $\frac{3}{4} \times \frac{5}{8}$.
28. $\frac{3}{4} \times \frac{7}{8}$.
29. $\frac{1}{2} \times \frac{1}{16}$.
30. $\frac{1}{2} \times \frac{3}{16}$.

31. If Henry walks 1 mi. in $\frac{1}{4}$ hr., how long will it take him, at the same rate, to walk $\frac{1}{2}$ mi.? to walk $\frac{1}{3}$ mi.?

32. If an express train runs 1 mi. in $\frac{3}{4}$ min., how long, at the same rate, will it take the train to run $\frac{5}{8}$ mi.?

33. If a train runs $\frac{2}{3}$ of a certain distance in 1 hr., what part of the distance can it run in $\frac{3}{4}$ hr.?

34. If it takes a train $\frac{3}{4}$ hr. to reach a certain station, in what fraction of an hour will it go $\frac{5}{6}$ of the distance?

35. If Frank lives $\frac{4}{5}$ mi. from school, and George lives $\frac{5}{8}$ as far, how far does George live from school?

36. If $\frac{5}{8}$ yd. of lace is needed to trim two neckbands, how much lace will be needed to trim each neckband?

37. In a certain race an automobile ran 1 mi. in 49 sec.; that is, in $\frac{49}{60}$ min. What part of a minute did it take the car to run $\frac{5}{8}$ mi.?

38. If Jane lives $\frac{7}{8}$ mi. from school, and Madge lives $\frac{1}{3}$ as far, how far from school does Madge live? If Isabel lives $\frac{1}{2}$ as far as Madge, how far from school does Isabel live?

39. Mrs. Sinclair used $\frac{7}{8}$ yd. of velvet in making one hat and $\frac{2}{5}$ as much in making another hat. How much velvet did she use for both hats?

40. Find the value of $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$; of $\frac{3}{8}$ of $\frac{4}{5}$ of $\frac{5}{12}$.

Multiply as follows:

41. $\frac{1}{8} \times \frac{4}{5}$.
42. $\frac{4}{5} \times \frac{5}{8}$.
43. $\frac{3}{5} \times \frac{5}{9}$.
44. $\frac{5}{9} \times \frac{3}{5}$.
45. $\frac{3}{8} \times \frac{7}{8}$.
46. $\frac{3}{4} \times \frac{8}{15}$.
47. $\frac{2}{3} \times \frac{9}{10}$.
48. $\frac{2}{3} \times \frac{15}{16}$.
49. $\frac{3}{4} \times \frac{5}{24}$.
50. $\frac{15}{32} \times \frac{4}{5}$.
51. $\frac{7}{16} \times \frac{5}{7}$.
52. $\frac{7}{16} \times \frac{8}{21}$.

Multiplying a Mixed Number by a Fraction. 1. If I buy $4\frac{1}{2}$ lb. of butter and use half of it, how much do I use?

First, $\frac{1}{2}$ of 4 lb. = 2 lb.
Then $\frac{1}{2}$ of $\frac{1}{2}$ lb. = $\frac{1}{4}$ lb.
Adding, 2 lb. + $\frac{1}{4}$ lb. = $2\frac{1}{4}$ lb., the amount used.

2. If I have $5\frac{1}{4}$ yd. of ribbon and use $\frac{2}{3}$ of it, how much ribbon do I use?

Write $\frac{2}{3}$ of $5\frac{1}{4}$, or $\frac{2}{3}$ of $\frac{21}{4}$, in the form for canceling; that is, as

$$\frac{2 \times 21}{3 \times 4}.$$

The result is $3\frac{1}{2}$; that is, $3\frac{1}{2}$ yd.

$$\boxed{\frac{\cancel{2} \times \overset{7}{\cancel{21}}}{\cancel{3} \times \underset{2}{\cancel{4}}} = \frac{7}{2} = 3\frac{1}{2}}$$

MULTIPLYING BY A FRACTION

1. If you have $8\frac{1}{2}$ lb. of sugar and use $\frac{3}{4}$ of it, how many pounds do you use?

2. How much is $\frac{3}{8}$ of $7\frac{1}{2}$ lb. of sugar?

Find the value in each of the following cases:

3. $\frac{2}{3}$ of $6\frac{1}{2}$ in.
4. $\frac{5}{8}$ of $3\frac{3}{4}$ ft.
5. $\frac{3}{4}$ of $5\frac{7}{8}$ in.
6. $\frac{3}{8}$ of $4\frac{3}{4}$ yd.
7. $\frac{1}{2}$ of $17\frac{3}{4}$ mi.
8. $\frac{2}{3}$ of $25\frac{1}{2}$ mi.
9. $\frac{1}{8}$ of $14\frac{1}{2}$ in.
10. $\frac{3}{8}$ of $21\frac{1}{2}$ lb.
11. $\frac{1}{4}$ of $23\frac{1}{4}$ in.
12. $\frac{3}{4}$ of $15\frac{1}{2}$ yd.
13. $\frac{3}{8}$ of $20\frac{1}{4}$ yd.
14. $\frac{5}{8}$ of $15\frac{3}{4}$ mi.

15. On an automobile trip of $24\frac{1}{2}$ mi., how far have you gone when you have covered $\frac{5}{8}$ of the distance?

Find the value in each of the following cases:

16. $\frac{2}{3}$ of $\frac{5}{8}$ of 15 ft.
17. $\frac{3}{8}$ of $\frac{2}{3}$ of $48\frac{1}{2}$ ft.

Multiplying a Mixed Number by a Mixed Number. In knitting a scarf, if Mary knits $3\frac{1}{2}$ in. in 1 hr., how many inches can she knit in $2\frac{1}{2}$ hr.?

What do you do first?
What do you do next?
Then what do you do?
What is the result?

$$3\frac{1}{2} = \frac{7}{2} \qquad 2\frac{1}{2} = \frac{5}{2}$$
$$\frac{5}{2} \times \frac{7}{2} = \frac{35}{4} = 8\frac{3}{4}$$

To multiply a mixed number by a mixed number, reduce each to an improper fraction and multiply these fractions.

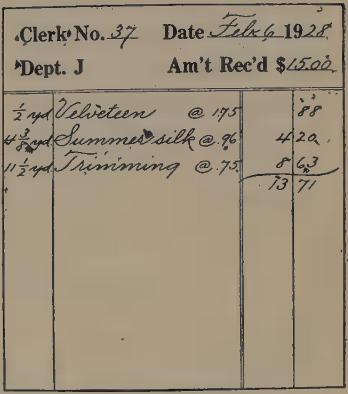
MULTIPLYING BY A MIXED NUMBER

Multiply as follows:

1. $1\frac{1}{2} \times 2\frac{1}{2}$.
2. $2\frac{1}{2} \times 3\frac{1}{2}$.
3. $3\frac{1}{2} \times 3\frac{1}{2}$.
4. $2\frac{1}{2} \times 2\frac{1}{4}$.
5. $2\frac{1}{2} \times 3\frac{3}{4}$.
6. $3\frac{1}{2} \times 1\frac{1}{3}$.
7. $3\frac{1}{2} \times 4\frac{2}{3}$.
8. $5\frac{1}{2} \times 1\frac{1}{8}$.
9. $3\frac{1}{2} \times 2\frac{5}{8}$.
10. $4\frac{1}{4} \times 2\frac{1}{8}$.
11. $3\frac{3}{4} \times 3\frac{3}{8}$.
12. $5\frac{3}{4} \times 7\frac{5}{8}$.
13. $2\frac{3}{4} \times 6\frac{7}{8}$.
14. $2\frac{1}{3} \times 6\frac{7}{8}$.
15. $3\frac{2}{3} \times 5\frac{7}{8}$.
16. $3\frac{1}{2} \times 2\frac{5}{6}$.
17. $2\frac{1}{3} \times 3\frac{5}{6}$.
18. $3\frac{2}{3} \times 4\frac{5}{6}$.
19. $2\frac{1}{2} \times 3\frac{1}{10}$.
20. $5\frac{1}{2} \times 4\frac{3}{10}$.
21. $3\frac{1}{2} \times 5\frac{7}{10}$.
22. $8\frac{1}{2} \times 2\frac{9}{10}$.
23. $3\frac{1}{3} \times 3\frac{9}{10}$.
24. $2\frac{2}{3} \times 2\frac{9}{10}$.
25. $3\frac{1}{4} \times 1\frac{3}{10}$.
26. $2\frac{3}{4} \times 3\frac{3}{10}$.
27. $3\frac{3}{4} \times 5\frac{7}{10}$.
28. $5\frac{3}{4} \times 6\frac{9}{10}$.
29. $3\frac{1}{8} \times 2\frac{9}{10}$.
30. $5\frac{7}{8} \times 3\frac{9}{10}$.
31. $3\frac{1}{2} \times 1\frac{1}{12}$.
32. $3\frac{1}{2} \times 3\frac{5}{12}$.
33. $2\frac{1}{4} \times 2\frac{1}{12}$.
34. $2\frac{3}{4} \times 3\frac{5}{12}$.
35. $3\frac{3}{4} \times 5\frac{7}{12}$.
36. $2\frac{1}{2} \times 1\frac{1}{16}$.
37. $3\frac{1}{2} \times 2\frac{3}{16}$.
38. $3\frac{1}{4} \times 2\frac{3}{16}$.
39. $4\frac{3}{4} \times 5\frac{5}{16}$.
40. $4\frac{3}{8} \times 4\frac{3}{8}$.
41. $2\frac{7}{8} \times 6\frac{7}{8}$.
42. $1\frac{1}{4} \times 3\frac{1}{12}$.

Sales Slips. In large stores the clerk who makes a sale is usually required to fill out a *sales slip*, which shows the details of the sale. In filling out these slips, the clerk frequently has to multiply with fractions, as is shown in this model form. A good clerk is able to do this work rapidly and accurately.

The symbol @, which is used in the model form, means "at" and refers to the price of 1 yd., 1 lb., or whatever other unit is used.



Clerk No. 37 Date Feb 6 1928

Dept. J Am't Rec'd $15.00

$\frac{1}{2}$ yd.	Velveteen @ 1.75		88
$4\frac{3}{8}$ yd.	Summer silk @ .96	4	20
$11\frac{1}{2}$ yd.	Trimming @ .75	8	63
		13	71

SALES SLIPS

Supposing that you are Clerk No. 81, make out sales slips for the following goods sold today:

1. $26\frac{1}{2}$ yd. silk @ \$1.75, 8 yd. cotton @ $37\frac{1}{2}$¢, $14\frac{1}{2}$ yd. madras @ 55¢. Amount received, \$60.

2. $5\frac{1}{2}$ yd. cheviot @ \$3.40, $6\frac{1}{2}$ yd. velveteen @ \$2.60, $5\frac{3}{8}$ yd. silk @ \$1.75. Amount received, \$50.

3. $8\frac{1}{4}$ yd. India linen @ 65¢, $4\frac{5}{8}$ yd. dimity @ 45¢, $12\frac{1}{8}$ yd. linen suiting @ \$1.20. Amount received, \$25.

4. $9\frac{1}{2}$ yd. linen @ 64¢, $4\frac{1}{8}$ yd. cheviot @ \$3.75, $12\frac{1}{2}$ yd. cotton @ 40¢. Amount received, \$30.

5. $3\frac{1}{2}$ yd. linen @ 72¢, $5\frac{1}{2}$ yd. ribbon @ 42¢, $2\frac{7}{8}$ yd. velvet @ \$3.20. Amount received, \$20.

6. In Exs. 1–5, how much change is due each customer?

III. REVIEW AND DRILL

MINIMUM ESSENTIALS

1. Write in words the numbers 4,201,702 and MDCVI.

2. Write in figures the number nineteen million six.

3. Add $3296 + 49{,}248 + 674 + 3462 + 58{,}779 + 6364$.

4. Add $\$38.75 + \$93.46 + \$7.75 + \$329.50 + \$4.36$.

5. Subtract \$98.78 from $\$942.65 + \$9.89$.

6. Multiply $997 + 53{,}095$ by 3278.

7. Multiply $\$69.30 + \$478.70$ by 964.

8. Divide $87{,}960 + 48{,}290$ by $70\frac{3}{4} + 179\frac{1}{4}$.

9. Divide 774,932 by 302, and 945,260 by 604.

10. Reduce to lowest terms: $\frac{6}{8}$, $\frac{12}{16}$, $\frac{16}{32}$, $\frac{18}{32}$, $\frac{24}{32}$, $\frac{32}{32}$.

11. How many fourths are there in 7? in $7\frac{1}{4}$? in $9\frac{3}{4}$?

12. Reduce $25\frac{3}{4}$ to fourths; to eighths.

13. Reduce to whole or to mixed numbers: $\frac{34}{2}$, $\frac{47}{4}$, $\frac{61}{8}$.

14. Find the value of $7\frac{1}{2} + \frac{1}{8} + 3\frac{3}{4} + 9\frac{3}{4} + 3\frac{5}{8} - \frac{1}{2}$.

15. From $9\frac{1}{4}$ in. take $3\frac{1}{2}$ in. $+ 4\frac{7}{8}$ in.

16. How many strips of molding 8 in. long can be cut from a strip 5 ft. long, and how much will be left over?

Multiply or divide as indicated:

17. 625×425.

18. 772×996.

19. $1728 \div 12$.

20. $5184 \div 24$.

21. $\frac{1}{2} \times \frac{3}{4}$.

22. $2\frac{1}{2} \times 3\frac{1}{2}$.

23. $3\frac{1}{2} \times 14\frac{7}{8}$.

24. $4128 \div 32$.

25. $\frac{1}{2} \times \frac{3}{4} \times \frac{5}{8}$.

26. $3\frac{1}{2} \times 2\frac{1}{2} \times 3\frac{1}{4}$.

27. $4\frac{1}{2} \times 3\frac{2}{3} \times 4\frac{3}{4}$.

28. $4\frac{1}{2} \times 3\frac{3}{4} \times 2\frac{5}{8}$.

PROBLEMS WITHOUT NUMBERS

All work oral

1. How do you subtract a number of two figures from one of three figures when the units' figure to be subtracted is larger than the units' figure above it, and the same is true of the figure in tens' place? Give your reasons.

2. How do you multiply by a number of three figures when the last two figures are zeros? Give your reasons.

3. How do you divide by a number of three figures when the last two figures are zeros? Give your reasons.

4. How do you find the square of a number? How do you find the cube of a number?

5. How do you cancel factors from the terms of a fraction? What are the advantages of cancellation? Upon what principle does cancellation depend?

6. If you know the cost of some land and the cost per acre, how do you find the number of acres?

7. If you know the weight of a cubic foot of water and the number of cubic feet of water in a cistern, how do you find the weight of the water in the cistern?

8. If you know one of two numbers and how much less it is than the other, how do you find the other number?

9. If a number has been divided by another and you are given the number divided and the result, how can you find the number by which it was divided?

10. If a ticket seller has a large number of quarters and dimes, how can he find their value in dollars?

PROBLEMS FOR COMPLETION

1. Ruth can buy a gingham play dress for \$2.50 and a blouse for \$2.75. She has \$1.75. Complete the problem in any reasonable way, of which there are usually several, and solve it.

Complete each of the following problems and solve it:

2. Marion has \$2 and wishes to buy some toys for her little brother and sister. The price of a box of alphabet blocks is \$1; of a Teddy bear, \$1.15; of a red ball, 30¢; of a rubber doll, 40¢; and of a box of paper dolls, 15¢.

3. Edwin has \$1.50 to spend for toys. A toy dog costs 89¢; a mask, 30¢; a ball, 25¢; and a trumpet, 39¢.

4. A man takes his family on a 4-day automobile trip in which they plan to cover 548 mi. The first day they go 142 mi.; the second day, 131 mi.; the third day, $136\frac{3}{4}$ mi.

5. The expenses of a certain school last year amounted to \$1883.15. This sum included four items: the teacher's salary, \$146.90 for books and supplies, \$139.50 for fuel and lights, and \$96.75 for miscellaneous expenses.

6. A certain school is in session from a quarter before nine to noon, and from a quarter past one to a quarter before four.

7. A sweater which Ida wishes to knit takes $2\frac{3}{4}$ hanks of yarn. Another kind will take only $2\frac{1}{2}$ hanks.

8. Ruth needs $9\frac{3}{4}$ yd. of cloth which costs \$1.20 a yard.

9. Our car uses 1 gal. of gasoline every 18 mi. Yesterday we drove 156 mi.

MISCELLANEOUS PROBLEMS

1. If John picks on an average $3\frac{3}{4}$ qt. of raspberries in an hour, how many quarts of berries does he pick in 2 hr.?

2. If Elizabeth picks 3 qt. of raspberries per hour, how many quarts of berries does she pick in $2\frac{1}{4}$ hr.?

3. The width of a certain rectangle is known to be $\frac{3}{4}$ of the length. If the length of the rectangle is 64 in., what is the width?

4. To make $2\frac{1}{2}$ times a recipe, which calls for 2 cups of milk, how many cups of milk must you use?

5. If it takes $2\frac{5}{8}$ yd. of cloth to make a boy's suit, how many yards will it take to make a dozen such suits?

6. When a tin cup which holds $\frac{15}{16}$ pt. is $\frac{2}{3}$ full, what part of a pint is there in the cup?

7. If a glass jar holds $\frac{15}{16}$ qt. of preserves, what part of a quart will it contain when $\frac{4}{5}$ full? What part of a quart will it contain when $\frac{2}{5}$ full? when $\frac{3}{5}$ full?

8. How long will it take an airplane that is traveling at the rate of 1 mi. in 27 sec. (1 mi. in $\frac{27}{60}$ min.) to go 5 mi.? to go $9\frac{1}{2}$ mi.? to go $10\frac{1}{2}$ mi.?

9. A dressmaker bought $8\frac{3}{4}$ yd. of silk @ \$2.60, $6\frac{5}{8}$ yd. of voile @ 75¢, $3\frac{1}{2}$ yd. of fancy lining @ \$1.85, and $5\frac{1}{4}$ yd. of braid @ $37\frac{1}{2}$¢. How much did it all cost?

PROBLEMS IN THRIFT

1. Mrs. Johnson finds that her cook is wasting one slice of bread every day. If a loaf of bread is cut into 16 slices, in how many days will she waste 3 loaves of bread?

2. If Mrs. Wallace buys potatoes during the winter, she pays $2 a bushel for them. In the fall she can buy them for $1.10 a bushel. Find how much she saves by buying 10 bu. in the fall.

3. Mrs. Brown decided to save money by buying groceries in larger quantities. She made a list showing what the grocer charged for certain goods when bought in quantity and the prices she paid when buying by the pound or by the can. If she could use the larger amounts, find how much she could save on the following order:

Bacon, piece of 8 lb., $3.20; 45¢ per pound
Canned corn, case of 24 cans, $3.60; 18¢ per can
Crackers, box of 7 lb., $1.40; 23¢ per pound
Soap, box of 100 bars, $8.75; 10¢ per bar

4. By using a fireless cooker, Mrs. Brown reduced her gas bills for January, February, and March from a total of $8.95 for these months in the previous year to $7.25. At this rate, how much will Mrs. Brown save if she uses the fireless cooker throughout the year?

5. In July a coal dealer told Mr. Field that he could buy his winter's coal for $11.60 a ton. Mr. Field waited, however, until October, and then had to pay $12.85 a ton for the 16 T. which he needed. How much could he have saved on his coal bill by buying in July?

IV. LITTLE EXAMINATIONS

I. 1. MDCXXXI = (?).
2. $742{,}982 - 638{,}863$.
3. $702\frac{1}{8} + 338\frac{3}{4}$.
4. $901\frac{3}{4} - 348\frac{7}{8}$.
5. $72 \times 3\frac{7}{8}$.
6. $57\frac{1}{2} \times 140$.
7. $\frac{3}{8} \times \frac{3}{4}$.
8. $2\frac{1}{8} \times 4\frac{1}{2}$.

II. 1. MDXCVI = (?).
2. $877{,}859 + 358{,}583$.
3. $700{,}203 - 386{,}774$.
4. $6076 \times 48{,}305$.
5. $3\frac{1}{2} + 2\frac{7}{8}$.
6. $3\frac{1}{2} - 2\frac{7}{8}$.
7. $3\frac{1}{2} \times 2\frac{7}{8}$.
8. $5\frac{3}{8} \times 9\frac{7}{10}$.

III. 1. MCCXCVI = (?).
2. $229{,}628 + 69{,}837$.
3. $402{,}120 - 36{,}879$.
4. $7008 \times 82{,}635$.
5. $72 + 9\frac{7}{8}$.
6. $72 - 9\frac{7}{8}$.
7. $72 \times 9\frac{7}{8}$.
8. $9\frac{7}{8} \times 72\frac{1}{2}$.

IV. 1. CLXXXIV = (?).
2. $893{,}434 + 98{,}877$.
3. $911{,}034 - 38{,}048$.
4. $489 \times \$662.75$.
5. $15 \times 14\frac{3}{8}$.
6. $15\frac{1}{2} + 14\frac{3}{8}$.
7. $15\frac{1}{2} - 14\frac{3}{8}$.
8. $15\frac{1}{2} \times 14\frac{3}{8}$.

V. 1. Reduce $8\frac{2}{3}$ to thirds.
2. $8\frac{1}{2} + 3\frac{3}{8} + 2\frac{1}{4}$.
3. $4\frac{7}{8} + 8\frac{3}{4} + 2\frac{1}{2}$.
4. $702\frac{1}{8} - 338\frac{3}{4}$.
5. $\frac{5}{8} + \frac{3}{8} + 2\frac{1}{4}$.
6. $3\frac{3}{4} + \frac{1}{2} - \frac{1}{8}$.
7. $8\frac{3}{4} + 3\frac{1}{2}$.
8. $33\frac{2}{5} \times 18\frac{3}{8}$.

VI. 1. Reduce $\frac{25}{5}$ to units.
2. $7\frac{3}{4} + 6\frac{3}{4} + 4\frac{7}{8}$.
3. $3\frac{2}{3} + 8\frac{3}{4} - 7\frac{1}{2}$.
4. $901\frac{3}{4} + 348\frac{7}{8}$.
5. $\$64.96 \div 8$.
6. $8\frac{3}{8}$ ft. + $6\frac{7}{16}$ ft.
7. $9\frac{3}{8}$ in. − $2\frac{1}{2}$ in.
8. $79\frac{1}{2} \times 121\frac{3}{4}$.

CHAPTER II

I. FRACTIONS

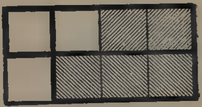
DIVIDING BY A WHOLE NUMBER

All work oral

State the answers in each of the following cases:

1. 9 in. ÷ 3, and $\frac{9}{32} \div 3$.
2. 15 in. ÷ 5, and $\frac{15}{16} \div 5$.
3. 21 yd. ÷ 7, and $\frac{21}{32} \div 7$.
4. 25 qt. ÷ 5, and $\frac{25}{32} \div 5$.

5. Dividing the numerator of a fraction by a whole number does what to the value of the fraction?



6. If you divide $\frac{1}{2}$ of this rectangle into two equal parts, each is what part of the rectangle? How much is $\frac{1}{2} \div 2$?

7. If you divide $\frac{3}{4}$ of the rectangle into two equal parts, how many eighths of the rectangle do you have? How much is $\frac{3}{4} \div 2$? What have you done to the denominator?

8. Multiplying the denominator of a fraction by a whole number does what to the value of the fraction?

By dividing the numerator, divide as follows:

9. $\frac{2}{3} \div 2$. **10.** $\frac{18}{16} \div 3$. **11.** $\frac{5}{8} \div 5$. **12.** $\frac{15}{32} \div 5$.

By multiplying the denominator, divide as follows:

13. $\frac{1}{2} \div 3$. **14.** $\frac{2}{3} \div 5$. **15.** $\frac{3}{4} \div 4$. **16.** $\frac{3}{5} \div 2$.

Dividing Fractions. The following examples show how we divide a fraction by a whole number:

1. Divide $\frac{4}{7}$ by 2.

Simply divide the numerator of the fraction by 2, the denominator remaining unchanged.

In dividing a fraction by a whole number, always divide the numerator, if it is exactly divisible, instead of multiplying the denominator. It shortens the work!

$$\boxed{\frac{4 \div 2}{7} = \frac{2}{7}}$$

2. Divide $\frac{5}{8}$ by 3.

Simply multiply the denominator by 3, the numerator remaining unchanged, as shown in the work at the right.

$$\boxed{\frac{5}{3 \times 8} = \frac{5}{24}}$$

3. Divide $\frac{15}{16}$ by 10.

Indicate the multiplication of the denominator and cancel, as shown.

This is shorter than to multiply the denominator before canceling. It saves time and avoids errors!

$$\boxed{\frac{\overset{3}{\cancel{15}}}{\underset{2}{\cancel{10}} \times 16} = \frac{3}{32}}$$

4. Divide $45\frac{1}{3}$ by 3.

Since 45 is exactly divisible by 3, we have $45 \div 3 = 15$, and $\frac{1}{3} \div 3 = \frac{1}{9}$, which is shorter than first reducing $45\frac{1}{3}$ to $\frac{136}{3}$.

$$\boxed{\frac{45\frac{1}{3}}{3} = 15\frac{1}{9}}$$

5. Divide $244\frac{1}{2}$ by 3.

First reduce $244\frac{1}{2}$ to the improper fraction $\frac{489}{2}$. Then divide the numerator by 3, since it is exactly divisible, and reduce the result as here shown.

$$\boxed{\begin{aligned} \frac{244\frac{1}{2}}{3} &= \frac{489 \div 3}{2} \\ &= \frac{163}{2} \\ &= 81\frac{1}{2} \end{aligned}}$$

DIVIDING BY A WHOLE NUMBER

1. To make a certain quantity of fudge, Mary's cookbook tells her to divide $\frac{1}{2}$ lb. of chocolate into 3 equal parts. What part of a pound is there in each part?

2. Clara's mother buys $13\frac{1}{8}$ yd. of cloth with which to make 5 children's dresses. How many yards of cloth should she allow for making each dress?

3. A sewing club needs $6\frac{3}{4}$ yd. of flannel for making 4 long wrappers for babies. How much flannel should be allowed for making each wrapper?

4. If a girl can knit 4 in. on a sweater sleeve in $1\frac{1}{4}$ hr., what part of an hour does it take to knit 1 in.?

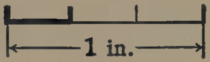
5. Helen finds that she uses 1 lb. 9 oz., or $1\frac{9}{16}$ lb., of wool in knitting 5 socks. What part of a pound does she use for 1 sock? for a pair of socks?

6. If an automobile goes $67\frac{1}{2}$ mi. in 3 hr., what is the average distance traveled per hour?

Divide as follows:

7. $\frac{1}{2} \div 2.$
8. $\frac{1}{2} \div 3.$
9. $\frac{2}{5} \div 2.$
10. $\frac{5}{8} \div 2.$
11. $\frac{3}{5} \div 3.$
12. $\frac{4}{5} \div 2.$
13. $\frac{3}{8} \div 3.$
14. $\frac{3}{4} \div 2.$
15. $\frac{1}{3} \div 2.$
16. $\frac{2}{3} \div 2.$
17. $\frac{4}{5} \div 2.$
18. $\frac{5}{8} \div 3.$
19. $\frac{5}{8} \div 4.$
20. $\frac{3}{8} \div 4.$
21. $\frac{3}{8} \div 6.$
22. $\frac{8}{5} \div 4.$
23. $\frac{1}{4} \div 2.$
24. $\frac{1}{4} \div 3.$
25. $\frac{1}{8} \div 2.$
26. $\frac{7}{8} \div 3.$
27. $\frac{7}{8} \div 4.$
28. $\frac{7}{8} \div 7.$
29. $\frac{5}{8} \div 5.$
30. $\frac{3}{4} \div 9.$
31. $27\frac{1}{2} \div 3.$
32. $29\frac{1}{2} \div 2.$
33. $36\frac{3}{5} \div 3.$
34. $28\frac{1}{8} \div 9.$
35. $24\frac{4}{5} \div 4.$
36. $28\frac{3}{4} \div 5.$
37. $75\frac{15}{16} \div 5.$
38. $134\frac{1}{2} \div 5.$

Dividing by a Unit Fraction. How many thirds of an inch are there in 1 in.? Then $1 \div \frac{1}{3}$ is how many? Compare the values of $1 \div \frac{1}{3}$ and 1×3.



In the same way, what can you say about the values of $3 \div \frac{1}{2}$ and 3×2?

What can you say about the values of $\frac{3}{4} \div \frac{1}{3}$ and $\frac{3}{4} \times 3$?

To divide a number by a unit fraction, multiply the number by the denominator of the fraction.

Thus, $1 \div \frac{1}{8} = 1 \times 8 = 8,$

$3 \div \frac{1}{8} = 3 \times 8 = 24,$

and $\frac{3}{4} \div \frac{1}{8} = \frac{3}{4} \times 8 = 6.$

DIVIDING BY A UNIT FRACTION

1. If a sheet of veneer is $\frac{1}{16}$ in. thick, how many sheets are needed to make a total thickness of 1 in.? of 2 in.? of $\frac{3}{16}$ in.? of $\frac{5}{8}$ in.?

2. If a sheet of cardboard is $\frac{1}{32}$ in. thick, how many sheets are there in a pile that is $12\frac{7}{8}$ in. high?

3. If a wagon wheel makes $\frac{1}{12}$ of a revolution in going 1 ft., how many feet will it go in making 144 revolutions?

Divide as follows:

4. $32 \div \frac{1}{8}$.
5. $24 \div \frac{1}{16}$.
6. $15 \div \frac{1}{12}$.
7. $24 \div \frac{1}{12}$.
8. $36 \div \frac{1}{16}$.
9. $\frac{5}{6} \div \frac{1}{3}$.
10. $\frac{1}{2} \div \frac{1}{2}$.
11. $\frac{3}{4} \div \frac{1}{4}$.
12. $\frac{7}{8} \div \frac{1}{8}$.
13. $\frac{4}{5} \div \frac{1}{5}$.
14. $2\frac{2}{5} \div \frac{1}{5}$.
15. $3\frac{5}{8} \div \frac{1}{8}$.
16. $4\frac{3}{4} \div \frac{1}{12}$.
17. $7\frac{3}{8} \div \frac{1}{16}$.
18. $5\frac{3}{16} \div \frac{1}{16}$.
19. $2\frac{3}{4} \div \frac{1}{24}$.
20. $3\frac{2}{3} \div \frac{1}{24}$.
21. $5\frac{7}{8} \div \frac{1}{32}$.
22. $2\frac{5}{8} \div \frac{1}{64}$.
23. $3\frac{7}{8} \div \frac{1}{64}$.

DIVIDING BY A FRACTION

All work oral

1. In a room 18 ft. wide, suppose that you wish to find how many strips of carpet $\frac{3}{4}$ yd. wide are needed to cover the floor. First you must find how many yards there are in 18 ft. How many yards are there?

2. If you have to divide 6 yd. by $\frac{3}{4}$ yd., you may think of the 6 yd. as fourths of a yard. Express 6 as fourths.

3. In dividing $\frac{24}{4}$ by $\frac{3}{4}$, think of the cases of 24¢ ÷ 3¢ and 24 in. ÷ 3 in. What are the results in these cases?

4. Then how much is $\frac{24}{4} \div \frac{3}{4}$? How many strips of carpet do you need to cover the floor in Ex. 1?

5. Divide $\frac{6}{5}$ by $\frac{2}{5}$. Multiply $\frac{6}{5}$ by $\frac{5}{2}$.

6. Show that $\frac{4}{5} \div \frac{2}{5}$ gives the same result as $\frac{4}{5} \times \frac{5}{2}$.

7. Study the following examples:

$$15 \div \frac{3}{5} = \overset{5}{\cancel{15}} \times \frac{5}{\cancel{3}} = 25 \qquad \frac{3}{8} \div \frac{5}{16} = \frac{3}{\cancel{8}} \times \frac{\overset{2}{\cancel{16}}}{5} = \frac{6}{5} = 1\tfrac{1}{5}$$

Divide as follows:

8. $2 \div \frac{2}{3}$.
9. $4 \div \frac{2}{3}$.
10. $3 \div \frac{3}{4}$.
11. $2 \div \frac{4}{5}$.
12. $3 \div \frac{3}{8}$.
13. $6 \div \frac{3}{8}$.
14. $5 \div \frac{5}{8}$.
15. $9 \div \frac{3}{4}$.
16. $4 \div \frac{4}{5}$.
17. $15 \div \frac{5}{4}$.
18. $20 \div \frac{5}{4}$.
19. $21 \div \frac{7}{8}$.
20. $10 \div \frac{5}{8}$.
21. $20 \div \frac{5}{6}$.
22. $\frac{7}{8} \div \frac{7}{8}$.
23. $5 \div \frac{1}{2}$.
24. $5 \div \frac{2}{3}$.
25. $5 \div \frac{1}{4}$.
26. $5 \div \frac{3}{4}$.
27. $5 \div \frac{1}{8}$.
28. $5 \div \frac{3}{8}$.
29. $\frac{5}{3} \div \frac{2}{3}$.
30. $\frac{3}{5} \div \frac{2}{3}$.
31. $\frac{3}{5} \div \frac{3}{2}$.
32. $\frac{2}{3} \div \frac{2}{3}$.
33. $\frac{2}{3} \div \frac{5}{3}$.
34. $\frac{2}{3} \div \frac{3}{5}$.
35. $\frac{3}{2} \div \frac{3}{5}$.

Dividing by a Fraction. From page 109 we have the rule, *To divide by a fraction multiply by the fraction inverted.*

That is, $\frac{3}{4} \div \frac{2}{3} = \frac{3}{4} \times \frac{3}{2} = \frac{9}{8} = 1\frac{1}{8}$,

and $$2\frac{5}{8} \div \frac{3}{4} = \frac{\overset{7}{\cancel{21}}}{\underset{2}{\cancel{8}}} \times \frac{\cancel{4}}{\cancel{3}} = \frac{7}{2} = 3\frac{1}{2}.$$

DIVIDING BY A FRACTION

In Exs. 1–16 first divide as indicated, and then divide the second fraction by the first:

1. $\frac{7}{8} \div \frac{2}{3}$.
2. $\frac{5}{6} \div \frac{3}{4}$.
3. $\frac{3}{8} \div \frac{5}{6}$.
4. $\frac{5}{8} \div \frac{1}{5}$.
5. $\frac{5}{8} \div \frac{3}{5}$.
6. $\frac{3}{8} \div \frac{3}{4}$.
7. $\frac{3}{4} \div \frac{5}{8}$.
8. $\frac{5}{6} \div \frac{5}{8}$.
9. $\frac{9}{10} \div \frac{4}{5}$.
10. $\frac{5}{12} \div \frac{7}{8}$.
11. $\frac{3}{8} \div \frac{5}{12}$.
12. $\frac{7}{16} \div \frac{3}{32}$.
13. $\frac{5}{64} \div \frac{7}{32}$.
14. $\frac{3}{8} \div \frac{5}{16}$.
15. $\frac{3}{40} \div \frac{6}{25}$.
16. $\frac{7}{48} \div \frac{7}{12}$.

17. Divide 6 by $\frac{1}{2}$. How many half-yard pieces can be cut from 6 yd. of ribbon? from 14 yd. of ribbon?

18. If a recipe for cake calls for $\frac{1}{2}$ lb. of sugar, $6\frac{1}{2}$ lb. of sugar is enough for how many times as much cake?

19. How many towels, each requiring $\frac{5}{8}$ yd. of toweling, can be made from a piece $11\frac{7}{8}$ yd. long?

20. How many jelly glasses holding $\frac{1}{2}$ pt. each does Alice's mother need for 8 qt. of jelly?

21. How many bottles holding $\frac{7}{8}$ qt. each does Alice's mother need for 14 qt. of sirup?

22. How many mats, each $\frac{3}{4}$ yd. long, can be made from a strip of carpet 15 yd. long?

Dividing by a Mixed Number. In dividing by a mixed number, simply write the mixed number as a fraction, invert it, and multiply.

For example, suppose that you wish to divide $4\frac{2}{3}$ by $2\frac{4}{5}$. In this case write both mixed numbers as fractions, invert the second one, as here shown, and cancel.

$$\boxed{\begin{aligned} 4\tfrac{2}{3} \div 2\tfrac{4}{5} &= \tfrac{14}{3} \div \tfrac{14}{5} \\ &= \frac{\cancel{14}}{3} \times \frac{5}{\cancel{14}} \\ &= \tfrac{5}{3} = 1\tfrac{2}{3} \end{aligned}}$$

The improper fraction $\frac{5}{3}$ is then reduced to the mixed number $1\frac{2}{3}$.

DIVIDING BY A MIXED NUMBER

1. Divide $48\frac{3}{8}$ by $5\frac{3}{8}$. How many strips of cloth, each $5\frac{3}{8}$ yd. long, can you cut from a strip $48\frac{3}{8}$ yd. long?

2. How many strips of ribbon, each $2\frac{3}{4}$ yd. long, can be cut from a piece of ribbon that is $13\frac{3}{4}$ yd. long?

3. At $66\frac{2}{3}$¢ a yard, how many yards of cloth can be bought for $8? for $17? for $36?

Divide as follows:

4. $5 \div 2\frac{1}{2}$.
5. $3 \div 1\frac{1}{2}$.
6. $3 \div 2\frac{1}{2}$.
7. $4\frac{1}{2} \div 1\frac{1}{2}$.
8. $4\frac{1}{2} \div 2\frac{1}{4}$.
9. $5\frac{1}{2} \div 2\frac{3}{4}$.
10. $7\frac{1}{2} \div 2\frac{1}{2}$.
11. $7 \div 3\frac{3}{8}$.
12. $2 \div 3\frac{1}{4}$.
13. $3 \div 3\frac{1}{2}$.
14. $5 \div 2\frac{2}{3}$.
15. $2 \div 5\frac{1}{3}$.
16. $28 \div 4\frac{2}{3}$.
17. $46 \div 3\frac{5}{6}$.
18. $2\frac{1}{8} \div 3\frac{1}{16}$.
19. $4\frac{3}{16} \div 3\frac{3}{8}$.
20. $3\frac{3}{8} \div 4\frac{3}{16}$.
21. $16\frac{4}{5} \div 4\frac{1}{5}$.
22. $15\frac{1}{3} \div 3\frac{5}{6}$.
23. $17\frac{1}{2} \div 2\frac{1}{2}$.
24. $9\frac{7}{12} \div 5\frac{3}{4}$.

Relations of Numbers. Suppose that 3 lemons cost 10¢. How much will 1 doz. lemons cost? That is, the cost of 1 doz. is how many times the cost of 3 lemons? What is the final result? How did you find it?

Could you have given the answer by first finding the cost of 1 lemon and then multiplying it by 12? Would this have been a better method than the one suggested above? State your reasons.

RELATIONS OF NUMBERS

All work oral

1. If 4 bananas cost 15¢, how much will 1 doz. cost?

2. If you can buy 5 apples for a quarter, how much must you pay for 10 apples? for 15 apples?

3. At 3 collars for 50¢, how much must John pay for ½ doz. collars? for 9 collars? for 2 doz. collars?

4. At 4 cans for 70¢, how much must Mrs. Reed pay for 8 cans of tomatoes? for 1 doz. cans? for 1½ doz. cans?

5. If you can buy tennis balls at the rate of 3 for $1, how much must you pay for ½ doz.? for 9? for a dozen?

In each of the following pairs of numbers state how many times as large as the second the first is:

6. 20, 5.
7. 20, 2½.
8. 20, 1¼.
9. 30, 7½.
10. 35, 5.
11. 35, 2½.
12. 35, 1¼.
13. 35, 3½.
14. 40, 10.
15. 40, 20.
16. 50, 25.
17. 40, 2½.
18. 60, 30.
19. 60, 7½.
20. 80, ½.
21. 70, 3½.

Fractional Parts. If a baseball team plays 28 games and wins 16 of them, what fraction of the games does it win?

Here we wish to find what fraction 16 is of 28.

Just as 1 is $\frac{1}{2}$ of 2, and 2 is $\frac{2}{4}$ of 4, so

$$16 \text{ is } \tfrac{16}{28}, \text{ or } \tfrac{4}{7}, \text{ of } 28.$$

That is, the team wins $\frac{4}{7}$ of the games played.

To find what fraction the first of two numbers is of the second, divide the first number by the second.

FINDING FRACTIONAL PARTS

Numbers 1 to 6, oral

1. There are 20 girls in a class of 40 pupils. The girls are what fraction of the class?

2. After walking 2 mi. on a hike of 6 mi., a troop of Boy Scouts has walked what part of the total distance?

Find what fraction the first of these numbers is of the second:

3. 5, 15.
4. 8, 24.
5. 16, 32.
6. 12, 18.

7. When you have gone 30 mi. on a railway journey of 240 mi., what part of the journey have you finished?

8. In Ex. 7, what part of the journey have you finished when you have gone 60 mi.? 80 mi.? 120 mi.?

Find what fraction the first of these numbers is of the second:

9. 35, 140.
10. 62, 310.
11. 45, 360.
12. \$90, \$270.
13. \$85, \$340.
14. \$67, \$335.
15. 27 in., 54 in.
16. 17 ft., 68 ft.
17. 19 ft., 133 ft.

USING WHAT YOU HAVE LEARNED

1. Martha has $\frac{3}{4}$ yd. of lace and needs $\frac{3}{8}$ yd. for trimming a waist. What part of the $\frac{3}{4}$ yd. does she need?

2. Anna has $2\frac{1}{2}$ yd. of velvet and needs $\frac{5}{8}$ yd. of it for trimming a hat. What part of the velvet does she need?

Find what part the first of these numbers is of the second:

3. $\frac{2}{3}$, $\frac{3}{4}$.
4. $\frac{3}{4}$, $\frac{7}{8}$.
5. $\frac{2}{5}$, $\frac{3}{4}$.
6. $\frac{1}{16}$, $\frac{3}{8}$.
7. $\frac{3}{16}$, $\frac{5}{8}$.
8. $\frac{5}{16}$, $\frac{7}{8}$.
9. $1\frac{1}{2}$, $4\frac{1}{2}$.
10. $2\frac{2}{3}$, 4.
11. $3\frac{5}{8}$, $4\frac{5}{6}$.
12. $16\frac{2}{3}$, $66\frac{2}{3}$.
13. $14\frac{1}{6}$, $42\frac{1}{2}$.
14. $16\frac{2}{3}$, $22\frac{1}{2}$.

Reduce to lowest terms these fractions found in divisions:

15. $\frac{30}{45}$.
16. $\frac{90}{120}$.
17. $\frac{105}{120}$.
18. $\frac{90}{288}$.
19. $\frac{225}{400}$.

20. Frank is 12 yr. old and his brother is 18 yr. old. Six years ago Frank's age was what part of his brother's? Six years from now it will be what part?

By canceling common factors, as in the case of ordinary fractions, reduce the following to lowest terms:

21. $\dfrac{7 \times 21 \times 77 \times 6}{49 \times 33 \times 42}$.

22. $\dfrac{20 \times 25 \times 36 \times 48}{27 \times 40 \times 15 \times 12}$.

23. $\dfrac{32 \times 27 \times 44 \times 39}{55 \times 72 \times 36 \times 8 \times 3}$.

24. $\dfrac{125 \times 108 \times 51 \times 8}{40 \times 45 \times 85 \times 12 \times 3}$.

By reducing to improper fractions, and by using cancellation as in Exs. 21–24, multiply as follows:

25. $13\frac{1}{3} \times 32\frac{1}{5} \times 16\frac{7}{8}$.
26. $27\frac{1}{5} \times 24\frac{1}{2} \times 37\frac{1}{2}$.
27. $42\frac{2}{3} \times 66\frac{2}{3} \times 74\frac{1}{4}$.
28. $7\frac{3}{5} \times 2\frac{1}{7} \times 5\frac{1}{4} \times 21\frac{1}{3}$.

REVIEW OF FRACTIONS

Add as follows:

1. $\frac{1}{2} + \frac{1}{2}$.

2. $\frac{1}{2} + \frac{1}{3}$.

3. $2\frac{1}{2} + 2\frac{1}{2}$.

4. $2\frac{1}{2} + 3\frac{1}{3}$.

5. $2\frac{1}{4} + 3\frac{3}{8}$.

6. $3\frac{1}{3} + 3\frac{2}{3}$.

7. $2\frac{3}{4} + 3\frac{2}{3}$.

8. $3\frac{5}{6} + 3\frac{2}{3}$.

Subtract as follows:

9. $\frac{7}{8} - \frac{1}{2}$.

10. $\frac{1}{3} - \frac{1}{4}$.

11. $3\frac{3}{8} - 2\frac{1}{2}$.

12. $7\frac{3}{4} - 4\frac{2}{5}$.

13. $5\frac{2}{3} - 3\frac{1}{2}$.

14. $2\frac{2}{3} - 1\frac{7}{8}$.

15. $7\frac{3}{4} - 4\frac{7}{8}$.

16. $6\frac{1}{2} - 3\frac{7}{8}$.

Multiply as follows:

17. $3 \times 3\frac{1}{3}$.

18. $2 \times 3\frac{7}{8}$.

19. $2\frac{3}{4} \times 8$.

20. $2\frac{1}{3} \times 3\frac{1}{3}$.

21. $2\frac{3}{8} \times 3\frac{1}{2}$.

22. $3\frac{1}{3} \times 4\frac{1}{2}$.

23. $3\frac{7}{8} \times 2\frac{1}{2}$.

24. $2\frac{1}{2} \times 4\frac{1}{4}$.

Divide as follows:

25. $\frac{5}{8} \div \frac{1}{5}$.

26. $\frac{2}{3} \div \frac{1}{3}$.

27. $2\frac{5}{8} \div \frac{3}{4}$.

28. $2 \div \frac{2}{3}$.

29. $5\frac{2}{3} \div 4\frac{2}{3}$.

30. $\frac{3}{4} \div 3\frac{1}{2}$.

31. $2\frac{3}{8} \div 2\frac{3}{4}$.

32. $3\frac{5}{8} \div 1\frac{1}{8}$.

33. Allowing $3\frac{3}{8}$ yd. to a suit of clothes, how many suits can be made from a roll of cloth containing 108 yd.?

34. To divide a sheet of paper $7\frac{7}{8}$ in. wide into 4 equal columns, what distance must be spaced off for each one?

35. After Edith has walked $\frac{3}{4}$ mi., she has gone $\frac{3}{8}$ of the way to Ida's house. Find the distance to Ida's house.

Copy and complete these statements:

36. The expression "$\frac{2}{3} \times$ " means to multiply by —— and divide by ——.

37. The expression "$\div \frac{2}{3}$" means to multiply by —— and divide by ——.

REVIEW PROBLEMS

1. In School No. 17 in Rochester there were recently 265 children in the kindergarten. If $\frac{3}{5}$ of them were boys, how many boys were there?

2. Ben's big brother Tom used 13 gal. of gasoline to drive his truck from Binghamton to Albany, a distance of 143 mi. At this rate, how far could he go on $17\frac{1}{2}$ gal.?

3. Before he moved to the farm Ben sold newspapers. He made $\frac{1}{2}$¢ on each paper. How much did he make in a week in which he sold 336 papers?

4. If it costs Helen 20¢ per week to buy cabbage for 3 rabbits, how much will it cost to feed 18 rabbits?

5. Last summer on Chautauqua Lake it took us 3 hr. to go 25 mi. in Mr. Clark's motor boat. At that rate, how long would it take us to go $37\frac{1}{2}$ mi.?

6. If the distance around the outside of an automobile tire is $8\frac{2}{3}$ ft., how many times must it turn to go 1 mi.?

7. If you pay 75¢ for 30 lb. of potatoes, how much must you pay for 50 lb.?

8. Mr. Clark paid $16.80 for gasoline and oil for his motor boat. If $\frac{1}{6}$ of this amount was for oil, what was the cost of the oil? of the gasoline?

9. How many boxes holding $\frac{3}{4}$ lb. each can be filled from 9 lb. of candy?

10. If $\frac{3}{4}$ lb. of cheese costs 30¢, how much do 3 lb. cost?

11. Find the cost of $3\frac{1}{4}$ bu. of potatoes @ $1.20, $2\frac{1}{2}$ qt. of milk @ 14¢, $4\frac{1}{2}$ doz. eggs @ 60¢, and $5\frac{1}{2}$ lb. of meat @ 32¢.

II. DECIMALS

A SILENT READING LESSON

We have already learned that \$3.25 means 3 dollars and 25 cents, which is the same as $\$3 + \$\frac{25}{100}$, or $\$3\frac{1}{4}$.

From this we see that a length of 3.25 ft. must mean a length of 3 ft. + $\frac{25}{100}$ ft., or $3\frac{1}{4}$ ft.

In writing \$3.25 we place the decimal point between the dollars and the fraction of a dollar, and in writing 3.25 ft. we place the decimal point between the feet and the fraction of a foot.

If on a time-table a certain place is given as 148.75 mi. from Albany, we know that the distance is $148\frac{3}{4}$ mi. That is, if we wish to write $2\frac{3}{4}$ mi. without using the fraction $\frac{3}{4}$, we may write it as 2.75 mi.

From the cases above we see that

4.75 in. means 4 in. + $\frac{75}{100}$ in., or $4\frac{3}{4}$ in.;
2.5 means $2 + \frac{5}{10}$, or $2\frac{1}{2}$;
0.5 (or .5) means $\frac{5}{10}$, $\frac{50}{100}$, 0.50, or $\frac{1}{2}$;
0.06 (or .06) means $\frac{6}{100}$, or $\frac{3}{50}$;
and 0.125 (or .125) means $\frac{125}{1000}$, or $\frac{1}{8}$.

We may write either 0.5 or .5 for five tenths, but we shall be more certain to notice the decimal point if we write 0.5.

We see that one decimal place means tenths, two decimal places mean hundredths, and three decimal places mean thousandths.

Similarly, tenths mean one decimal place, hundredths mean two decimal places, and thousandths mean three decimal places.

A LITTLE WRITING LESSON

Copy the following, filling in the blank spaces:

1. The fraction $\frac{1}{2}$ is called a *common* ——, or simply a *fraction*, and the fraction 0.5 is called a *decimal fraction*, or simply a *decimal*. The usual way of writing one half is ——, but the decimal way of writing it is 0.5.

2. As a common fraction, three tenths is written $\frac{3}{10}$; as a decimal, it is written as 0.3. As a common fraction, three hundredths is written $\frac{3}{100}$; as a decimal, it is written as ——.

3. The decimals 0.5, 0.75, and 0.05 may be written as common fractions; that is, $0.5 = \frac{5}{10}$, or $\frac{1}{2}$; $0.75 =$ ——, or $\frac{\ }{4}$; and $0.05 =$ ——, or ——.

4. The first place to the right of the decimal point is called tenths, the second decimal place is called ——, the third —— —— is called thousandths, and the —— decimal place is called ten-thousandths.

5. The common business way of reading the decimal 1.37 is "one point three-seven" or "one and thirty-seven ——," and 32.43 may be read "—— —— point —— ——," or it may be read "—— and —— ——."

Write both the common fraction and the decimal for

6. Four tenths.

7. Six tenths.

8. Seven tenths.

9. Nine tenths.

10. Five hundredths.

11. Seven hundredths.

12. Twenty-five hundredths.

13. Sixty-two hundredths.

A SILENT READING LESSON

The names of the decimal places that are most frequently used are shown in the following illustration:

Thousands	Hundreds	Tens	Units	(Decimal point)	Tenths	Hundredths	Thousandths
8	2	0	5	.	3	7	5

This number is read " eight thousand two hundred five *and* three hundred seventy-five thousandths," or " eight-two-o-five point three-seven-five."

The places beyond thousandths are ten-thousandths, hundred-thousandths, millionths, and so on.

In a decimal the denominator is shown by the position of the decimal point. Thus, 0.9 is nine tenths, the point indicating that the denominator is 10. Similarly, 0.25 is twenty-five hundredths, and 0.05 is five hundredths.

In writing decimals, write only the numerator of the fraction, inserting such zeros as may be necessary to indicate the denominator, and place the decimal point before the tenths.

Thus, the number two hundred four thousandths in figures is .204, or, as usually written, it is 0.204. Similarly, the number seventeen thousandths is 0.017.

READING AND WRITING DECIMALS

Numbers 1 to 35, oral

Read the following statements and tell the meaning of each:

1. On a motor trip Ethel went 102.7 mi. in one day.

2. The average height of the boys in our class is 55.2 in.

3. The average lifting power of the boys in our class is 52.9 lb. for the right forearm and 48.5 lb. for the left.

4. Cork is 0.24 times as heavy as water and granite is 2.78 times as heavy as water.

Read the following numbers:

5. 0.1.
6. 0.3.
7. 0.7.
8. 1.7.
9. 3.7.
10. 4.8.
11. 5.6.
12. 0.9.
13. 2.9.
14. 7.9.
15. 9.2.
16. 9.7.
17. 0.15.
18. 2.15.
19. 7.15.
20. 5.17.
21. 5.71.
22. 1.75.
23. 2.25.
24. 4.10.
25. 5.50.
26. 6.75.
27. 0.15.
28. 2.15.
29. 7.125.
30. 5.712.
31. 5.217.
32. 1.125.
33. 1.250.
34. 2.375.

35. What is the name of the first decimal place? of the second? of the third?

Write the following numbers as decimals:

36. Three hundredths.
37. Seven hundredths.
38. One and three tenths.
39. Two and seven tenths.
40. Twenty-six hundredths.
41. 9 tenths.
42. 9 hundredths.
43. 19 hundredths.
44. 19 thousandths.
45. 195 thousandths.

A SILENT READING LESSON

With a steel tape which was marked in feet and tenths of a foot, a class measured the sides of a large triangular flower bed in a park. The sides were found to be 48.7 ft., 53.8 ft., and 62.9 ft.

Writing the numbers in a column, and adding as with United States money, the class found that it was 165.4 ft. around the bed.

No measurement of length, area, volume, or weight is ever exact.

$$\begin{array}{r} 48.7 \\ 53.8 \\ \underline{62.9} \\ 165.4 \end{array}$$

With instruments such as ordinary rulers and tapes the error may be small, and with finer instruments still smaller; but the measurement is never exact.

In the above case the class measured to the nearest tenth of a foot (0.1 ft.). With such a tape we cannot measure to the nearest 0.01 ft.; that is, any measurement made with it is a measurement *to the nearest 0.1 ft. only.*

The class found an old automobile highway that was being extended a distance of 4.10 mi. After 0.26 mi. is completed, how much is still to be built?

Since one distance, 0.26 mi., is given to the nearest 0.01 mi., the other, 4.10 mi., is given in the same way, and not as 4.1 mi. If one measurement can be found to hundredths, the other can also be so found.

$$\begin{array}{r} 4.10 \\ \underline{.26} \\ 3.84 \end{array}$$

Subtracting, as with United States money, the class finds that there are still 3.84 mi. to be built.

No result can be more nearly exact than the measurements from which it is found.

DRILL CHART IN DECIMALS

Add as follows:

1. $7.46	**2.** $3.28	**3.** 58.98	**4.** 18.29	**5.** 72.74
8.62	6.42	26.88	.09	31.08
3.88	8.00	6.09	13.50	7.89

Subtract as follows:

6. 80.22	**7.** 60.28	**8.** 69.60	**9.** 75.32	**10.** 47.04
20.08	31.99	22.98	38.96	8.60

Add as follows:

11. 62.86	**12.** 72.36	**13.** 3.627	**14.** 612.4	**15.** 8.289
2.98	3.98	3.684	328.2	.498
51.82	85.63	2.928	287.7	.489
33.59	39.36	4.004	428.6	4.060
41.23	42.05	2.800	371.8	7.000
4.89	20.68	3.304	366.6	.076

Subtract as follows:

16. 17.00	**17.** 27.00	**18.** 27.00	**19.** 270.0	**20.** 5.175
6.12	8.12	18.12	182.7	2.299

Add the following:

21. 18.27 ft., 32.65 ft.

22. 11.32 ft., 26.99 ft.

23. 28.4 mi., 46.7 mi.

24. 16.92 mi., 28.37 mi.

25. 8.7 ft., 6.9 ft., 7.2 ft.

26. 5.8 in., 7.2 in., 9.0 in.

27. 7.9 in., 8.3 in., 8.4 in.

28. 8.1 mi., 6.9 mi., 8.7 mi.

ADDING AND SUBTRACTING DECIMALS

1. Henry's bicycle has a cyclometer for measuring the distance traveled. When he started out one morning, the cyclometer registered 127.6, which means 127.6 mi. When he returned, it registered 141.3. How far did he go?

2. On a four-day automobile trip the distances traveled were 127.8 mi., 146.3 mi., 162.9 mi., and 96.4 mi. What was the total distance traveled?

3. The mileage dial of an automobile speedometer registered 2134.8 mi. in the morning and 2307.4 mi. at night. How many miles did the car go during the day?

4. The mileage dial of an automobile speedometer registered 1872.9 mi. at the beginning of a trip of 298.8 mi. How much did it register at the end of the trip?

5. The normal temperature of the body when a person is well is 98.6 degrees. When Clara had the measles, her temperature rose to 102.1 degrees. How much was this above normal?

6. An airplane made 126.3 mi. in the first hour of a trip, 112.8 mi. in the second hour, and 94.9 mi. in the third hour. Find the total distance covered.

7. The weather bureau reported the rainfall in New York City for seven successive days to be 1.35 in., 1.80 in., 1.25 in., 0 in., 0.08 in., 0 in., 1.07 in. Find the total rainfall.

8. The distance from New York City to Peekskill is given in the time-table as 40.67 mi. From Peekskill to Hudson, which is on the same line, the distance is given as 73.05 mi. How far is it from New York City to Hudson?

SPEED TEST CHART IN ADDING DECIMALS

Copy and add the following in 1 min.:

1. 2.9	**2.** 13.4	**3.** 14.69	**4.** 28.30	**5.** 7.734
4.6	5.9	7.06	33.98	2.976

Copy and add the following in 4 min.:

6. 4.5	**8.** 17.3	**10.** 33.48	**12.** 36.83	**14.** 8.824
3.6	14.6	7.80	3.08	2.499
6.9	7.6	18.09	.79	.696

7. 6.9	**9.** 42.8	**11.** 13.68	**13.** 74.86	**15.** 9.008
7.8	14.9	9.56	.86	7.860
2.5	8.0	4.46	38.00	.009
3.2	7.6	1.27	8.02	.016

Copy and add the following in 8 min.:

16. 68.74	**18.** 49.06	**20.** 63.78	**22.** 39.86	**24.** 6.768
4.36	8.68	36.86	4.23	5.390
21.09	44.00	8.08	29.68	6.009
37.62	6.70	43.50	42.00	.620
.63	10.07	3.32	.62	4.371

17. 33.49	**19.** 42.48	**21.** 61.69	**23.** 72.98	**25.** 7.956
62.68	.90	47.98	4.48	3.754
81.89	35.00	52.79	67.00	9.683
.73	3.98	4.28	6.93	8.426
43.06	42.09	47.60	33.98	9.900

Changing a Decimal to a Fraction. At a bargain sale Edward finds that he can buy a radio set for 0.25 less than the regular price. At another store he is told that he can buy a similar set for $\frac{1}{5}$ less than the same regular price. Can you tell which is the larger reduction?

You know that 0.25 means $\frac{25}{100}$, which is equal to $\frac{1}{4}$, and so the first reduction is the larger. That is,

To change a decimal to a fraction, write the decimal as a fraction with the denominator that is indicated by the number of decimal places and reduce to lowest terms.

In a case like that of $0.33\frac{1}{3}$ we proceed as follows:

$$0.33\tfrac{1}{3} = \frac{33\frac{1}{3}}{100} = \frac{\frac{100}{3}}{100} = \frac{100}{300} = \frac{1}{3}.$$

A number like 3.75, which consists of a whole number and a decimal, is sometimes called a *mixed decimal.*

CHANGING DECIMALS TO FRACTIONS

1. Which is the better for the buyer, a reduction of $\frac{1}{5}$ from the regular price or one of 0.15? Why is it better?

Change to fractions or to mixed numbers:

2. 0.6. **3.** 0.30. **4.** 0.825. **5.** 8.25. **6.** 0.625.

7. Express 0.125 as eighths. Find 0.125 of $3200.

8. Express $0.33\frac{1}{3}$ as thirds. Find $0.33\frac{1}{3}$ of $7110.75.

Express the following decimals as thirds:

9. $0.6\frac{2}{3}$. **11.** $1.3\frac{1}{3}$. **13.** $1.6\frac{2}{3}$. **15.** $2.66\frac{2}{3}$. **17.** $3.3\frac{1}{3}$.

10. $0.66\frac{2}{3}$. **12.** $1.33\frac{1}{3}$. **14.** $1.66\frac{2}{3}$. **16.** $6.66\frac{2}{3}$. **18.** $7.33\frac{1}{3}$.

Annexing Zeros to a Decimal. Edward said that he paid twenty-four and a half dollars for his radio set. If he wrote \$24.5 on the board, did he write the amount in the customary business way? How should it be written?

While \$24.5 means $\$24\frac{1}{2}$, neither of these is the correct business form, since we usually write \$24.50.

Is there any difference in value between \$1.5 and \$1.50? between \$3.2 and \$3.20? We thus see that

Annexing zeros to a decimal does not change the value.

Decimals which have the same number of decimal places are sometimes called *similar decimals.*

To write decimals as similar decimals, annex or cut off zeros at the right until they have the same number of decimal places.

For example, we may write 0.3, 0.25, and 2.125 as 0.300, 0.250, and 2.125.

ANNEXING ZEROS TO DECIMALS

1. In subtracting 1.75 from 2.6, it is easier to write the numbers as similar decimals. Write them in this way.

Express the following as similar decimals:

2. 0.5, 0.35.
3. 0.2, 0.02.
4. 0.5, 0.008.
5. 0.8, 0.648.
6. 0.8, 0.900.
7. 0.7, 0.080.
8. 0.12, 0.508.
9. 0.35, 0.008.
10. 0.89, 0.060.

11. Write in the easiest form for subtraction, and then subtract 18.25 from 34.3; 29.004 from 35.06; 13.006 from 17.23.

Changing a Fraction to a Decimal. The average height of the girls in a certain class was found to be $55\frac{4}{5}$ in. In reporting this fact the teacher used a decimal. How is $\frac{4}{5}$ written as a decimal?

$$\boxed{\begin{array}{r} 0.8 \\ 5\overline{)4.0} \end{array}}$$

Since $\frac{4}{5}$ means the same as $4 \div 5$, or $4.0 \div 5$, we simply divide 4.0, placing the decimal point as we do in dividing United States money.

To change a fraction to a decimal, place a decimal point after the numerator and divide by the denominator.

A fraction cannot always be changed to an exact decimal. A fraction or a series of dots is sometimes used to indicate this fact; for example, we may write, $\frac{1}{3} = 0.33\frac{1}{3}$, or $\frac{1}{3} = 0.333 \cdots$. More often we write simply 0.33 or 0.333, discarding the fraction.

In the case of $\frac{2}{3}$ we have $\frac{2}{3} = 0.6\frac{2}{3} = 0.66\frac{2}{3} = 0.666\frac{2}{3}$. We usually write 0.67 for $0.66\frac{2}{3}$ or 0.667 for $0.666\frac{2}{3}$; that is, we discard the fraction if it is less than $\frac{1}{2}$, and increase the last figure by 1 if the fraction is $\frac{1}{2}$ or greater.

In some cases we write $0.333+$ and $0.667-$, indicating that in one case the exact value is more than 0.333 and in the other less than 0.667.

CHANGING FRACTIONS TO DECIMALS

1. John reads in a book that lead is $11\frac{7}{20}$ times as heavy as water. Write $11\frac{7}{20}$ with a decimal.

Change to decimals having not more than three places:

2. $\frac{3}{4}$.
3. $\frac{5}{8}$.
4. $\frac{4}{5}$.
5. $\frac{7}{8}$.
6. $\frac{1}{16}$.
7. $\frac{3}{16}$.
8. $\frac{13}{16}$.
9. $\frac{1}{32}$.
10. $\frac{5}{32}$.
11. $\frac{9}{25}$.
12. $\frac{5}{6}$.
13. $\frac{8}{9}$.

A SILENT READING LESSON

In a book about France, Mary read of a tower that was 35 meters high, and she wanted to know how many feet this was. Her father told her that a meter was equivalent to 3.28 ft. Mary saw that she would have to multiply 3.28 ft. by 35.

She multiplied just as she would have multiplied \$3.28 by 35, and found that the height of the tower was 114.80 ft.

$$\boxed{\begin{array}{r} 3.28 \\ 35 \\ \hline 1640 \\ 984 \\ \hline 114.80 \end{array}}$$

If we are multiplying $\frac{7}{1000}$ by 5, we see that we have

$$5 \times \frac{7}{1000} = \frac{35}{1000},$$

and so

$$5 \times 0.007 = 0.035.$$

That is, we can tell where to put the decimal point by comparing the decimal with the common fraction.

John read in a machinery handbook that the number of inches around a pulley, which is 7 in. across, is 7 × 3.142.

John multiplied as with whole numbers. He saw that if he multiplied 0.002 by 7, the result would be thousandths. He thus saw that there were three decimal places in the result, and that the distance was 21.994 in., or nearly 22 in.

$$\boxed{\begin{array}{r} 3.142 \\ 7 \\ \hline 21.994 \end{array}}$$

In multiplying decimals, there are as many decimal places in the product (result) as there are in the two numbers together.

That is, in multiplying 3.28 by 35 Mary had 2 + 0, or 2 decimal places, and in multiplying 3.142 by 7 John had 3 + 0, or 3 decimal places.

DRILL CHART IN MULTIPLYING DECIMALS

Multiply as follows:

1. 5 × 8.06 ft.
2. 7 × 5.38 in.
3. 4 × 5.32 mi.
4. 6 × 3.08 in.
5. 9 × 8.75 ft.
6. 7 × 0.96 oz.
7. 8 × 0.73 ft.
8. 5 × 3.77 yd.
9. 7 × 8.08 ft.
10. 2 × 4.26 in.
11. 3 × 5.08 ft.
12. 7 × 6.35 lb.
13. 19 × 4.77.
14. 36 × 5.32.
15. 15 × 4.28.
16. 42 × 7.22.
17. 94 × 3.28.
18. 81 × 4.48.
19. 85 × 8.23.
20. 92 × 3.32.
21. 34 × 5.81.
22. 78 × 8.22.
23. 66 × 4.92.
24. 62 × 2.48.
25. 28 × 20.09.
26. 94 × 66.82.
27. 73 × 81.62.
28. 55 × 70.03.
29. 15 × 44.34.
30. 19 × 19.96.
31. 50 × 27.85.
32. 36 × 49.82.
33. 40 × 17.77.
34. 32 × 28.65.
35. 80 × 39.37.
36. 92 × 64.28.

In multiplying 2.8 by $3\frac{1}{2}$, multiply as if you had $3\frac{1}{2} \times 28$, and point off 0 + 1 decimal places. Multiply in the same way in each of these cases:

37. $2\frac{1}{2} \times 12.4$.
38. $6\frac{3}{4} \times 12.8$.
39. $7\frac{1}{2} \times 22.8$.
40. $9\frac{1}{8} \times 64.8$.
41. $8\frac{2}{5} \times 24.5$.
42. $6\frac{1}{3} \times 18.6$.
43. $9\frac{2}{3} \times 12.9$.
44. $8\frac{7}{8} \times 25.6$.
45. $3\frac{1}{2} \times 68.38$.
46. $3\frac{1}{2} \times 46.64$.
47. $2\frac{1}{2} \times 41.14$.
48. $6\frac{1}{2} \times 51.10$.
49. $2\frac{1}{4} \times 88.60$.
50. $4\frac{3}{4} \times 70.08$.
51. $2\frac{3}{4} \times 86.04$.
52. $7\frac{1}{4} \times 92.36$.
53. $5\frac{2}{3} \times 8.763$.
54. $7\frac{1}{3} \times 9.645$.
55. $4\frac{5}{8} \times 7.096$.
56. $3\frac{7}{8} \times 3.496$.
57. $5\frac{3}{8} \times 2.736$.
58. $8\frac{1}{6} \times 1.728$.
59. $3\frac{5}{6} \times 3.456$.
60. $3\frac{1}{8} \times 6.912$.

A SILENT READING LESSON

Edward's father is building a cylindric silo to hold feed for his cattle. The silo is 15.3 ft. in diameter; that is, it is 15.3 ft. across. He asks Edward to find the length of an iron band that will just go round the silo, telling him that he can do this by multiplying 15.3 ft. by 3.1.

Multiplying in the ordinary way, Edward finds that the result is 4743. He then places the decimal point as explained on page 128. He thus finds that the length of the band, to the nearest 0.1 ft., is 47.4 ft.

$$\begin{array}{r} 15.3 \text{ ft.} \\ 3.1\phantom{\text{ ft.}} \\ \hline 153\phantom{\text{ ft.}} \\ 459\phantom{0\text{ ft.}} \\ \hline 47.43 \text{ ft.} \end{array}$$

In another case Edward had to multiply 0.42 by 0.35. He first multiplied 42 by 35 and found that the result was 1470. Then thinking of the fractions, Edward saw that

$$\frac{42}{100} \times \frac{35}{100} = \frac{1470}{10000},$$

and so $0.42 \times 0.35 = 0.1470$;

that is, he pointed off four places in the product (result).

$$\begin{array}{r} 0.42 \\ 0.35 \\ \hline 210 \\ 126 \\ \hline 0.1470 \end{array}$$

In multiplying by a decimal, multiply as with whole numbers, and then, beginning at the right, point off as many decimal places in the product as there are decimal places in the two numbers together.

Thus, in the first case above Edward had 1 + 1 decimal places, and so pointed off two places in the result; and in the second case he had 2 + 2 decimal places, and so pointed off four places.

USING DECIMALS

1. Taking the rod as 16.5 ft., how many feet are there in 19 rd.? in 27 rd.? in 33.6 rd.?

2. In city surveying, decimal parts of a foot are generally used, and a "front foot" means 1 ft. along the street on which the land fronts. At $36.50 per front foot, find the cost of a piece of land with a frontage of 33.3 ft.

3. If we multiply 1.7 by $12.76, the result is $216.92, $2.1692, $21.693, or $21.692. Without actually multiplying, state which it is.

4. The circumference of (the distance around) a circle is 3.1416 times the diameter. If the diameter is 13.4 ft., what is the circumference? Since the diameter is given to the nearest 0.1 ft., give the result in the same way.

5. A cubic foot of water weighs 62.5 lb., and ice is 0.92 times as heavy as water. How much does 1 cu. ft. of ice weigh? Why should you give the result only to the nearest 0.1 lb.?

6. Oak is 1.17 times as heavy as water (see Ex. 5). How much does 1 cu. ft. of oak weigh?

Find the value in each of the following cases:

7. $3.14 + 1.89$.
8. $3.14 - 1.89$.
9. 3.14×1.89.
10. 3.14×18.9.
11. 3.14×0.189.
12. 3.14×1890.
13. $0.318 + 0.125$.
14. $0.318 - 0.125$.
15. 0.318×0.125.
16. 0.318×1.25.
17. 0.318×12.5.
18. 0.318×1250.
19. $4.19 + 3.09$.
20. $4.19 - 3.09$.
21. 4.19×3.09.
22. 4.19×30.9.
23. 4.19×0.309.
24. 4.19×3090.

DRILL CHART IN MULTIPLYING DECIMALS

Numbers 1 to 24, oral

Multiply as follows:

1. 0.6 × 3.
2. 0.3 × 6.
3. 0.1 × 5.
4. 0.3 × 4.
5. 0.6 × 7.
6. 0.7 × 3.
7. 0.3 × 7.
8. 0.1 × 2.
9. 0.3 × 0.5.
10. 0.6 × 0.8.
11. 0.9 × 0.7.
12. 0.7 × 0.9.
13. 0.2 × 0.3.
14. 0.4 × 0.4.
15. 0.7 × 0.8.
16. 0.8 × 0.8.
17. 0.8 × 0.12.
18. 0.3 × 0.15.
19. 0.7 × 0.11.
20. 0.10 × 0.12.
21. 0.10 × 0.34.
22. 0.11 × 0.11.
23. 0.12 × 0.12.
24. 0.06 × 0.24.

Multiply, recording your time for each column:

25. 4 × 0.49.
26. 0.32 × 7.
27. 3 × 2.75.
28. 3 × 0.97.
29. 0.87 × 8.
30. 7 × 2.42.
31. 3 × 0.78.
32. 0.83 × 6.
33. 8 × 7.23.
34. 9 × 0.93.
35. 0.27 × 9.
36. 6 × 9.23.
37. 0.9 × 74.
38. 0.6 × 7.2.
39. 7.7 × 85.
40. 3.1 × 8.1.
41. 58 × 0.8.
42. 6.7 × 7.8.
43. 0.6 × 96.
44. 0.7 × 7.8.
45. 1.8 × 67.
46. 2.9 × 3.4.
47. 0.6 × 74.
48. 0.8 × 4.9.
49. 1.4 × 35.
50. 1.4 × 2.6.
51. 0.5 × 48.
52. 0.5 × 4.8.
53. 6.3 × 4.86.
54. 96 × 9.65.
55. 5.4 × 3.53.
56. 73.2 × 98.6.
57. 0.42 × 390.

Multiplying by 10, 100, and 1000. We multiply a whole number by 10 by simply annexing a zero, as follows:

$10 \times 35 = 350 \qquad 10 \times 2750 = 27{,}500$

In multiplying by 100 we annex two zeros, and in multiplying by 1000 we annex three zeros, as follows:

$100 \times 125 = 12{,}500 \qquad 1000 \times 350 = 350{,}000$

Multiplying decimals by 10, 100, and 1000 is similar to multiplying dollars and cents; that is, we move the decimal point to the right one place for 10, two places for 100, three places for 1000, and so on, as in the following cases:

$100 \times 32.5 = 3250 \qquad 1000 \times 2.752 = 2752$

$1000 \times \$12.75 = \$12{,}750 \qquad 1000 \times 37.25 = 37{,}250$

MULTIPLYING BY 10, 100, AND 1000

Multiply the following numbers by 10:

1. 35.
2. 42.
3. 3.5.
4. 4.2.
5. 0.35.
6. 1.42.
7. 23.7.
8. 2.37.
9. 625.
10. 62.5.

Multiply the following by 100:

11. 125.
12. 12.5.
13. 1.25.
14. 6.125.
15. 61.25.
16. 612.5.
17. 3.75.
18. 37.5.
19. 375.
20. 150.
21. 15.0.
22. 1.50.
23. 2725.
24. 27.25.
25. 0.2725.

Multiply the following by 1000:

26. 275.
27. 27.5.
28. 2.75.
29. 0.275.
30. 627.5.
31. 62.75.
32. 727.5.
33. 8275.
34. 92.25.
35. 0.02.
36. 0.12.
37. 0.125.
38. 0.0001.
39. 0.0011.
40. 0.0111.

Multiplying by Multiples of 10. We have just found how to multiply by 10, 100, 1000, and so on for other powers of 10. We also know how to multiply by whole numbers like 40, 700, and so on. Multiplying decimals by such numbers is similar to multiplying dollars and cents.

That is, in multiplying 487.7 by 600, move the decimal point two places to the right and multiply by 6.

In multiplying 89.07 by 740, move the decimal point one place to the right and multiply by 74.

MULTIPLYING BY MULTIPLES OF 10

Numbers 1 to 16, oral

State the value in each of the following cases:

1. $5 \times \frac{1}{5}$.
2. $4 \times \frac{1}{4}$.
3. $7 \times \frac{1}{7}$.
4. $10 \times \frac{1}{10}$.
5. 10×0.1.
6. 10×6.
7. 10×0.6.
8. 20×0.6.
9. $100 \times \frac{1}{100}$.
10. 100×0.01.
11. 100×0.02.
12. 100×0.65.
13. $1000 \times \frac{1}{1000}$.
14. 1000×0.001.
15. 1000×0.125.

16. How do you multiply a decimal by 40? by 600? by 1000? by 2000? by 70? by 700? by 7000?

Multiply as follows:

17. 50×8.74.
18. 40×6.89.
19. 70×7.81.
20. 30×86.4.
21. 40×36.6.
22. 80×45.49.
23. 60×725.8.
24. 700×67.09.
25. 500×61.6.
26. 1000×2.25.
27. 4970×83.2.
28. 2000×3.725.
29. 7250×67.28.
30. 6220×750.4.
31. 5600×895.7.
32. 3500×774.66.
33. 4500×508.07.
34. $25,000 \times 1.25$.

Short Division by a Whole Number. Suppose that you buy two melons at the same price, paying 30¢, or $0.30, for the two. How much do they cost apiece?

$$\begin{array}{r}.\\2\overline{)0.30}\\[1em]0.15\\2\overline{)0.30}\end{array}$$

First place the decimal point just above the decimal point in the number divided.

Then divide as with whole numbers.

The result is 0.15; that is, the cost is 15¢.

Check. $2 \times 0.15 = 0.30.$

Similarly, in dividing 27.125 by 5, we have

$$\begin{array}{r}\\5\overline{)27.125}\end{array} \quad \text{and} \quad \begin{array}{r}5.425\\5\overline{)27.125}\end{array}$$

SHORT DIVISION BY A WHOLE NUMBER

Divide as indicated:

1. $\begin{array}{r}.\\2\overline{)2.48}\end{array}$
2. $\begin{array}{r}.\\3\overline{)2.49}\end{array}$
3. $\begin{array}{r}.\\4\overline{)2.48}\end{array}$
4. $5\overline{)2.65}$
5. $6\overline{)2.52}$
6. $7\overline{)2.45}$
7. $8\overline{)2.56}$
8. $9\overline{)25.2}$
9. $2\overline{)32.4}$
10. $3\overline{)0.87}$
11. $\begin{array}{r}.\\2\overline{)0.136}\end{array}$
12. $\begin{array}{r}.\\2\overline{)1.360}\end{array}$
13. $\begin{array}{r}.\\3\overline{)13.6}\end{array}$
14. $4\overline{)13.6}$
15. $5\overline{)1.360}$
16. $5\overline{)1.36}$
17. $6\overline{)13.8}$
18. $6\overline{)1.38}$
19. $6\overline{)0.138}$
20. $6\overline{)0.0138}$
21. $7\overline{)1540}$
22. $\begin{array}{r}.\\7\overline{)1.540}\end{array}$
23. $\begin{array}{r}.\\7\overline{)15.40}\end{array}$
24. $7\overline{)154.7}$
25. $7\overline{)1561}$
26. $7\overline{)156.1}$
27. $7\overline{)15.61}$
28. $7\overline{)1.561}$
29. $7\overline{)0.1561}$
30. $7\overline{)1.5561}$
31. $8\overline{)656}$
32. $\begin{array}{r}.\\8\overline{)65.6}\end{array}$
33. $\begin{array}{r}.\\8\overline{)65.68}\end{array}$
34. $8\overline{)6.568}$
35. $8\overline{)0.6568}$
36. $8\overline{)2.5826}$
37. $8\overline{)22.568}$
38. $8\overline{)225.68}$
39. $8\overline{)2256.8}$
40. $9\overline{)22.563}$

Dividing by a Whole Number. On an automobile trip Walter and his father went 925.9 mi. in 13 da. Find to the nearest 0.1 mi. the average distance which they went per day.

Dividing as with United States money, we find that the average daily distance was a little over 71.2 mi.

If we wish to show that the result is greater than 71.2 but less than 71.25, we may write 71.2 +. If the result is 71.25 or greater, but less than 71.3, we may write 71.3 −. More often we write simply 71.2 or 71.3, omitting the signs.

$$\begin{array}{r}
71.2\\
13\overline{)925.9}\\
\underline{91}\\
15\\
\underline{13}\\
2\,9\\
\underline{2\,6}\\
3
\end{array}$$

To find the nearest tenth, find the second decimal figure and change the first decimal figure if necessary.

DIVIDING BY A WHOLE NUMBER

In each of the following cases state the number of decimal places to which the result should be given, and then divide:

1. $2\overline{)3.4\text{ in.}}$
2. $6\overline{)4.72\text{ mi.}}$
3. $9\overline{)0.127\text{ in.}}$
4. $3\overline{)81.6\text{ in.}}$
5. $4\overline{)81.6\text{ in.}}$
6. $5\overline{)81.6\text{ in.}}$
7. $6\overline{)81.6\text{ in.}}$
8. $15\overline{)12.68\text{ mi.}}$
9. $14\overline{)1.278\text{ in.}}$
10. $32\overline{)12.3142\text{ in.}}$
11. $25\overline{)62.75\text{ in.}}$
12. $25\overline{)627.5\text{ in.}}$
13. $25\overline{)6.275\text{ in.}}$
14. $28\overline{)596.4\text{ in.}}$
15. $7\overline{)2.75\text{ ft.}}$
16. $18\overline{)6.70\text{ in.}}$
17. $23\overline{)0.592\text{ mi.}}$
18. $23\overline{)59.20\text{ mi.}}$
19. $23\overline{)5.920\text{ mi.}}$
20. $23\overline{)592.0\text{ mi.}}$
21. $36\overline{)0.144\text{ in.}}$

22. The distance around a triangle which has its sides all equal is 23.1 in. What is the length of each side?

23. The distance around a six-sided figure with equal sides is 15.24 in. What is the length of each side?

24. Four boys who are going camping pay $21.48 for their outfit. How much is each boy's share?

25. The distance between two places is known to be 12.5 mi. What is one third of the distance between them?

26. The distance from second base straight to home plate is 127.3 ft. Find the distance from home plate to a point one fourth of the way to second base.

Divide, giving each result to the nearest tenth:

27. $6\overline{)6277.8}$
28. $8\overline{)2748.1}$
29. $33\overline{)262.9}$
30. $68\overline{)702.7}$
31. $22\overline{)434.8}$
32. $37\overline{)517.6}$
33. $72\overline{)44.2}$
34. $29\overline{)36.7}$
35. $12\overline{)89.8}$

Divide, giving each result to the nearest hundredth:

36. 52.83 ÷ 31.
37. 42.06 ÷ 44.
38. 39.34 ÷ 73.
39. 72.09 ÷ 28.
40. 67.34 ÷ 33.
41. 40.73 ÷ 46.
42. 50.06 ÷ 57.
43. 63.42 ÷ 67.
44. 22.37 ÷ 43.
45. 44.86 ÷ 38.
46. 42.73 ÷ 16.
47. 68.42 ÷ 37.
48. 72.26 ÷ 35.
49. 80.71 ÷ 53.
50. 39.09 ÷ 32.
51. 37.02 ÷ 42.
52. 53.09 ÷ 72.
53. 60.27 ÷ 28.
54. 50.09 ÷ 762.
55. 81.24 ÷ 306.
56. 93.03 ÷ 600.

Divide, giving each result to the nearest thousandth:

57. 21.143 ÷ 23.
58. 63.856 ÷ 47.
59. 9.030 ÷ 237.

Dividing by Powers of 10. Just as in multiplying by powers of 10 we move the decimal point to the right, so in dividing we move it to the left.

To divide by 10, move the decimal point one place to the left.

Thus, 7.2 ÷ 10 = 0.72, and 69.872 ÷ 10 = 6.9872.

To divide by 100, move the decimal point two places to the left, and to divide by 1000, move it three places to the left.

Thus, 8.7 ÷ 100 = 0.087, and 123.62 ÷ 1000 = 0.12362.

Dividing by Multiples of Powers of 10. *To divide by a number ending in zeros, move the decimal point in the number divided as many places to the left as there are zeros in the divisor (the number by which you divide). Then divide by the number preceding the zeros.*

That is, in dividing 216 by 600, proceed as if you were to divide 2.16 by 6. Then, as here shown, 216 ÷ 600 = 0.36.

$$\boxed{\begin{array}{r} 0.36 \\ 6\cancel{0}\cancel{0}\overline{)2.16} \end{array}}$$

DIVIDING BY TENS

Divide each of the following by 10 and also by 100:

1. 375. 2. 37. 3. 3. 4. 0.2. 5. 3.7. 6. 13.5.

Divide each of the following by 1000:

7. 300. 8. 30. 9. 3. 10. 25. 11. 250. 12. 275.

Divide as follows:

13. 70)39.9
14. 60)70.8
15. 290)5191
16. 470)296.1
17. 4000)2984
18. 3000)8436
19. 21,000)882
20. 37,000)888
21. 1250)63.75

Dividing a Whole Number by a Decimal. Before learning how to divide by a decimal it is necessary to notice that we get the same results in each of these four divisions:

$$\begin{array}{r}3\\2\overline{)6}\end{array}\qquad\begin{array}{r}3\\20\overline{)60}\end{array}\qquad\begin{array}{r}3\\200\overline{)600}\end{array}\qquad\begin{array}{r}3\\2000\overline{)6000}\end{array}$$

Multiplying both the number divided and the divisor by the same number does not change the result.

Thus, if we wish to divide 1.255 by 0.05, we move the decimal point two places to the right in each case so that we can divide by a whole number, and have

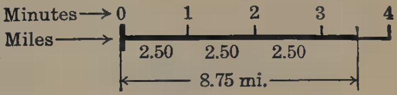
$$0.05\overline{)1.255}\qquad 5\overline{)125.5}\qquad\begin{array}{r}25.1\\5\overline{)125.5}\end{array}$$

DIVIDING A WHOLE NUMBER BY A DECIMAL

Divide as follows:

1. $2\overline{)4}$
2. $0.2\overline{)4}$
3. $0.02\overline{)4}$
4. $2\overline{)400}$
5. $0.2\overline{)400}$
6. $2\overline{)7}$
7. $0.2\overline{)7}$
8. $0.02\overline{)7}$
9. $2\overline{)70}$
10. $0.2\overline{)70}$
11. $3\overline{)6}$
12. $0.3\overline{)6}$
13. $3\overline{)60}$
14. $0.3\overline{)60}$
15. $0.03\overline{)6}$
16. $0.03\overline{)60}$
17. $0.003\overline{)6}$
18. $0.004\overline{)6}$
19. $0.005\overline{)6}$
20. $0.05\overline{)600}$
21. $5\overline{)10}$
22. $5\overline{)1.0}$
23. $5\overline{)1}$
24. $0.5\overline{)1}$
25. $0.5\overline{)100}$
26. $0.05\overline{)10}$
27. $0.05\overline{)1}$
28. $0.005\overline{)1}$
29. $0.005\overline{)10}$
30. $0.005\overline{)100}$

Getting Ready to Divide by a Decimal. If an airplane can fly 2.50 mi. a minute, how long will it take the plane to go 8.75 mi.?



As shown by this line, it will take as many minutes as the distance covered in 1 min. is contained in 8.75 mi. That is, we must find the result of dividing 8.75 by 2.50.

We move the decimal point as many places to the right in both the divisor and the number divided as is necessary to make the divisor a whole number, placing the decimal point for the result in the proper place. This should be done when writing the example.

$$\begin{array}{r} 3.5 \\ 25\overline{)87.5} \\ \underline{75} \\ 12\,5 \\ \underline{12\,5} \end{array}$$

In dividing by a decimal, move the decimal points as many places to the right as is necessary to make the divisor a whole number, place the decimal point for the result above the point in the number divided, and divide as with whole numbers.

DIVIDING BY A DECIMAL

State how many places you will move the decimal points to the right in each of these divisions:

1. $0.6\overline{)30}$
2. $0.6\overline{)3.0}$
3. $0.6\overline{)0.3}$
4. $0.6\overline{)0.03}$
5. $0.06\overline{)0.03}$
6. $1.2\overline{)1.44}$
7. $0.12\overline{)1.44}$
8. $0.12\overline{)14.4}$
9. $1.2\overline{)14.4}$
10. $0.12\overline{)0.144}$
11. $1.25\overline{)1.375}$
12. $12.5\overline{)13.75}$
13. $1.25\overline{)137.5}$
14. $0.125\overline{)13.75}$
15. $0.125\overline{)0.1375}$

DIVISION WITH DECIMALS

1. How long will it take a train traveling 0.8 mi. a minute to go 25.7 mi.? to go 32.8 mi.? How far does common sense tell you to carry the result?

2. How long will it take a train traveling 0.9 mi. a minute to go 36.9 mi.? to go 47 mi.?

3. How long will it take an airplane traveling at the rate of 1.2 mi. a minute to go 20.4 mi.?

4. The speedometer of an automobile shows that the car has gone 37.4 mi. on a trip of 187 mi. What part of the total distance has the car gone?

5. A workman finds that a pile of 500 metal plates is 10.00 in. high. The measurement 10.00 in. indicates that he measured the pile to the nearest 0.01 in. What is the average thickness of each plate?

Divide as follows, first estimating each result:

6. 67.25 ÷ 500. **8.** 887.6 ÷ 700. **10.** 54.54 ÷ 9000.
7. 5.248 ÷ 200. **9.** 96.48 ÷ 8000. **11.** 0.9366 ÷ 7000.

Divide as follows, first estimating each result and then finding it to the nearest thousandth:

12. 17.78 ÷ 1.5. **15.** 19.26 ÷ 3.1. **18.** 33.43 ÷ 6.8.
13. 62.23 ÷ 0.23. **16.** 21.43 ÷ 3.7. **19.** 423.6 ÷ 0.27.
14. 5.423 ÷ 0.38. **17.** 584.3 ÷ 5.9. **20.** 8.297 ÷ 7.9.

21. Traveling at the rate of 42.7 mi. an hour, how long will it take a train to go 128.1 mi.?

22. Divide 0.7 by 0.91 to the nearest ten-thousandth.

ORAL REVIEW DRILL CHART IN DECIMALS

Perform the following additions:

1.	**2.**	**3.**	**4.**	**5.**
2.1	4.3	3.1	4.2	4.2
3.2	5.6	7.9	.8	1.9

Perform the following subtractions:

6.	**7.**	**8.**	**9.**	**10.**
3.4	3.4	3.4	6.0	7.3
1.3	1.4	.5	2.7	.5

Perform the following multiplications:

11.	**13.**	**15.**	**17.**	**19.**
12	1.2	20	2.0	.2
2	2	3	3	0.3

12.	**14.**	**16.**	**18.**	**20.**
12	1.2	20	2.0	0.2
0.2	0.2	0.3	0.3	0.3

Perform the following divisions:

21. 12 ÷ 3.
22. 1.2 ÷ 3.
23. 1.2 ÷ 0.3.
24. 12 ÷ 0.3.
25. 25 ÷ 5.
26. 2.5 ÷ 5.
27. 2.5 ÷ 0.5.
28. 25 ÷ 0.5.
29. 20 ÷ 10.
30. 2.0 ÷ 10.
31. 2.0 ÷ 1.0.
32. 2 ÷ 1.0.
33. 24 ÷ 12.
34. 2.4 ÷ 12.
35. 2.4 ÷ 1.2.
36. 0.24 ÷ 1.2.
37. 4.8 ÷ 2.4.
38. 4.8 ÷ 24.
39. 48 ÷ 0.24.
40. 0.48 ÷ 0.24.
41. 2.25 ÷ 0.15.
42. 22.5 ÷ 1.5.
43. 0.225 ÷ 1.5.
44. 22.5 ÷ 0.15.

Decimals and Fractions Combined. If we buy 2 collars for 25¢, the price per collar is half of 25¢, or $12\frac{1}{2}$¢. We may write $12\frac{1}{2}$¢ as $\$0.12\frac{1}{2}$.

If we divide 0.25 by 2, we have either

$$\begin{array}{r} 0.12\frac{1}{2} \\ 2\overline{)0.25} \end{array} \quad \text{or} \quad \begin{array}{r} 0.125 \\ 2\overline{)0.250} \end{array}$$

That is,

$$0.12\frac{1}{2} = 0.125,$$
$$1.2\frac{1}{2} = 1.25,$$

and so on for other numbers like $37.4\frac{1}{2}$ and $4.26\frac{1}{2}$.

We also know that $\frac{1}{4} = 1 \div 4 = 0.25$, and so

$$1.2\frac{1}{4} = 1.225 \qquad 3\frac{3}{4} = 3.75$$
$$42\frac{1}{4} = 42.25 \qquad 0.3\frac{3}{4} = 0.375$$

and so on. That is, we may combine decimals and fractions. Such a number may be reduced to the form of a decimal, at least approximately, or to that of a fraction.

DECIMALS AND FRACTIONS COMBINED

Write the following in the decimal form:

1. $2\frac{1}{2}$.
2. $0.2\frac{1}{2}$.
3. $2\frac{1}{4}$.
4. $32\frac{1}{2}$.
5. $3.2\frac{1}{2}$.
6. $0.32\frac{1}{2}$.
7. $1.3\frac{1}{2}$.
8. $1.3\frac{1}{4}$.
9. $1.3\frac{3}{4}$.
10. $\frac{1}{2}$.
11. $1\frac{1}{2}$.
12. $\frac{1}{4}$.
13. $2.75\frac{1}{2}$.
14. $2.05\frac{1}{2}$.
15. $3.5\frac{1}{4}$.

At the following prices per 100, find the cost of one article:

16. \$5. **17.** \$7. **18.** \$2.50. **19.** \$7.50. **20.** \$17.50.

At the following prices per 1000, find the cost of one article:

21. \$15. **22.** \$25. **23.** \$125. **24.** \$250. **25.** \$3750.

A SILENT READING LESSON

Mrs. Edwards can buy canned pineapples at 35¢ a can or at 3 cans for \$1. If she pays \$1 for 3 cans, she pays $\frac{1}{3}$ of \$1 for 1 can. Since $\frac{1}{3}$ of \$1 is $33\frac{1}{3}$¢, Mrs. Edwards saves a little when she buys 3 cans for \$1.

We often have to use parts of \$1, such as $\frac{1}{3}$ of \$1, although we could not pay exactly $33\frac{1}{3}$¢ in a single amount, because there is no coin with the value $\frac{1}{3}$¢.

We need to know certain fractional parts of a dollar for just such cases as the one given in the above example. The following table shows fifteen different parts of \$1, which, together with such cases as 1¢ = $\frac{1}{100}$ of \$1 and 10¢ = $\frac{1}{10}$ of \$1, are the ones most used in practical work:

5¢ = $\frac{1}{20}$ of \$1
$6\frac{1}{4}$¢ = $\frac{1}{16}$ of \$1
$8\frac{1}{3}$¢ = $\frac{1}{12}$ of \$1
$12\frac{1}{2}$¢ = $\frac{1}{8}$ of \$1
$16\frac{2}{3}$¢ = $\frac{1}{6}$ of \$1

20¢ = $\frac{1}{5}$ of \$1
25¢ = $\frac{1}{4}$ of \$1
$33\frac{1}{3}$¢ = $\frac{1}{3}$ of \$1
$37\frac{1}{2}$¢ = $\frac{3}{8}$ of \$1
50¢ = $\frac{1}{2}$ of \$1

$62\frac{1}{2}$¢ = $\frac{5}{8}$ of \$1
$66\frac{2}{3}$¢ = $\frac{2}{3}$ of \$1
75¢ = $\frac{3}{4}$ of \$1
$83\frac{1}{3}$¢ = $\frac{5}{6}$ of \$1
$87\frac{1}{2}$¢ = $\frac{7}{8}$ of \$1

There are other parts, such as 60¢ = $\frac{3}{5}$ of \$1, but they are not often used.

Suppose that Fred can buy paper pads in quantities of four or more at $12\frac{1}{2}$¢ each. To find the cost of 16 pads, it is much easier to find the value of 16 × \$$\frac{1}{8}$, which is \$2, than to find the value of 16 × $12\frac{1}{2}$¢. If you do not think so, multiply in each case.

At the rate of 6 tablets for \$1, Fred would pay $16\frac{2}{3}$¢ for each tablet, and 30 × \$$\frac{1}{6}$, or \$5, for 30 tablets.

Using Fractional Parts. 1. At the rate of 3 for \$1, or $33\frac{1}{3}$¢ each, how much will 20 collars cost?

We have $20 \times \$\frac{1}{3} = \$\frac{20}{3} = \$6\frac{2}{3} = \$6.66\frac{2}{3}$.

We give such a result to the nearest cent; that is, the cost of the 20 collars is \$6.67.

2. At 6 for \$1, how much will 15 bottles of ink cost?

We have $15 \times \$\frac{1}{6} = \$2\frac{1}{2} = \$2.50$.

3. At $\$1.12\frac{1}{2}$ each, how much will 7 doormats cost?

Since $\$1.12\frac{1}{2} = \$1\frac{1}{8}$,

we have $7 \times \$1\frac{1}{8} = \$7\frac{7}{8} = \$7.87\frac{1}{2}$.

The cost of the 7 mats is therefore \$7.88.

Always count $\frac{1}{2}$¢ or more as a whole cent in such cases.

PARTS OF A DOLLAR

1. At the rate of 3 for \$1, how much must a school pay for 26 spelling books? for 228 spelling books?

2. At the rate of 6 for \$1, how much must a school pay for 160 boxes of crayons? for 112 boxes of crayons?

3. At $33\frac{1}{3}$¢ each, find the cost of 75 handkerchiefs.

4. At $\$1.33\frac{1}{3}$ a book, find the cost of 124 books.

5. At $\$1.12\frac{1}{2}$ a yard, find the cost of 128 yd. of velvet.

Multiply $\$0.33\frac{1}{3}$ and $\$0.66\frac{2}{3}$ by each of these numbers:

6. 42. 7. 66. 8. 78. 9. 96. 10. 540.

Multiply $\$0.16\frac{2}{3}$ and $\$0.12\frac{1}{2}$ by each of these numbers:

11. 48. 12. 96. 13. 144. 14. 192. 15. 240.

DRILL CHART ON PARTS OF A DOLLAR

Numbers 1 to 17, oral

1. At 3 for \$2, how much will 1 necktie cost?
2. At 8 for \$5, how much will 1 rosebush cost?
3. At 8 for \$7, how much will 1 arithmetic cost?

Multiply as follows:

4. 6 × $33\frac{1}{3}$¢.
5. 8 × $6\frac{1}{4}$¢.
6. 12 × $8\frac{1}{3}$¢.
7. 16 × $12\frac{1}{2}$¢.
8. 18 × $16\frac{2}{3}$¢.
9. 45 × 20¢.
10. 9 × $33\frac{1}{3}$¢.
11. 8 × $37\frac{1}{2}$¢.
12. 48 × 50¢.
13. 8 × $62\frac{1}{2}$¢.
14. 9 × $66\frac{2}{3}$¢.
15. 6 × $83\frac{1}{3}$¢.
16. At $87\frac{1}{2}$¢ each, how much will 16 plates cost?
17. At $83\frac{1}{3}$¢ each, how much will 36 pitchers cost?

Multiply as follows:

18. 120 × 5¢.
19. 32 × $6\frac{1}{4}$¢.
20. 48 × $8\frac{1}{3}$¢.
21. 48 × $12\frac{1}{2}$¢.
22. 48 × $16\frac{2}{3}$¢.
23. 75 × 20¢.
24. 96 × 25¢.
25. 64 × $37\frac{1}{2}$¢.
26. 75 × 50¢.
27. 72 × $62\frac{1}{2}$¢.
28. 93 × $66\frac{2}{3}$¢.
29. 64 × 75¢.
30. 78 × $83\frac{1}{3}$¢.
31. 72 × $87\frac{1}{2}$¢.
32. 60 × $8\frac{1}{3}$¢.
33. 72 × $12\frac{1}{2}$¢.
34. 99 × $33\frac{1}{3}$¢.
35. 88 × $37\frac{1}{2}$¢.
36. 176 × $37\frac{1}{2}$¢.
37. 126 × $83\frac{1}{3}$¢.
38. 75 × $62\frac{1}{2}$¢.

Find the number of cents in

39. $\frac{1}{16}$ of \$1.
40. $\frac{5}{12}$ of \$1.
41. $\frac{5}{16}$ of \$1.
42. $\frac{7}{16}$ of \$1.
43. $\frac{1}{32}$ of \$1.
44. $\frac{3}{32}$ of \$1.

Short Cuts in Division. From the work on pages 144 and 145 we can now learn a few more short cuts that will make our work easier.

Just as $5¢$ is \$0.05, or $\frac{1}{20}$ of \$1,
so 0.05 is the same as $\frac{1}{20}$,
and $0.06\frac{1}{4}$ is the same as $\frac{1}{16}$.

That is, the table on page 144 may be written like this:

$0.05 = \frac{1}{20}$	$0.12\frac{1}{2} = \frac{1}{8}$	$0.25 = \frac{1}{4}$	$0.50 = \frac{1}{2}$	$0.75 = \frac{3}{4}$
$.06\frac{1}{4} = \frac{1}{16}$	$.16\frac{2}{3} = \frac{1}{6}$	$.33\frac{1}{3} = \frac{1}{3}$	$.62\frac{1}{2} = \frac{5}{8}$	$.83\frac{1}{3} = \frac{5}{6}$
$.08\frac{1}{3} = \frac{1}{12}$	$.20 = \frac{1}{5}$	$.37\frac{1}{2} = \frac{3}{8}$	$.66\frac{2}{3} = \frac{2}{3}$	$.87\frac{1}{2} = \frac{7}{8}$

Then, instead of dividing by 0.05, we may divide by $\frac{1}{20}$, which is the same as multiplying by 20. Thus, it is much easier to think $18 \times 20 = 360$ than $18 \div 0.05 = 360$.

SHORT CUTS IN DIVISION

1. Instead of dividing by $0.08\frac{1}{3}$, we may simply multiply by what whole number?

2. Instead of dividing by $0.37\frac{1}{2}$, or 0.375, we may multiply by what fraction? Divide 3 by $0.37\frac{1}{2}$ mentally.

Divide mentally each of the following numbers by $0.33\frac{1}{3}$:

3. 7. **4.** 9. **5.** 11. **6.** 2.1. **7.** 1.2. **8.** 70.

Divide mentally each of the following numbers by 0.75:

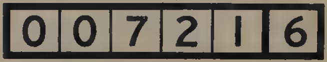
9. 3. **10.** 6. **11.** 9. **12.** 12. **13.** 30. **14.** 60.

Divide as rapidly as you can:

15. $0.2\overline{)26}$ **16.** $0.16\frac{2}{3}\overline{)72}$ **17.** $0.83\frac{1}{3}\overline{)75}$ **18.** $0.87\frac{1}{2}\overline{)63}$

WRITTEN REVIEW

1. When an automobile speedometer registers as here shown, it means that the car has gone 721.6 mi. Since a mile is 5280 ft., this is how many feet more than 721 mi.?



2. If our speedometer registered 721.6 mi. at the start of a day's trip, and 850.8 mi. at the end, how far did the car go?

3. If a speedometer registered 1078.2 mi. this morning and 1203.6 mi. tonight, how far has the car gone today?

4. If our speedometer registers 2079.4 mi. when our car is 98.7 mi. from a city, what should the speedometer register when we reach the city?

At the following prices per 100, find the cost of one article:

5. $24.
6. $240.
7. $2400.
8. $45.00.
9. $4500.
10. $1.25.
11. $1.
12. $1.1.
13. $1250.
14. $5000.
15. $50.00.
16. $500.00.

At the following prices per 1000, find the cost of one article:

17. $350.
18. $3.50.
19. $7500.
20. $820.00.

Perform the following multiplications:

21. $\begin{array}{r} 62.5 \\ \underline{25} \end{array}$

22. $\begin{array}{r} 1.25 \\ \underline{10} \end{array}$

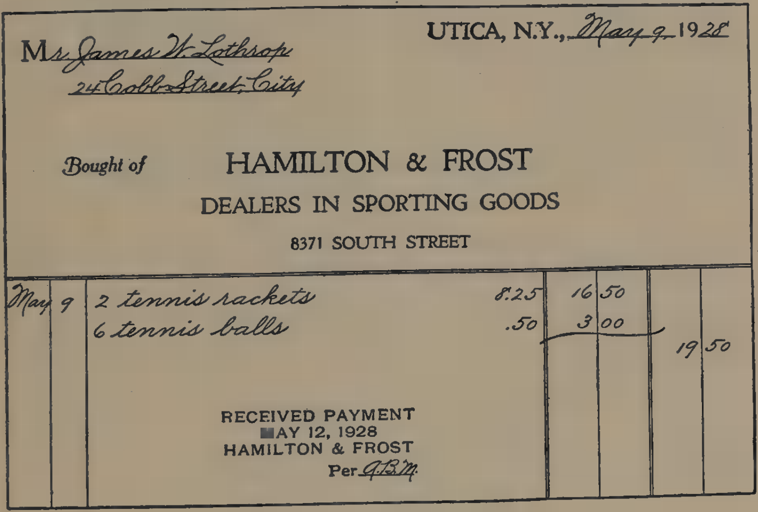
23. $\begin{array}{r} 1350 \\ \underline{3.14} \end{array}$

24. $\begin{array}{r} 22{,}636 \\ \underline{0.33\frac{1}{3}} \end{array}$

Perform the following divisions:

25. 625 ÷ 25.
26. 0.625 ÷ 2.5.
27. 144 ÷ 1.2.
28. 1.44 ÷ 0.12.
29. 37.5 ÷ 75.
30. 3.75 ÷ 0.75.

Bill. If goods are not paid for at the time they are purchased, the dealer usually sends a *bill*. Here is a model bill, receipted by the dealer to show that it has been paid:



UTICA, N.Y., May 9, 1928

Mr. James W. Lathrop
24 Cobb Street, City

Bought of HAMILTON & FROST
DEALERS IN SPORTING GOODS
8371 SOUTH STREET

May	9	2 tennis rackets	8.25	16	50		
		6 tennis balls	.50	3	00	19	50

RECEIVED PAYMENT
MAY 12, 1928
HAMILTON & FROST
Per G.B.M.

MAKING OUT BILLS

Make out bills for the following items, inserting dates and names of purchasers and dealers, and receipt the bills:

1. 1 baseball cap, 80¢; 1 catcher's mitt, $2.80; 1 baseball mask, $2.75; 1 catcher's body protector, $3.85.

2. 1 league ball, $1.35; 3 bats @ $1.20; 2 fielder's mitts @ $1.53; 3 canvas bases @ $2.40; 1 pitcher's toe plate, 45¢.

3. 4 hunting caps @ 80¢; 1 hunting vest, $2.25; 2 pr. corduroy hunting pants @ $4.45; 2 hunting coats @ $4.80.

4. 3 tennis rackets @ $7.75; 3 racket cases @ $1.35; 2 racket presses @ $1.75; $\frac{3}{4}$ doz. balls @ $4.80.

Price List. Here is a price list of articles which boys and girls may wish to buy:

Baseball bat	$1.25	Hammock	$4.50
Basket ball	5.25	Rubber-stamp outfit	1.75
Bicycle	32.75	School shoes	4.25
Book satchel	1.25	Scout camera	5.25
Catcher's mask	3.50	Skating cap	2.00
Catcher's mitt	2.50	Storm-proof boots	4.75
Drawing set	.60	Sweater	2.90
Football	2.50	Typewriter	12.50
Football pants	3.25	Watch	4.25

Use this price list in the problems on this page or in other problems that you make up.

BUYING FROM A PRICE LIST

Make out bills for the following purchases:

1. 2 sweaters, 3 skating caps, and a watch.

2. A bicycle, a sweater, a baseball bat, a catcher's mask, and 2 catcher's mitts.

3. A rubber-stamp outfit, a typewriter, a scout camera, and a drawing set.

4. 4 pairs of football pants and 2 footballs.

5. A watch, a pair of storm-proof boots, a book satchel, 2 pairs of school shoes, and a hammock.

6. 4 baseball bats, 2 catcher's masks, a basket ball, a catcher's mitt, and 2 pairs of football pants.

7. A watch, a typewriter, a book satchel, a scout camera, and a skating cap.

Marketing. Here is a price list of groceries such as you might use if you were running a house:

Alaska salmon	22¢ per can
Corn sirup	26¢ per 1½-pound can
Cornstarch	16¢ per package
Graham flour	25¢ per package
Hominy	7¢ per pound
Potato flour	18¢ per package
Rye flour	6¢ per pound
White flour	$1.35 per sack
Yellow corn meal	6¢ per pound

You may use this price list in the problems on this page. A better way is to find the prices of these articles at some grocery and write them on the board for the class to use.

MARKETING

Make out bills for the following purchases:

1. 1 sack of white flour and 3 packages of potato flour.
2. 10 lb. of rye flour and 6 lb. of yellow corn meal.
3. 8 packages of cornstarch and 12 lb. of hominy.
4. 2 packages of cornstarch, 6 cans of Alaska salmon, 8 packages of potato flour, and a sack of white flour.
5. 5 cans of corn sirup, 5 lb. of rye flour, 10 lb. of hominy, and 9 lb. of yellow corn meal.
6. 11 lb. of hominy, 7 packages of potato flour, 12 cans of Alaska salmon, and a sack of white flour.
7. 3 packages of graham flour, 2 sacks of white flour, and 4 packages of cornstarch.

Buying at a Sale. Often, at special sales, merchants offer goods at “$\frac{1}{4}$ off,” at “$\frac{1}{3}$ off,” or at other reductions in price. For example, at “$\frac{1}{4}$ off,” a tennis racket marked \$3.60 will cost \$3.60 less $\frac{1}{4}$ of \$3.60, which is \$3.60 – \$0.90, or \$2.70.

$$\begin{array}{lr} & \$3.60 \\ \text{Less } \frac{1}{4}, & .90 \\ \hline & \$2.70 \end{array}$$

Similarly, a nickel watch that is advertised at \$4.50, with “$\frac{1}{3}$ off,” can be bought for \$4.50 less $\frac{1}{3}$ of \$4.50, which is \$4.50 – \$1.50, or \$3.

BUYING AT A SALE

Find the cost at “$\frac{1}{4}$ off” of articles marked as follows:

1. Phonograph, \$52.40.
2. Typewriter, \$36.80.
3. Boy-Scout suit, \$6.40.
4. Nickel watch, \$4.80.
5. Baseball mitt, \$1.60.
6. Fishing rod, \$2.40.

Find the cost at “$\frac{1}{3}$ off” of articles marked as follows:

7. Fishing reel, \$2.40.
8. Baseball bat, \$1.50.
9. Boy's sweater, \$3.15.
10. Camera, \$6.75.
11. Phonograph, \$48.
12. Basket ball, \$5.25.

13. A dealer pays \$330 for some percaline at 25¢ a yard. How many yards of percaline does he buy?

14. If 4 tennis rackets cost \$17.20, how much will 2 rackets cost? Solve by one division.

15. If 12 fishing reels cost \$31.20, how much will 4 fishing reels cost? How much will 6 fishing reels cost?

16. If 3 cameras cost \$27.75, how much will 4 cameras cost? Solve by one multiplication by a mixed number.

USING WHAT YOU HAVE LEARNED

1. How much will 25 phonographs cost at $52.40 each?

2. How much will 38 typewriters cost at $72.50 each?

3. How much must a school pay for 64 manual-training benches at $18.50 each?

4. How much will 96 yd. of silk cost at $2.25 a yard?

5. How much will 75 yd. of voile cost at $1.60 a yard?

6. How much must a school pay for 20 jack planes at $2.60 each? at $2.75 each? at $3.25 each?

7. How much will 24 yd. of cloth cost at 87½¢ a yard?

8. How much will 240 door catches cost at 35¢ each?

9. How much will 96 yd. of cloth cost at $1.25 a yard?

10. A dealer pays $360 for some gingham at 60¢ a yard. How many yards of gingham does he buy?

11. In going from New York to Albany, a distance of 143.2 mi., the train passes through Poughkeepsie, which is 72.8 mi. from New York. How far is it from Poughkeepsie to Albany?

12. A train leaving Albany at 11 50 P.M. reaches Buffalo, which is 296.2 mi. from Albany, at 7 10 A.M. the next day. Find the average speed at which the train travels per hour, remembering that 11 50 P.M. means 11 hr. 50 min. in the evening, and 7 10 A.M. means 7 hr. 10 min. in the morning.

13. In traveling from Albany to Niagara Falls, a distance of 318.2 mi., we pass through Syracuse, which is 147.4 mi. from Albany. From Albany to Syracuse is how much less than half the distance from Albany to Niagara Falls?

14. One train leaves New York at 8 45 A.M. and arrives at Buffalo, which is 438.4 mi. from New York, at 8 20 P.M. Another train leaves Buffalo at 11 57 P.M. and arrives at New York at 4 15 the next afternoon. Find the average speed of each train per hour.

15. On an automobile trip of 120.4 mi. a man stops for luncheon when the speedometer registers 46.9 mi. What part of the trip has he made when he stops for luncheon?

16. Two cousins, Frank and Tom, are visiting their uncle in Schenectady. Frank lives in Little Falls, which is 56.6 mi. from Schenectady; Tom lives in Batavia, which is 243.3 mi. from Schenectady. Tom says Batavia is five times as far from Schenectady as Little Falls is, but Frank says it lacks 40 mi. of it. Which boy is right?

17. A boy who earns $8.50 a week pays 12¢ for car fares and 30¢ for luncheon each week day. Once a week he pays 15¢ for a ticket to the movies, and on Saturdays he pays 50¢ for a ticket to the ball game. His other expenses are 60¢ each week. How much does he save in 8 wk.?

18. Except for 8 holidays, the schools of a certain city are in session 5 da. a week for 39 wk. and 4 da. If a boy goes to school every day it is in session, how many days is he in school?

19. During the summer vacation a boy works 10 wk. in an office. He goes to and from the office in the street cars, and the fare is 7¢ each way. If he works every week day except the Fourth of July, to how much do his car fares amount? The previous summer the car fare was 5¢ each way. How much does the increase cost him?

III. MEASURES

A SILENT READING LESSON

We know how the common measures, such as feet, inches, pounds, and so on, are used. We measure cloth by the yard, the length of a page like this in inches, the length of a room in feet and inches, and the length of an automobile trip in miles. We measure our weight in pounds, milk by the quart, apples in an orchard by the bushel, and time by minutes, hours, years, and so on.

Sometimes we speak of numbers like 4 lb., 16 ft., and $10 as *denominate numbers,* and of numbers like 4 ft. 2 in. and 10 lb. 3 oz. as *compound numbers.* To know how to use compound numbers is important in only a few cases, because they are now rarely used in practical work.

When we change the name of a number from feet to inches, we say that we *reduce* from feet to inches.

The following cases show how we make such reductions; for example, suppose that we are required to change $8\frac{1}{2}$ ft. to inches.

We first see that

$$1 \text{ ft.} = 12 \text{ in.}$$

Then, proceeding as shown above, the result is 102 in.

$$\boxed{\begin{aligned} 1 \text{ ft.} &= 12 \text{ in.} \\ 8\tfrac{1}{2} \text{ ft.} &= 8\tfrac{1}{2} \times 12 \text{ in.} \\ &= 102 \text{ in.} \end{aligned}}$$

Similarly, to change 102 in. to feet, we first see that

$$1 \text{ in.} = \tfrac{1}{12} \text{ ft.}$$

We then proceed as shown in the work at the right.

$$\boxed{\begin{aligned} 1 \text{ in.} &= \tfrac{1}{12} \text{ ft.} \\ 102 \text{ in.} &= 102 \times \tfrac{1}{12} \text{ ft.} \\ &= 8\tfrac{1}{2} \text{ ft.} \end{aligned}}$$

Table of Length. Review the following table of length:

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet = 1 rod (rd.)
320 rods, or 5280 feet = 1 mile (mi.)

REVIEW OF THE TABLE OF LENGTH

1. If you live $\frac{1}{4}$ mi. from your school, how many feet do you go in walking to school? in walking to school and walking home?

2. In Ex. 1 how many yards do you walk?

Find the number of inches in each of the following lengths:

3. $2\frac{1}{2}$ ft. **4.** $3\frac{3}{4}$ ft. **5.** $16\frac{1}{2}$ ft. **6.** $\frac{1}{2}$ yd. **7.** $1\frac{3}{4}$ yd.

Find the number of feet in each of the following lengths:

8. 9 rd. **9.** $6\frac{1}{2}$ yd. **10.** 15 yd. **11.** 2 rd. **12.** $\frac{3}{4}$ mi.

13. If I buy $3\frac{3}{4}$ yd. of tape, how many inches do I buy?

14. Express in feet the length of a field 40 rd. long.

15. The distance between two railway stations is known to be $9\frac{3}{4}$ mi. Express the distance in miles and feet.

16. In time-tables the distances between stations are sometimes given in decimals to the nearest 0.01 mi. A distance of 22.75 mi. is how many feet more than 22 mi.?

Express the following as miles and feet:

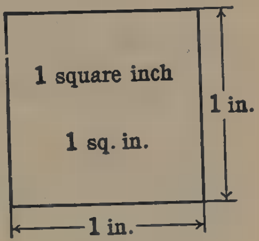
17. 2.8 mi. **19.** 3.65 mi. **21.** 5.85 mi.
18. 2.25 mi. **20.** 4.35 mi. **22.** 0.72 mi.

23. Which is the longest, 17 rd., $93\frac{1}{2}$ yd., or $280\frac{1}{2}$ ft.?

Table of Square Measure. Review the following table:

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
30¼ square yards = 1 square rod (sq. rd.)
160 square rods = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)

If a square is 1 in. on a side, it is 1 in. square, and we say that its *area* is 1 sq. in.



If a square is 1 ft. on a side, it is 1 ft. square, and its area is 1 sq. ft.

If a square is 1 yd. on a side, its area is 1 sq. yd., and so on.

Carpenters, architects, and draftsmen often write 8″ for 8 in., 5′ for 5 ft., and 6′ 7″ for 6 ft. 7 in.

REVIEW OF SQUARE MEASURE

1. Find the number of square inches in the top of a desk which has an area of 3½ sq. ft.

2. A school blackboard has an area of 4½ sq. yd. What is the area in square feet?

3. A flower bed has an area of 2 sq. rd. What is the area in square yards? in square feet?

4. Find the number of square feet in 7 sq. yd.

Copy, and supply the missing numbers:

5. 18 sq. ft. = —— sq. yd.
6. 18 sq. yd. = —— sq. ft.
7. 4 sq. rd. = —— sq. yd.
8. ½ A. = —— sq. rd.
9. 2½ sq. mi. = —— A.
10. 2½ A. = —— sq. rd.

MEASURING AND DRAWING

1. Measure this page of the arithmetic. Draw a plan of the page to the scale $\frac{1}{4}$. This means that we represent $1''$ on the page by $\frac{1}{4}''$ on the plan that we draw. If the page is $7\frac{1}{4}''$ long, the length of the plan will be $7\frac{1}{4} \times \frac{1}{4}''$. You can easily find this length on your ruler.

2. Measure the top of your desk. Draw a plan to the scale of $1''$ to $1'$.

3. Draw a plan of the top of your desk to the scale $\frac{1}{8}$.

4. Measure the length and width of the schoolroom. Draw a plan to scale and write below it the scale used.

5. Measure the size of the paper upon which you make your drawings. Draw a plan of a sheet to the scale $\frac{1}{8}$.

6. Measure the size of any other book you are studying. Draw a plan of a page to the scale $\frac{1}{4}$.

7. Roy made a drawing of a box cover. The cover was $14''$ long, and the drawing $3\frac{1}{2}''$. What scale was used?

8. A drawing of a floor $24''$ long and $20''$ wide is $6''$ long and $5''$ wide. What scale was used?

9. A rectangle $2''$ by $3''$ has an area of 6 sq. in. Draw such a rectangle. Then draw a rectangle $4''$ by $6''$, and see if you can tell the area of the second one. Using a scale twice as large does what to the area?

10. A very common scale is $3''$ to $1'$, which is the same as the scale $\frac{1}{4}$. It is often spoken of as "$\frac{1}{4}$ full size." The scale $1\frac{1}{2}''$ to $1'$ is called "$\frac{1}{8}$ full size." What part of an inch to $1'$ would be called "$\frac{1}{16}$ full size"?

Table of Weight. Review the following table of weight:

16 ounces (oz.) = 1 pound (lb.)
100 pounds = 1 hundredweight (cwt.)
2000 pounds = 1 ton (T.)

REVIEW OF MEASURES OF WEIGHT

Numbers 1 to 3, oral

1. If you buy $\frac{1}{2}$ lb. of tea, you buy how many ounces?

2. A man buys $\frac{1}{2}$ T. of coal. This is how many pounds?

3. In 4000 lb. of coal how many tons are there?

Find the number of pounds in

4. 2 cwt.
5. $\frac{1}{2}$ cwt.
6. $2\frac{1}{2}$ cwt.
7. $3\frac{1}{4}$ cwt.
8. $4\frac{3}{4}$ cwt.
9. $\frac{1}{2}$ T.
10. 2 T.
11. $2\frac{1}{2}$ T.
12. $3\frac{1}{2}$ T.
13. $5\frac{1}{2}$ T.
14. 0.1 T.
15. 2.1 T.
16. 2.2 T.
17. 3.5 T.
18. 4.5 T.
19. 0.25 T.
20. 2.25 T.
21. 9.25 T.
22. 8.75 T.
23. 6.75 T.

Find the number of tons in

24. 2000 lb.
25. 6000 lb.
26. 8000 lb.
27. 500 lb.
28. 4500 lb.
29. 6500 lb.
30. 1500 lb.
31. 3500 lb.
32. 5500 lb.
33. 20 cwt.
34. 10 cwt.
35. 15 cwt.

Find the number of ounces in

36. $\frac{1}{2}$ lb. **37.** $2\frac{1}{2}$ lb. **38.** $\frac{1}{4}$ lb. **39.** $3\frac{1}{4}$ lb. **40.** $\frac{3}{4}$ lb.

Find the number of pounds in

41. 32 oz. **42.** 8 oz. **43.** 40 oz. **44.** 64 oz. **45.** 72 oz.

Table of Liquid Measure. Review the following table:

2 pints (pt.) = 1 quart (qt.)
4 quarts = 1 gallon (gal.)

REVIEW OF LIQUID MEASURE

Numbers 1 to 4, oral

1. If Mr. Russell buys 1 qt. of oil for his car, how many pints of oil does he buy?

2. Name the measures in the above picture.

3. If Mr. Russell buys $1\frac{1}{2}$ qt. of oil, how many pints does he buy? This is what part of a gallon?

4. If a can holds 10 gal., how many quarts does it hold?

Find the number of pints in

5. $2\frac{1}{2}$ qt. **6.** $8\frac{1}{2}$ qt. **7.** $\frac{1}{2}$ gal. **8.** $7\frac{1}{2}$ gal. **9.** $4\frac{1}{4}$ qt.

Find the number of quarts in

10. 16 pt. **11.** 17 pt. **12.** $\frac{1}{2}$ gal. **13.** $9\frac{1}{2}$ gal. **14.** $10\frac{3}{4}$ gal.

Table of Dry Measure. Review this table of dry measure:

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

The present tendency is to sell all sorts of produce by weight. The weight of a bushel of a commodity like potatoes or beans is fixed by state laws.

REVIEW OF DRY MEASURE

1. Name the three measures in the above picture.

Find the number of pints in

2. 2 qt. **3.** $\frac{1}{2}$ qt. **4.** $7\frac{1}{2}$ qt. **5.** 1 pk. **6.** $3\frac{1}{2}$ pk.

Find the number of quarts in

7. 4 pt. **8.** 9 pt. **9.** $\frac{3}{4}$ pk. **10.** $7\frac{1}{2}$ pk. **11.** $2\frac{1}{2}$ bu.

Find the number of pecks in

12. 16 pt. **13.** 16 qt. **14.** $3\frac{1}{2}$ bu. **15.** $9\frac{1}{2}$ bu.

Find the number of bushels in

16. 32 qt. **17.** 32 pk. **18.** 32 pt. **19.** 64 pk.

Table of Time. Review the following table of time:

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
12 months (mo.) = 1 year (yr.)

Common years have 365 da., and *leap* years have 366 da. Years whose numbers are divisible by 4, but not by 100, and those divisible by 400, are leap years. Thus, 1928 and 1932, but not 2100, are leap years.

April, June, September, and November have 30 da. each; the others have 31 da., except February, which has 28 da. in a common year and 29 da. in a leap year.

REVIEW OF MEASURES OF TIME

Find the number of minutes in

1. $\frac{1}{2}$ hr. **2.** 120 sec. **3.** $1\frac{1}{4}$ hr. **4.** $1\frac{1}{2}$ hr.

Find the number of hours in

5. $1\frac{1}{2}$ da. **6.** 180 min. **7.** $2\frac{1}{2}$ da. **8.** $2\frac{1}{4}$ da.

9. How many minutes are there from 11 30 A.M. to 2 15 P.M.; that is, from 11 hr. 30 min. (half past 11) in the morning to 2 hr. 15 min. (quarter past 2) in the afternoon?

Find the number of minutes from

10. 10 20 A.M. to 11 10 A.M.; 11 45 A.M. to 12 15 P.M.
11. 10 min. before 10 P.M. to 20 min. after 10 P.M.
12. 10 min. after 9 A.M. to 10 min. before 11 A.M.
13. Find the number of minutes in your school day.

Table of Money. Review the following table of money:

10 mills = 1 cent (ct. or ¢)
5 cents = 1 nickel
10 cents = 1 dime
25 cents = 1 quarter dollar
50 cents = 1 half dollar
100 cents = 1 dollar
10 dimes = 1 dollar ($)

The mill is not coined, but is frequently used in business calculations. The ten-dollar gold piece is sometimes called an *eagle*.

Table of Dozens. This table is often used in counting:

12 units = 1 dozen (doz.)
12 dozen = 1 gross (gr.)

REVIEW OF MONEY AND COUNTING

1. How many oranges do you buy when you buy 3 doz.? $3\frac{1}{2}$ doz.? 5 doz.? $5\frac{1}{4}$ doz.?

2. How many eggs are $2\frac{1}{2}$ doz. eggs? $3\frac{3}{4}$ doz. eggs?

3. If a dressmaker buys 2 gr. of buttons, she buys how many dozen? how many buttons?

4. If, when you buy 1 doz. buttons, you hand the clerk a half dollar and receive 2¢ in change, how much do you pay for each button?

5. If you pay 3 dimes, a nickel, and a cent for $\frac{1}{2}$ doz. large buttons, how much do you pay for each button?

6. Which is more, 6 doz. dozen or a half a dozen dozen? How many units are there in each?

USING WHAT YOU HAVE LEARNED

1. How many 5-rod spaces are there in 1 mi. of telegraph poles placed 5 rd. apart? How many poles are there, including the first pole and the last?

2. Our train arrives at 1 07 P.M. It is now 11 56 A.M. How long have we to wait?

3. Instead of arriving at 11 45 A.M., a train arrived at 12 25 P.M. How late was it?

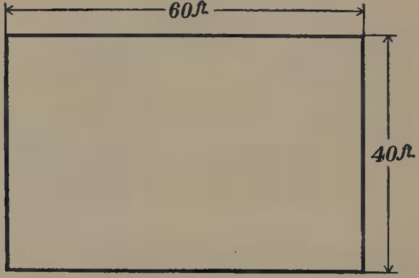
4. A school garden is 5 rd. long and 3 rd. wide. Find the perimeter (the distance around the garden) in feet.

5. At $9.60 a gross, how much will a dozen lamp shades cost? How much will 7½ doz. cost?

6. A dealer buys 45 gr. (gross) of blackboard erasers of which $\frac{1}{10}$ are damaged. How many erasers are damaged?

7. Which is cheaper, to buy a gross of articles for $108 or 12 doz. at $9.50 a dozen? How much cheaper is it?

8. A school garden is 60 ft. long and 40 ft. wide. This picture is a plan of the garden to the scale of $\frac{1}{32}$ in. to 1 ft. Express the size of the garden in yards.



9. How many feet of fencing will be needed to inclose the garden? How many yards?

10. Draw a plan of a garden that is 20 ft. long and 15 ft. wide, using the scale of 1 in. to 10 ft.

A SILENT READING LESSON

Most of the problems we have had thus far have been of the kind that we could solve without much difficulty. As the problems become a little harder, we shall need to know how to think them through so as to be able to solve them. It will help us to consider these things in each case:

1. Just what is given?
2. Just what are we to find?
3. With what is given, how shall we go to work?

Let us see how this works with an example. Suppose that a man buys 5 T. 1200 lb. of coal. If he uses 100 lb. a day, how many days will the coal last?

1. Just what is given? Two things: (1) the weight of all the coal, and (2) the weight of the coal used each day.

2. Just what are we to find? The number of days the coal will last.

3. How shall we go to work? If we had 10 lb. of coal and used 2 lb. a day, we should have $10 \div 2 = 5$; that is, the coal would last 5 da. We therefore see that we must divide 5 T. 1200 lb. by 100 lb.

If we had the weight in pounds, this would be easy. We therefore write 5 T. 1200 lb. as 10,000 lb. + 1200 lb., or 11,200 lb.

We then divide 11,200 lb. by 100 lb., or 11,200 by 100, and the result is 112, the number of days.

If a problem concerns measuring a rectangle, a triangle, a distance, or some similar case, we should draw a plan which shows clearly what is given and what is to be found.

MISCELLANEOUS PROBLEMS

1. How many tulip bulbs, placed 6 in. apart, will it take for 4 rows, each 8 ft. long? Draw a plan of the first row and find the number needed for that row.

2. A farmer raises 1950 bu. of wheat on 95 A. If he can increase the crop one half by better farming methods, what will be the yield per acre?

3. The boys in the class measured a building lot near the school. One side was 175.2 ft., the next was 83.4 ft., the third was 196.5 ft., and the fourth was also 83.4 ft. How far was it around the lot?

4. A train schedule provides 10 round trips to the city each week day and 4 round trips on Sunday from a certain suburban station. In 4 wk. the trains travel a total of 6144 mi. How far from the city is the suburban station from which the trains start?

5. A workman's time slip showed that at the end of the week he had worked 44 hr. He was paid at the rate of 62¢ an hour. How much money did he receive at the end of the week and how many 8-hour days had he worked?

6. A factory which employs 428 men runs 8 hr. a day 5 da. in the week and 4 hr. on Saturday. The men are paid at the rate of 75¢ an hour. Owing to poor business, the working hours had to be reduced to 24 hr. per week. How much did the men lose in wages each week?

7. A newsboy pays 3¢ for a weekly magazine that he sells for 5¢. How many must he sell in order to make $8 more than the cost of the magazines?

IV. REVIEW AND DRILL

MINIMUM ESSENTIALS

Copy, add, and check the results:

1. $941.35	**2.** $910.45	**3.** $590.05	**4.** $783.60
598.38	207.98	232.68	55.62
2.98	700.00	329.87	454.08
79.26	56.50	762.45	384.96
563.42	862.73	82.74	672.00
802.75	426.37	538.73	586.49
508.72	758.42	457.07	754.62

Perform the following operations involving fractions:

5. $\frac{3}{4}$ of $\frac{5}{9}$.

6. $\frac{3}{4} \div \frac{9}{16}$.

7. $\frac{7}{8} + \frac{3}{4}$.

8. $\frac{7}{8} - \frac{3}{4}$.

9. $6\frac{2}{3} \times 9\frac{3}{8}$.

10. $\frac{2}{5}$ of $\frac{15}{16}$.

11. $4\frac{1}{8} - \frac{1}{4}$.

12. $3\frac{5}{8} + 2\frac{15}{16}$.

13. $3\frac{5}{8} - 2\frac{15}{16}$.

14. $2\frac{1}{2} + 1\frac{7}{8}$.

15. $\frac{2}{3} \div \frac{5}{12}$.

16. $3 \times 12\frac{3}{4}$ in.

17. $12\frac{3}{4}$ in. $\div$ 3.

18. $16\frac{3}{4}$ ft. $\div$ 4.

19. $30\frac{3}{5}$ yd. $\div$ 9.

Perform the following operations involving decimals:

20. $2.68 + 1.22$.

21. $2.68 - 1.22$.

22. 1.65×1.5.

23. $1.65 \div 1.5$.

24. $1.27 + 0.68$.

25. $1.27 - 0.68$.

26. 1.27×0.68.

27. $1.50 + 0.02$.

28. $1.50 - 0.02$.

29. 1.50×0.02.

30. $1.50 \div 0.02$.

31. 1.62×2.4.

32. 16.2×24.

33. 162×0.24.

34. 6×3.2 ft.

35. 4×1.3 mi.

36. 12×2.6 in.

37. 24×3.5 ft.

38. 2.4×35 ft.

39. 2.4×0.35 ft.

40. 0.24×35 ft.

PROBLEMS WITHOUT NUMBERS

All work oral

1. If John picked a certain number of quarts of berries today and a certain part as many yesterday, how do you find how many quarts he picked in the two days?

2. If you know the number of decimal places in each of two numbers, how do you find how many decimal places to point off in the result when you multiply the numbers together?

3. If Ethel lives a certain distance from school, and this distance is a certain number of times as far as Kate lives, how do you find how far from school Kate lives?

4. If you know the cost of a certain number of boxes of oranges, how do you find the cost of a certain other number of boxes at the same price per box?

5. If you know the thickness of a sheet of blotting paper to the nearest hundredth of an inch, how can you, without measuring, find the height of a pile containing a certain number of these sheets?

6. If you know the area of a kitchen floor in square feet, how do you find the number of square yards of linoleum needed to cover the floor, making no allowances for waste?

7. If you know the number of square rods in the area of a field, how do you find the number of acres in the field?

8. If you know the capacity of a can in gallons, how do you find the number of quarts it will hold?

9. How do you express a decimal as a fraction? a fraction as a decimal?

PROBLEMS FOR COMPLETION

1. The daily route of a certain rural mail carrier is 36.4 mi. long. He goes over this route 305 times a year. Complete the problem in any reasonable way that you wish and then solve it.

2. On each trip the carrier in Ex. 1 uses 3 gal. of gasoline costing 17¢ a gallon. Complete and solve the problem.

Complete and solve the following problems:

3. In the 305 trips referred to in Ex. 1, the carrier delivered 12,256 letters, each bearing a 2-cent stamp.

4. The school expenses in a country school for 4 mo. were as follows: teacher's salary, \$400; fuel, \$42.80; textbooks, \$16.80; repairs, \$18.28; janitor, \$24.72. There were 20 pupils in the school.

5. Charles lives 1.6 mi. from his school. He walks both ways 4 da. in the week, and rides the other school day. There are 20 school days in a certain month.

6. Mr. Gray gave his son John the use of a small piece of land. John raised watermelons on it and sold his crop of 27 melons at an average price of $33\frac{1}{3}$¢ each. His father then gave him enough more to amount to \$10.

7. Mr. Shaw had a crib of corn containing 900 bu. Because he neglected to repair the crib, rats destroyed 25 bu. of corn. The corn was worth 90¢ a bushel.

8. We arrive at Syracuse, where we have to change cars, at 11 57 A.M. Ordinarily, the other train that we take leaves at 2 19 P.M., but it is 35 min. late.

Testing Your Alertness. In arithmetic you must be alert. This test will show how alert you really are.

In the three lines given below, each number is obtained in a certain way from the numbers coming before it. In each line four numbers are given, and you are to supply the fifth number, which is printed in heavy type:

2, 4, 6, 8, **10**
16, 8, 4, 2, **1**
15, 13, 10, 6, **1**

In the first row, 2 is added to each number; in the second, each number is divided by 2; in the third, 2 is subtracted from 15, 3 from 13, 4 from 10, and 5 from 6.

ALERTNESS TEST

Copy the following sets of numbers and insert the fifth number in each line, all in 3 min. from the time you start:

1. 9, 8, 7, 6.
2. 2, 4, 8, 16.
3. 3, 6, 9, 12.
4. 3, 6, 12, 24.
5. 4, 2, 1, $\frac{1}{2}$.
6. 2, 7, 12, 17.
7. 3, 7, 11, 15.
8. 9, 3, 1, $\frac{1}{3}$.
9. $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$, 1.
10. 4, 3, 2, 1.
11. 55, 61, 67, 73.
12. 39, 32, 25, 18.
13. 42, 51, 60, 69.
14. 10, 20, 40, 80.
15. 60, 30, 15, $7\frac{1}{2}$.
16. 3, 4, 3, 4.
17. 5, $7\frac{1}{2}$, 10, $12\frac{1}{2}$.
18. 1, 2, 4, 7.
19. 9, 10, 8, 9.
20. 18, 17, 15, 12.

MISCELLANEOUS PROBLEMS

1. If the fee charged for registering an automobile is 50¢ for each 100 lb. for cars weighing 3500 lb., or less, what will be the cost of registering a car weighing 2800 lb.? 3100 lb.? 3400 lb.? 2600 lb.?

2. For cars weighing over 3500 lb. the registration fee is 50¢ for each 100 lb. up to 3500 lb. and 75¢ for each 100 lb. over 3500 lb. What will be the cost of registering a car weighing 3600 lb.? 3800 lb.? 4100 lb.?

3. Allowing $31\frac{1}{2}$ gal. to a barrel, how many gallons are there in a tank that holds $23\frac{1}{2}$ barrels? How many barrels are there in a tank that holds 7875 gal.?

4. How many automobiles weighing 3200 lb. each will it take to make up a shipment that will weigh 16 T.?

5. If a motor truck uses on an average 75 gal. of gasoline every $3\frac{1}{2}$ da., how much should the driver order for 70 da.?

6. If a business house uses on an average 500 envelopes every 30 da., how long will 3500 envelopes last? How long will 5750 envelopes last?

7. A dressed hog weighed 180 lb. If this was $\frac{3}{4}$ of what it weighed when alive, how much did the hog cost at 20¢ a pound, live weight?

8. If 3 barrels of pork sold for $153, what is the value of $\frac{3}{4}$ of a barrel?

9. At $8\frac{1}{3}$¢ a pound, how much sugar can you get in exchange for 25 doz. eggs at 52¢ per dozen?

10. A baker bought 3 doz. eggs for $1.50. If six of them were bad, what did the good eggs cost him per dozen?

ROBERT'S CORN FIELD

1. Robert decides to earn money by raising corn. He rents from his father a field containing 320 sq. rd. At $10 an acre, how much should he charge to expenses as rent?

2. He estimates his labor at 40¢ an hour and the work of a horse at 60¢ an hour. Find the labor cost in a month when he puts in 42 hr., during 4 hr. of which he has a horse.

3. For seed he needs 16 ears to the acre. At $3\frac{1}{2}$¢ an ear, how much does the seed cost for 2 A.?

4. If he uses 1 qt. of seed corn to 200 hills, how many hills can he plant with 1 pk. of seed?

5. If he plants the hills 3 ft. 4 in. apart, and no hill is nearer than 4 ft. to either end, how many hills can he plant in a row which is 32 rd. long?

6. Robert's field is 10 rd. wide. If he plants the rows 3 ft. 6 in. apart, and no row is nearer to either side than 3 ft. 9 in., how many rows does he plant?

7. By testing his seed corn Robert increases his crop by one ear of corn to every four hills. Allowing 100 ears to a bushel, by how many bushels does he increase it? In such problems discard meaningless remainders; for example, take $202\frac{1}{4}$ ears as 202 ears and take $2\frac{1}{50}$ bu. as 2 bu.

8. If Robert sold his corn at 90¢ per bushel, how much more did he make by testing the seed in Ex. 7?

9. On one acre of his land, on which he used fertilizer costing $14, Robert raised 43 bu. On the other acre, without fertilizing, he raised only 24 bu. At 90¢ a bushel, how much did he gain by fertilizing?

A FARM CLUB

1. A boy in a farm club husked all the corn grown on an acre of land. He saved out 7 bu. for seed and sold the rest, which weighed 3225 lb. Allowing 75 lb. to a bushel of new ear corn, how many bushels did the acre yield?

2. In Ex. 1, suppose that the seed corn was worth $3 a bushel, that the ordinary field corn was worth 90¢ a bushel, and that the stalks were worth a quarter as much as the corn itself. Find the value of the crop.

3. The accounts kept by a club boy showed these items:

Rent of land,	$7.00	Husking,	$5.00
Plowing,	2.00	Fertilizing,	7.50
Harrowing,	1.25	Hoeing, 12 hr. @ 44¢,	(?)
Seed,	.91	Cultivating, 3 times @ 60¢,	(?)

If he raised $72\frac{1}{2}$ bu., what was the cost per bushel?

4. A boy finds that if he uses stable fertilizer, which he can get for nothing, a half acre produces corn that will sell for $35. Without the fertilizer the land will produce only $22. Find the gain in income per acre from fertilizing.

5. An average ear of white dent seed corn weighs 12 oz. Allowing 75 lb. to a bushel of new ear corn, how many ears of seed corn are there in a bushel?

6. Some farmers estimate that there are 100 ears of ordinary field corn in a bushel. If 15 ears will plant an acre, how many acres will $3\frac{1}{2}$ bu. plant?

7. The club found that 1 A. of land should produce $64 above all expenses. At this rate, how much should a piece of land containing 120 sq. rd. produce?

V. LITTLE EXAMINATIONS

I. 1. $5\frac{1}{2}$ ft. = (?) in.
2. 25 in. = (?) ft.
3. $7\frac{3}{4}$ lb. = (?) oz.
4. 44 qt. = (?) pk.
5. $3\frac{4}{5} \times 6\frac{2}{3}$.
6. $5 \times 9\frac{3}{4}$ ft.
7. $3\frac{1}{2} \times 7\frac{1}{2}$ ft.
8. $82\frac{1}{2}$ ft. ÷ 3.

II. 1. $6\frac{3}{4}$ ft. = (?) in.
2. 108 in. = (?) yd.
3. $4\frac{1}{2}$ sq. mi. = (?) A.
4. 61 sq. ft. = (?) sq. yd.
5. $9\frac{2}{3}$ yd. ÷ 3.
6. $4\frac{7}{8} \times 8\frac{3}{4}$ yd.
7. $8 \times 5\frac{3}{4}$ bu.
8. $103\frac{1}{3} \div 3\frac{1}{3}$.

III. 1. $46\frac{1}{2}$ ft. = (?) yd.
2. 2560 A. = (?) sq. mi.
3. $72\frac{1}{2}$ gal. = (?) qt.
4. 9 sq. ft. = (?) sq. in.
5. $9 \times 7\frac{1}{2}$ lb.
6. $7\frac{3}{4} \times 9\frac{1}{2}$ lb.
7. $16\frac{1}{2} \div 8\frac{1}{4}$.
8. $8\frac{1}{2}$ lb. + $\frac{1}{4}$ lb.

IV. 1. $61\frac{1}{2}$ ft. = (?) yd.
2. 24 A. = (?) sq. rd.
3. $2\frac{1}{4}$ T. = (?) lb.
4. 45 sq. ft. = (?) sq. yd.
5. $17\frac{1}{8} - 9\frac{3}{4}$.
6. $8 \times 5\frac{1}{4}$ ft.
7. $3\frac{7}{8} \times 6\frac{2}{3}$ ft.
8. $31\frac{1}{2}$ ft. ÷ 6.

V. 1. 16.28×9.79.
2. $16.28 \div 0.04$.
3. $16.28 + 9.79 - 3.99$.
4. $16.28 - 9.79 + 16.22$.
5. $36 \times 16\frac{2}{3}$¢.
6. $84 \times 8\frac{1}{3}$¢.
7. $96 \times 37\frac{1}{2}$¢.
8. $88 \times 62\frac{1}{2}$¢.

VI. 1. 82.75×1.125.
2. $16.25 \div 0.125$.
3. $78.23 + 16.85 - 23.08$.
4. $6.234 - 5.031 + 8.277$.
5. 3.14×6.72.
6. $2.25 \div 1.5$.
7. $2.25 \div 0.15$.
8. $2\frac{1}{2} \times 24.8$.

CHAPTER III

I. REVIEW OF OPERATIONS

A SILENT READING LESSON

We can easily add, subtract, multiply, and divide with whole numbers, or integers as they are also called. We know the combinations that are needed, such as $7 + 8$, $17 - 9$, 8×7, and $72 \div 8$. We also know that $72 \div 8$ may be written as $\frac{72}{8}$, or in other ways, and have learned some things about fractions and decimals.

We can read numbers as far as millions and know that a thousand million is called a billion.

We can solve easy problems and we know that it is necessary to read the problems carefully so as to find out just what is given and just what is required. We can then think out the solution.

We also know that it is a good thing for us to estimate a result in advance so as to avoid obtaining an answer that is absurdly large or absurdly small. We have learned how to check our results so as to be sure that they are right; for example, we know that $51 - 29 = 22$, because $29 + 22 = 51$.

We have just had a vacation, however, and are "a little rusty" in computing. It will be a good plan, therefore, to review our earlier work briefly in order to be sure that we can find results quickly and accurately.

ORAL DRILL CHART IN RAPID ADDITION

State the results of these additions rapidly:

1.	5 1	2 0	8 3	1 2	4 3	3 1	6 3	9 0	0 5	7 0
2.	9 1	1 0	6 2	0 0	4 1	8 2	2 2	7 1	5 2	3 0
3.	6 1	0 1	4 2	1 1	3 2	2 1	5 0	9 2	7 2	8 1
4.	4 4	6 0	0 2	5 3	1 3	3 4	2 3	8 0	0 6	7 3
5.	8 4	1 5	6 4	2 4	7 4	0 3	4 7	3 3	5 4	9 3
6.	9 4	3 5	0 4	7 5	2 5	6 5	1 6	4 5	8 7	5 5
7.	5 8	6 6	2 6	7 7	1 8	4 6	9 5	0 7	8 5	3 6
8.	3 7	5 6	1 7	0 8	6 7	7 6	2 7	9 6	8 6	4 9
9.	5 7	6 8	2 8	8 9	4 8	1 9	9 9	9 7	7 9	3 8
10.	6 9	1 4	9 8	3 9	8 8	2 9	0 9	7 8	5 9	9 8

DRILL CHART IN RAPID ADDITION

Place the edge of your paper under the first row of examples and add rapidly, writing only the results. Then fold the answers under, do the next row, and so on:

1.	7	8	9	8	7	9	8	9	7	8
	6	9	6	8	8	7	7	8	9	6
2.	6	9	6	6	5	5	5	5	9	6
	8	9	7	9	8	6	7	9	4	6
3.	5	7	6	5	9	8	8	5	9	4
	9	5	2	6	3	4	1	9	7	5
	3	7	9	8	5	1	7	6	9	7
4.	6	2	4	5	1	9	8	7	3	6
	6	3	5	7	4	4	3	2	4	9
	1	6	1	6	2	3	7	4	2	8
5.	1	3	5	9	6	0	8	9	7	8
	3	6	2	2	9	8	4	3	3	7
	4	9	7	3	4	7	9	4	5	4
6.	7	7	9	8	5	2	6	3	5	2
	6	4	5	9	6	3	3	9	2	8
	1	6	3	4	7	9	5	8	6	9
7.	8	3	5	9	6	9	5	8	7	9
	3	9	4	2	2	3	9	3	8	0
	5	7	0	9	3	6	1	9	2	2

DRILL CHART IN RAPID ADDITION

Placing your paper as suggested on page 177, write the results of these additions as rapidly as you can, and then check each one by adding in the opposite direction:

1.	122	247	372	198	617	940
	340	326	421	657	249	582
	268	129	506	423	583	317
2.	963	201	269	436	487	429
	89	485	437	67	289	450
	528	76	180	270	83	478
	147	967	48	855	248	38
3.	902	785	268	307	689	543
	497	184	962	30	734	278
	973	38	564	752	47	106
	965	382	67	879	208	30
4.	914	106	277	125	742	609
	147	752	347	90	382	752
	583	99	967	235	86	328
	627	384	6	965	972	34
5.	142	279	938	968	304	478
	629	886	870	579	478	569
	438	157	226	487	892	342
	16	197	3	18	29	164
	740	403	514	326	698	873

PROBLEMS IN ADDITION

1. Mr. Brown had to take an early train to go to the state fair, and Ben drove him to the station in the city where they ate breakfast. They ordered a baked apple, 25¢; a grape fruit, 30¢; an omelette, 35¢; ham and eggs, 60¢; wheat cakes and sirup, 20¢; a cup of coffee, 10¢; and a cup of cocoa, 10¢. How much was the bill?

2. At the fair, Mr. Brown bought a washing-machine for $125.50, a radio set for $39.75, a porch hammock for $11.75, a bicycle for Ben costing $37.50, and a cream separator for $113.35. How much did he spend?

3. The deposits in our school savings bank during the last school year were as follows: September, $28.25; October, $39.75; November, $43.69; December, $23.46; January, $45.76; February, $34.62; March, $39.41; April, $32.60; May, $22.61. What was the total amount?

4. Last August six car loads of soft coal were bought for our schools. The loads weighed 102,607 lb., 99,342 lb., 98,746 lb., 101,609 lb., 103,429 lb., and 101,027 lb. Find the total number of pounds of coal.

5. During the week before Christmas the sales in a store were as follows: Monday, $196.29; Tuesday, $236.42; Wednesday, $241.65; Thursday, $255.75; Friday, $262.81; and Saturday, $309.28. Find the total sales for the week.

6. At a sale a dealer sold six overcoats at the following prices: $18.50, $24.75, $36.50, $49.95, $59.50, and $69.75. How much did he receive for all the coats?

DRILL CHART IN ADDING MONEY

Placing your paper as suggested on page 177, write the results of these additions as rapidly as you can, and then check each one by adding in the opposite direction:

1.	$2.97	$3.66	$4.86	$5.39	$12.82
	3.42	.75	2.97	6.78	3.96
	6.08	2.68	3.42	.96	15.00
	9.25	3.47	7.88	3.07	19.29
2.	$7.56	$8.82	$8.80	$8.87	$15.88
	3.49	3.41	2.75	9.26	16.72
	2.73	5.80	3.06	6.45	25.31
	5.28	9.62	4.22	5.32	40.25
	3.42	3.46	7.89	2.01	34.26
3.	$8.72	$9.45	$3.97	$7.76	$27.92
	5.41	5.32	8.65	5.48	32.64
	1.23	3.24	5.34	8.32	58.76
	3.34	6.05	3.21	2.41	69.43
	5.65	5.78	5.09	5.75	80.72
	7.89	9.80	9.08	6.92	21.35
4.	$1.28	$8.77	$5.40	$6.50	$34.68
	2.96	6.42	6.90	2.81	7.92
	3.48	9.39	7.77	5.72	34.77
	9.30	5.45	6.45	3.46	18.92
	5.77	2.39	3.29	9.87	27.76
	6.62	1.07	4.76	8.09	45.75

ORAL DRILL CHART IN RAPID SUBTRACTION

State the results of these subtractions rapidly:

1.	1	5	1	6	8	10	19	12	10	12
	0	4	1	5	7	6	9	7	1	2
2.	3	7	0	5	4	10	17	10	11	10
	3	6	0	5	3	4	8	8	7	5
3.	3	5	6	3	2	20	16	14	11	11
	1	3	3	2	1	6	8	5	8	4
4.	4	6	7	9	4	20	12	10	12	10
	0	4	5	7	2	7	8	7	4	3
5.	7	8	5	7	5	18	10	15	13	11
	3	1	2	4	0	9	9	7	7	2
6.	6	9	6	8	8	17	14	15	10	11
	2	6	1	5	6	9	8	6	2	6
7.	4	8	9	5	7	16	11	14	11	10
	1	2	4	1	7	9	9	7	5	4
8.	8	9	9	7	6	15	20	12	13	11
	3	5	3	0	0	9	8	6	4	3
9.	9	9	8	9	7	14	12	14	12	12
	8	1	8	2	2	9	9	6	3	5
10.	9	9	7	8	8	13	13	12	13	13
	9	0	1	0	4	9	8	8	5	6

DRILL CHART IN RAPID SUBTRACTION

Placing your paper as suggested on page 177, subtract, writing the results rapidly, and check each result by adding it to the number subtracted:

1.	247	673	529	643	796	825
	125	251	318	401	685	302
2.	243	622	735	843	778	825
	114	507	226	219	149	119
3.	826	548	702	820	971	517
	132	356	131	340	280	256
4.	623	507	652	781	607	300
	439	278	396	294	528	129
5.	742	807	912	803	912	814
	698	348	637	436	377	698
6.	548	620	833	706	671	833
	239	157	288	529	182	699
7.	980	700	802	513	800	525
	69	77	160	96	69	197
8.	723	936	714	800	642	826
	190	800	96	7	394	468
9.	628	877	628	547	625	333
	194	222	99	123	389	99

DRILL CHART IN SUBTRACTING MONEY

Placing your paper as suggested on page 177, write the results of the following subtractions rapidly, and check each result by adding it to the number subtracted:

1.	$8.70 2.60	$9.45 1.25	$7.35 5.05	$6.92 3.70	$25.48 10.25
2.	$8.62 4.34	$9.75 2.56	$6.30 4.15	$5.75 2.38	$62.32 41.28
3.	$7.23 4.60	$6.41 3.50	$9.78 4.91	$4.25 1.37	$33.48 21.62
4.	$9.36 7.48	$8.02 4.36	$7.00 .98	$8.25 1.38	$69.42 29.53
5.	$3.67 1.98	$4.60 2.85	$7.03 5.67	$5.01 2.62	$83.73 62.84
6.	$8.72 3.96	$5.81 2.96	$9.20 3.09	$8.73 7.83	$37.04 26.92
7.	$9.28 .96	$7.22 5.34	$9.20 7.32	$7.64 1.99	$44.43 39.78
8.	$8.13 4.66	$6.29 6.17	$5.33 3.53	$9.01 .68	$38.90 27.98
9.	$4.25 3.78	$6.07 2.39	$5.13 2.93	$6.48 3.96	$72.73 37.47

PROBLEMS IN SUBTRACTION

1. When Ben and Tom started on an automobile trip with their father, the speedometer registered 5686.8 mi. When they arrived home, the speedometer registered 7564.5 mi. How far did they go?

2. Our school team deposited in the bank, during the football season, receipts amounting to $920.31, and drew out $892.84 for expenses. How much was left?

3. We had a school bazaar for the purpose of raising money to buy a printing press costing $275. After paying expenses, we cleared $189.34. How much more must we raise before we can buy the press?

4. On Jan. 1 the gas meter in School No. 17 registered 5,642,000 cu. ft. On Feb. 1 it registered 5,753,000 cu. ft. How many cubic feet were used in January?

5. The water meter registered 1,653,587 cu. ft. on Jan. 1. On Mar. 1 it registered 2,502,431 cu. ft. How many cubic feet of water were used from Jan. 1 to Mar. 1?

6. We wish to buy a velvet curtain costing $421.75 for our school auditorium. We have $298.87 in our treasury and plan to give a school pageant to raise the rest of the money. How much do we need to raise?

7. In 1790 the population of the United States was 3,929,214. In 1920 it was 105,710,620. How many more people were there in the country in 1920?

8. The total area of the farms in the United States in 1900 was 623,218,619 A. In 1920 it was 955,883,715 A. How much greater was the acreage in 1920?

ORAL DRILL CHART IN RAPID MULTIPLICATION

State the results of these multiplications rapidly:

1.	5	6	2	8	4	1	9	9	7	3
	7	8	8	9	8	9	9	7	9	8
2.	3	5	7	2	8	4	0	6	1	9
	0	2	1	2	2	1	0	2	0	1
3.	8	7	9	5	2	3	1	4	0	6
	1	2	2	0	1	2	1	2	1	1
4.	6	1	9	3	8	2	0	7	5	9
	9	4	8	9	8	9	9	8	9	8
5.	6	4	5	0	1	3	2	8	7	0
	0	4	3	2	3	4	3	0	3	6
6.	3	5	1	0	6	2	7	9	8	4
	7	6	7	8	7	7	6	6	6	9
7.	1	8	6	2	7	0	4	3	5	9
	5	4	4	4	4	3	7	3	4	3
8.	5	2	6	7	1	4	9	0	8	3
	8	6	6	7	8	6	5	7	5	6
9.	3	9	0	7	2	6	1	4	8	5
	5	4	4	5	5	5	6	5	7	5
10.	7	0	9	6	3	4	1	8	2	5
	0	5	0	3	1	3	2	3	0	1

DRILL CHART IN RAPID MULTIPLICATION

Placing your paper as suggested on page 177, write the results of these multiplications as rapidly as you can, seeing how long it takes you to do them all:

1.	22	27	33	38	47	41	46	67
	2	2	3	3	3	4	4	4
2.	31	39	40	46	57	21	38	47
	5	5	6	6	6	5	5	5
3.	50	56	81	89	97	91	64	98
	6	6	7	7	7	8	8	8
4.	20	21	26	48	54	87	73	96
	9	9	9	9	9	9	9	9
5.	60	68	67	59	66	73	82	97
	8	8	8	8	8	8	8	8
6.	80	88	79	83	82	54	96	93
	6	6	6	6	7	7	7	7
7.	30	50	70	30	40	60	76	38
	8	8	9	7	5	4	9	7
8.	43	51	71	35	48	42	67	48
	7	8	9	7	5	7	9	8
9.	36	57	75	39	58	68	87	58
	9	8	9	7	5	3	6	9

DRILL CHART IN MULTIPLYING MONEY

Perform the following multiplications rapidly:

1.	$4.17 6	$9.81 2	$4.73 6	$0.98 10	$17.26 13
2.	$5.31 7	$7.64 3	$6.74 7	$7.05 20	$19.39 15
3.	$8.02 9	$5.75 4	$9.29 5	$37.07 7	$42.17 11
4.	$7.43 5	$6.42 5	$3.02 9	$49.71 9	$76.92 23
5.	$6.38 4	$7.07 6	$6.00 6	$39.69 6	$88.72 39
6.	$2.93 8	$8.82 7	$7.32 3	$48.00 4	$49.63 82
7.	$5.30 3	$9.24 8	$8.08 8	$52.09 5	$79.78 69
8.	$6.00 8	$8.07 9	$9.39 4	$98.73 2	$98.79 89
9.	$7.00 9	$7.35 9	$8.60 8	$47.80 5	$78.60 32
10.	$6.83 7	$6.49 6	$5.22 7	$39.95 9	$86.22 76

PROBLEMS IN MULTIPLICATION

1. The cashier at our cafeteria made this table of the day's receipts. Fill in the columns and find both totals:

Paper Money			Coins		
0 Twenties			73 Halves		
2 Tens			141 Quarters		
3 Fives			367 Dimes		
7 Twos			1242 Nickels		
36 Ones			1769 Pennies		
Total			Total		

2. In Ex. 1, what was the total amount received?

3. We used 17,000 cu. ft. of gas last month at $1.20 per 1000 cu. ft. How much was the gas bill?

4. Ben took 39 doz. eggs to the store and received 47¢ a dozen for them. How much money did he receive?

5. Mr. Brown sold 936 bu. of tomatoes to the canning factory and was paid at the rate of 43¢ per bushel. How much did he receive?

Find the cost of the following purchases:

6. 37 blankets at $5.75 each.

7. 378 electric-light bulbs at 49¢ each.

8. 29 cases of tomatoes at $3.60 per case.

9. 93 qt. of milk at 12¢ a quart.

10. 376 gal. of gasoline at 23¢ per gallon.

ORAL DRILL CHART IN RAPID DIVISION

State the results of these divisions rapidly:

1.	$2\overline{)4}$	$5\overline{)5}$	$7\overline{)14}$	$6\overline{)12}$	$5\overline{)10}$	$7\overline{)35}$
2.	$7\overline{)0}$	$3\overline{)3}$	$7\overline{)28}$	$4\overline{)16}$	$9\overline{)18}$	$4\overline{)20}$
3.	$8\overline{)8}$	$4\overline{)4}$	$8\overline{)16}$	$9\overline{)36}$	$5\overline{)35}$	$5\overline{)15}$
4.	$3\overline{)6}$	$9\overline{)63}$	$6\overline{)18}$	$9\overline{)27}$	$4\overline{)12}$	$2\overline{)10}$
5.	$4\overline{)8}$	$8\overline{)72}$	$7\overline{)21}$	$4\overline{)24}$	$7\overline{)42}$	$8\overline{)48}$
6.	$6\overline{)6}$	$7\overline{)63}$	$8\overline{)32}$	$6\overline{)24}$	$2\overline{)12}$	$3\overline{)12}$
7.	$3\overline{)0}$	$5\overline{)0}$	$8\overline{)24}$	$5\overline{)20}$	$7\overline{)49}$	$2\overline{)14}$
8.	$9\overline{)9}$	$9\overline{)72}$	$6\overline{)30}$	$4\overline{)28}$	$3\overline{)15}$	$4\overline{)32}$
9.	$2\overline{)0}$	$6\overline{)54}$	$4\overline{)36}$	$6\overline{)36}$	$2\overline{)16}$	$3\overline{)18}$
10.	$1\overline{)0}$	$6\overline{)0}$	$8\overline{)40}$	$4\overline{)40}$	$7\overline{)56}$	$5\overline{)25}$
11.	$3\overline{)9}$	$9\overline{)54}$	$6\overline{)42}$	$7\overline{)77}$	$3\overline{)21}$	$2\overline{)18}$
12.	$4\overline{)0}$	$8\overline{)0}$	$9\overline{)45}$	$5\overline{)30}$	$8\overline{)56}$	$6\overline{)54}$
13.	$7\overline{)7}$	$8\overline{)56}$	$8\overline{)64}$	$6\overline{)48}$	$2\overline{)20}$	$3\overline{)24}$
14.	$2\overline{)8}$	$4\overline{)44}$	$5\overline{)35}$	$9\overline{)54}$	$3\overline{)27}$	$7\overline{)70}$
15.	$2\overline{)6}$	$9\overline{)18}$	$9\overline{)63}$	$9\overline{)81}$	$5\overline{)40}$	$8\overline{)80}$

DRILL CHART ON THE DIVISION TABLE

Copy and complete the following division tables:

1. $\frac{0}{1} = 0$, $\frac{1}{1} = 1$, $\frac{2}{1} = 2$, and so on to $\frac{10}{1} = 10$.

2. $\frac{0}{2} = 0$, $\frac{2}{2} = 1$, $\frac{4}{2} = 2$, and so on to $\frac{20}{2} = 10$.

3. $\frac{0}{3} = 0$, $\frac{3}{3} = 1$, $\frac{6}{3} = 2$, and so on to $\frac{30}{3} = 10$.

4. $\frac{0}{4} = 0$, $\frac{4}{4} = 1$, $\frac{8}{4} = 2$, and so on to $\frac{40}{4} = 10$.

5. $\frac{0}{5} = 0$, $\frac{5}{5} = 1$, $\frac{10}{5} = 2$, and so on to $\frac{50}{5} = 10$.

Continue the above tables from

6. $\frac{0}{6} = 0$ to $\frac{60}{6} = 10$.

7. $\frac{0}{7} = 0$ to $\frac{70}{7} = 10$.

8. $\frac{0}{8} = 0$ to $\frac{80}{8} = 10$.

9. $\frac{0}{9} = 0$ to $\frac{90}{9} = 10$.

Copy and complete the following statements:

10. 81 ÷ 9 =
11. 32 ÷ 8 =
12. 72 ÷ 9 =
13. 49 ÷ 7 =
14. 48 ÷ 6 =
15. 54 ÷ 9 =
16. 24 ÷ 8 =
17. 45 ÷ 9 =
18. 40 ÷ 8 =
19. 54 ÷ 6 =

20. 56 ÷ 7 =
21. 36 ÷ 9 =
22. 42 ÷ 6 =
23. 48 ÷ 8 =
24. 42 ÷ 7 =
25. 56 ÷ 8 =
26. 36 ÷ 6 =
27. 72 ÷ 8 =
28. 27 ÷ 9 =
29. 63 ÷ 7 =

30. 120 ÷ 10 =
31. 120 ÷ 12 =
32. 144 ÷ 12 =
33. 132 ÷ 11 =
34. 132 ÷ 12 =
35. 121 ÷ 11 =
36. 108 ÷ 12 =
37. 108 ÷ 9 =
38. 108 ÷ 27 =
39. 108 ÷ 36 =

ORAL DRILL CHART IN RAPID DIVISION

State the results rapidly, including the remainders:

1. $2\overline{)3}$
2. $2\overline{)5}$
3. $2\overline{)7}$
4. $2\overline{)9}$
5. $2\overline{)1}$
6. $3\overline{)4}$
7. $3\overline{)5}$
8. $3\overline{)8}$
9. $3\overline{)2}$
10. $5\overline{)2}$
11. $5\overline{)1}$
12. $5\overline{)4}$
13. $6\overline{)7}$
14. $6\overline{)5}$
15. $6\overline{)1}$
16. $6\overline{)22}$
17. $9\overline{)33}$
18. $9\overline{)31}$
19. $7\overline{)51}$
20. $7\overline{)54}$
21. $7\overline{)61}$
22. $7\overline{)68}$
23. $9\overline{)40}$
24. $9\overline{)60}$
25. $9\overline{)62}$
26. $9\overline{)22}$
27. $9\overline{)44}$
28. $6\overline{)50}$
29. $7\overline{)32}$
30. $9\overline{)64}$
31. $8\overline{)62}$
32. $8\overline{)61}$
33. $8\overline{)63}$
34. $9\overline{)70}$
35. $4\overline{)10}$
36. $9\overline{)16}$
37. $4\overline{)11}$
38. $7\overline{)11}$
39. $8\overline{)13}$
40. $8\overline{)52}$
41. $6\overline{)11}$
42. $9\overline{)11}$
43. $9\overline{)42}$
44. $9\overline{)50}$
45. $9\overline{)53}$
46. $9\overline{)43}$
47. $9\overline{)35}$
48. $9\overline{)40}$
49. $7\overline{)40}$
50. $4\overline{)31}$
51. $9\overline{)24}$
52. $6\overline{)55}$
53. $8\overline{)11}$
54. $8\overline{)60}$
55. $8\overline{)31}$
56. $9\overline{)13}$
57. $9\overline{)30}$
58. $8\overline{)53}$
59. $9\overline{)51}$
60. $9\overline{)71}$
61. $9\overline{)51}$
62. $9\overline{)26}$
63. $9\overline{)14}$
64. $6\overline{)41}$
65. $7\overline{)34}$
66. $9\overline{)20}$
67. $8\overline{)55}$
68. $9\overline{)60}$
69. $7\overline{)13}$
70. $9\overline{)32}$
71. $9\overline{)10}$
72. $7\overline{)12}$
73. $9\overline{)34}$
74. $8\overline{)15}$
75. $8\overline{)21}$

SPEED TEST CHART IN DIVISION

Perform these divisions on paper, writing the number of minutes it takes for each column:

1. $7\overline{)4910}$
2. $6\overline{)3624}$
3. $5\overline{)7002}$
4. $8\overline{)4807}$
5. $9\overline{)8102}$
6. $3\overline{)3901}$
7. $2\overline{)4801}$
8. $3\overline{)6120}$
9. $4\overline{)4803}$
10. $5\overline{)7511}$
11. $6\overline{)6636}$
12. $7\overline{)5614}$
13. $8\overline{)4848}$
14. $9\overline{)8118}$
15. $3\overline{)3618}$
16. $4\overline{)3216}$
17. $5\overline{)4551}$
18. $6\overline{)5407}$
19. $7\overline{)5609}$
20. $8\overline{)6417}$
21. $15\overline{)165}$
22. $15\overline{)1650}$
23. $21\overline{)2310}$
24. $21\overline{)2331}$
25. $31\overline{)3441}$
26. $41\overline{)4551}$
27. $19\overline{)2090}$
28. $18\overline{)1998}$
29. $17\overline{)3570}$
30. $17\overline{)3587}$
31. $33\overline{)6996}$
32. $33\overline{)9900}$
33. $33\overline{)9999}$
34. $12\overline{)1440}$
35. $12\overline{)1452}$
36. $23\overline{)3496}$
37. $23\overline{)4830}$
38. $23\overline{)4853}$
39. $27\overline{)2970}$
40. $27\overline{)2997}$
41. $42\overline{)6048}$
42. $42\overline{)6468}$
43. $42\overline{)6510}$
44. $47\overline{)3807}$
45. $53\overline{)3763}$
46. $55\overline{)3355}$
47. $55\overline{)3850}$
48. $57\overline{)4047}$
49. $82\overline{)6642}$
50. $83\overline{)4233}$
51. $92\overline{)2576}$

PROBLEMS IN DIVISION

1. It is about 5302 mi. from New York to San Francisco through the Panama Canal. At an average speed of 21 mi. per hour, in how many days will a steamer make the trip?

2. The distance from New York to San Francisco by the route around South America is about 13,244 mi. At an average speed of 21 mi. per hour, how many days will it take a steamer to make the trip?

3. In one week 50 vessels passed through the Panama Canal, and paid a total of $211,150 for tolls. What was the average toll per vessel?

4. If it costs $104,760 to run a village school system for 180 da., what is the average cost per day?

5. If the coal used in heating our school for a period of 120 da. costs $81.60, what is the cost per day?

6. Ben's father found that he used 330 gal. of gasoline to drive his car a total of 5280 mi. This was an average of how many miles per gallon?

7. A man buys $1729.44 worth of household furniture. If he pays for it in 18 equal monthly installments, what is the amount of each payment?

8. The sun is about 93,000,000 mi. from the earth. If you could travel by airplane from the earth to the sun at the rate of 100 mi. an hour, how many days would it take you to get there? This is about how many years?

9. In a recent year it cost the City of Rochester about $3,049,000 to educate the 27,000 children in the elementary schools. About how much did it cost to educate each pupil?

PROBLEMS FOR GIRLS

1. If you wish to buy material to curtain two windows and need 4 yd. for each window, how much will the material cost at 90¢ a yard?

In solving this problem, think to yourself: "I am given (1) the number of windows, and (2) the number of yards for each window. I can therefore find the total number of yards. I am also given (3) the price per yard, and so can find the total cost. That is, there are two steps: (1) to find the total number of yards, and (2) to find the cost."

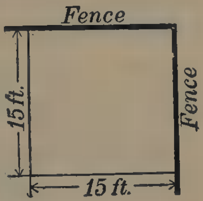
2. If you wish to buy material to curtain four windows and need 5 yd. for each window, how much will the material cost at 80¢ a yard?

3. Suppose that you wish to buy material to curtain two windows, and that each window is 5 ft. high and requires two widths of material. If it is necessary to allow 9 in. on each strip for hemming, how much will the material cost at 90¢ a yard?

In solving this problem, think to yourself: "I am given (1) the number of windows, (2) the height of each, (3) the number of strips to a window, (4) that each strip requires 5 ft. 9 in. of material, and (5) the price per yard. Hence by taking 2 × 2 × 5 ft. 9 in. I can find the number of yards. I can multiply 5 ft. 9 in. by 2 × 2, or 4, by simply writing 4 × 5 ft. = 20 ft. = $6\frac{2}{3}$ yd., and 4 × 9 in. = 36 in. = 1 yd. I therefore need $6\frac{2}{3}$ yd. + 1 yd., or $7\frac{2}{3}$ yd. Then, since I know the price per yard, I can find the total cost by multiplying the price per yard by the number of yards; that is, $7\frac{2}{3}$ × 90¢ will give me the answer."

PROBLEMS FOR BOYS

1. Ben and Tom built a new pen for their rabbits. They built it in a corner of the yard close to a tight-board fence, and so had to build only two sides, each 15 ft. long. The boys used small-meshed chicken wire costing 6¢ a running foot. How much did the wire cost?



Think to yourself: "I am given (1) the number of feet for each side, (2) the number of sides to be built, and (3) the cost per foot. There are therefore two steps: (1) to find the total number of feet, and (2) to find the cost."

2. Walter and Sam also built a rabbit pen, but had no fence against which to build. They therefore had to build four sides, each of which they made 10 ft. long. Their wire was wider than that used in Ex. 1 and cost 8¢ a running foot. How much did the wire for this pen cost?

3. The rabbits got out of the pens by digging under the wire. To stop this the boys had to put a strip of chicken wire down into the ground all round the pens. This wire was not so wide and cost only 4¢ a foot. How much did this wire cost for the pen belonging to Walter and Sam?

4. At 4¢ a foot, what did it cost Ben and Tom for wire to put into the ground on all four sides of their pen?

5. The cats and dogs jumped over the top of the wire and tried to catch the rabbits. To cover their pen Ben and Tom used five strips of wire, each 15 ft. long, and Walter and Sam used four strips, each 10 ft. long. At 11¢ a foot, how much did it cost to cover each pen?

PROBLEMS ABOUT A CAMPING TRIP

1. The Brown family is getting ready for an automobile camping trip. They buy the following articles: a tent, $11.25; 5 folding cots at $2.98 each; a cook-stove, $4.95; camp dishes, $11.75; a folding table, $4.75; 5 camp stools at 98¢ each; and 5 water-proof blankets at $5.95 each. How much do all these articles cost?

2. For camp clothing Mr. Brown spends $5.75, Mrs. Brown spends $6.30, and they spend $2.95 each for Ben, Tom, and Tony. How much does the clothing cost?

3. Their car averages 15 mi. to 1 gal. of gasoline and uses 1 qt. of oil every 100 mi. At 25¢ a gallon for gasoline and 30¢ a quart for the oil, how much will the gasoline and oil cost if they travel 1500 mi.?

4. If they should travel by rail instead of by car, the fare would be 3¢ per mile. If Tony goes for half fare, but the other four pay full fare, how much would their tickets cost for a trip of 1500 mi.?

5. Before they start on the trip they also buy a new battery for the car at $34.50, 3 new tires at $29.50 each, and 3 new inner tubes at $4.25 each. How much do all these articles cost?

6. All three boys need bathing suits, which cost $2.75 each. Ben and Tom need sweaters, which cost $3.25 each, and Tony needs a pair of water wings, which cost 59¢. Mr. Brown plans to buy a new fishing pole for $5.95 and a reel for $2.45. Mrs. Brown needs a raincoat, which costs $14.75. How much will all these articles cost?

Invoice. An *invoice* is [redacted] bill which is sent by a wholesale business house to a retail dealer at the time that goods are shipped. The following model shows a simple invoice:

SCOTT AND FIELD
Wholesale Grocers

Sold to
H. J. Smith
64 Main St.
Olean, N. Y.

ROCHESTER, N. Y., Jan. 10, *1928*

10 [redacted] tomatoes	@	3.50	35	00		
6 boxes soap	@	7.80	46	80		
500 lb. sugar	@	7¢	35	00		
					116	80

MAKING INVOICES

Make invoices for the following orders:

1. 2 Wilton rugs @ $87.50, 15 yd. linoleum @ $1.75, 12 yd. oil cloth @ 85¢, 6 dining-room chairs @ $8.75.

2. 3 bed-room sets @ $390.50, 6 dining tables at $37.50, 3 doz. dining-room chairs @ $96.50 per dozen.

3. 5 player pianos @ $875.50, 25 mandolins @ $5.75, 20 clarionets @ $17.25, 30 saxophones @ $10.25.

4. 15 washing machines @ $114.95, 8 clothes hampers @ $6.75, 6 doz. mop-wringers @ $27.50 per dozen, 16 vacuum cleaners @ $45.50.

5. 15 radio sets @ $23.50, 25 sets @ $39.75, 27 sets @ $59.00, 5 sets @ $119.95, 35 loud speakers @ $20.75.

6. Make up three invoice problems in which you use prices found, if possible, from stores in your city or town.

Using Large Numbers. The number shown below is read "one hundred forty-seven billion three hundred seven million four hundred twenty-seven thousand two hundred eighty."

$$\overbrace{147}^{\text{Billions}},\overbrace{307}^{\text{Millions}},\overbrace{427}^{\text{Thousands}},\overbrace{280}^{\text{Units}}$$

Numbers as large as this are used chiefly in speaking of sums borrowed by nations. Numbers in millions are often used in speaking of the population of cities, states, and countries, or in speaking of the money needed to carry on large industries or in paying the expenses of a city or state. The exercises below show some of the ways in which large numbers are used. These figures are not exactly correct because the populations change from year to year, and even from day to day.

READING AND WRITING NUMBERS

Read aloud the following numbers which represent the populations of the countries named:

1. United States, 105,711,000.
2. Germany, 59,858,000.
3. England, 35,678,000.
4. France, 39,403,000.
5. China, 302,110,000.
6. India, 247,140,000.

Copy the following, writing the numbers in words:

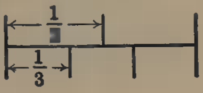
7. The population of the entire British Empire in a recent year was 441,595,965.

8. The public debt of the United States before the World War was $1,908,635,000, and during the war it rose to $26,596,701,648.

II. FRACTIONS

REVIEW OF EASY WORK

1. Here is a line 1 in. long. It is divided into halves and also into thirds. We see that $\frac{1}{2}$ is greater than $\frac{1}{3}$. Draw a line 3 in. long, divide it into thirds and also into fourths, and find which is greater, $\frac{1}{3}$ or $\frac{1}{4}$. Write your answer.



2. From Ex. 1 find which is greater, $\frac{2}{3}$ or $\frac{3}{4}$.

3. As in Ex. 1, show which is greater, $\frac{2}{3}$ or $\frac{3}{5}$.

4. Draw a line and divide it to show that $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$.

Draw lines and divide them to show that

5. $\frac{3}{4} = \frac{6}{8}$.
6. $\frac{1}{5} = \frac{2}{10}$.
7. $\frac{2}{5} = \frac{4}{10}$.
8. $\frac{3}{5} = \frac{6}{10}$.
9. $\frac{1}{6} = \frac{2}{12}$.
10. $\frac{5}{6} = \frac{10}{12}$.
11. $\frac{1}{3} = \frac{2}{6} = \frac{4}{12}$.
12. $\frac{1}{4} = \frac{2}{8} = \frac{4}{16}$.

Copy and complete the following statements:

13. $\frac{3}{4} + \frac{1}{4} =$
14. $\frac{3}{4} - \frac{1}{4} =$
15. $\frac{3}{4} \times \frac{1}{4} =$
16. $\frac{1}{4} \times \frac{3}{4} =$
17. $\frac{3}{4} \div \frac{1}{4} =$
18. $\frac{1}{4} \div \frac{3}{4} =$

Count forward to and then backward from

19. 3 by $\frac{1}{4}$, saying $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, $\frac{4}{4}$, $\frac{5}{4}$, and so on. Then count again, saying $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1, $1\frac{1}{4}$, and so on.

20. 2 by $\frac{1}{8}$, saying $\frac{1}{8}$, $\frac{2}{8}$, $\frac{3}{8}$, $\frac{4}{8}$, $\frac{5}{8}$, and so on. Then count again, saying $\frac{1}{8}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{1}{2}$, $\frac{5}{8}$, and so on.

21. 3 by $\frac{1}{3}$, saying $\frac{1}{3}$, $\frac{2}{3}$, 1, and so on.

22. 2 by $\frac{1}{6}$, saying $\frac{1}{6}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, and so on.

23. 1 by $\frac{1}{12}$, saying $\frac{1}{12}$, $\frac{1}{6}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{5}{12}$, and so on.

DRILL CHART IN ADDING FRACTIONS

Numbers 1 to 20, oral

State the results of the following additions:

1. $\frac{1}{2} + \frac{1}{2}$.
2. $\frac{1}{4} + \frac{3}{4}$.
3. $\frac{3}{4} + \frac{1}{4}$.
4. $\frac{1}{8} + \frac{7}{8}$.
5. $\frac{7}{8} + \frac{1}{8}$.
6. $\frac{3}{8} + \frac{5}{8}$.
7. $\frac{5}{8} + \frac{3}{8}$.
8. $\frac{1}{5} + \frac{4}{5}$.
9. $\frac{2}{5} + \frac{3}{5}$.
10. $\frac{1}{6} + \frac{5}{6}$.
11. $\frac{2}{4} + \frac{1}{4}$.
12. $\frac{1}{2} + \frac{1}{4}$.
13. $\frac{4}{8} + \frac{3}{8}$.
14. $\frac{1}{2} + \frac{3}{8}$.
15. $\frac{1}{4} + \frac{1}{8}$.
16. $\frac{1}{2} + \frac{3}{4}$.
17. $\frac{1}{2} + \frac{5}{8}$.
18. $\frac{3}{4} + \frac{3}{8}$.
19. $\frac{3}{4} + \frac{5}{8}$.
20. $\frac{3}{4} + \frac{7}{8}$.

Write the following fractions as indicated:

21. $\frac{3}{4}$ as twelfths.
22. $\frac{2}{3}$ as twelfths.
23. $\frac{5}{8}$ as sixteenths.
24. $\frac{5}{6}$ as twelfths.
25. $\frac{1}{2}$ as sixteenths.
26. $\frac{3}{5}$ as twentieths.

Find the results of the following additions:

27. $\frac{3}{8} + \frac{1}{2}$.
28. $\frac{1}{2} + \frac{7}{8}$.
29. $\frac{1}{2} + \frac{1}{3}$.
30. $\frac{1}{2} + \frac{1}{4} + \frac{1}{3}$.
31. $\frac{1}{2} + \frac{3}{4} + \frac{1}{3}$.
32. $\frac{1}{2} + \frac{3}{4} + \frac{2}{3}$.
33. $\frac{1}{3} + \frac{1}{3} + \frac{1}{5}$.
34. $\frac{1}{3} + \frac{2}{3} + \frac{2}{5}$.
35. $\frac{1}{3} + \frac{1}{4} + \frac{1}{5}$.
36. Which is the larger, $2\frac{3}{4} + 5\frac{7}{8}$ or $3\frac{7}{8} + 4\frac{3}{4}$?

Perform the following additions:

37. $1\frac{1}{2} + 2\frac{3}{4}$.
38. $3\frac{3}{4} + 2\frac{3}{4}$.
39. $2\frac{5}{8} + 3\frac{1}{4}$.
40. $3\frac{7}{8} + 2\frac{1}{4}$.
41. $3\frac{1}{3} + 2\frac{1}{2}$.
42. $3\frac{2}{3} + 2\frac{1}{2}$.
43. $4\frac{1}{3} + 3\frac{1}{4}$.
44. $4\frac{2}{3} + 3\frac{3}{4}$.
45. $3\frac{5}{6} + 3\frac{1}{2}$.
46. $2\frac{5}{6} + 5\frac{1}{4}$.
47. $5\frac{5}{6} + 3\frac{3}{4}$.
48. $2\frac{1}{6} + 1\frac{1}{8}$.

DRILL CHART IN SUBTRACTING FRACTIONS

Numbers 1 to 20, oral

State the results of the following subtractions:

1. $\frac{1}{2} - \frac{1}{4}$.
2. $\frac{3}{4} - \frac{1}{4}$.
3. $\frac{3}{4} - \frac{1}{2}$.
4. $\frac{7}{8} - \frac{1}{8}$.
5. $\frac{7}{8} - \frac{5}{8}$.
6. $\frac{5}{8} - \frac{3}{8}$.
7. $\frac{7}{8} - \frac{3}{8}$.
8. $\frac{4}{5} - \frac{1}{5}$.
9. $\frac{4}{5} - \frac{2}{5}$.
10. $\frac{3}{5} - \frac{2}{5}$.
11. $\frac{3}{8} - \frac{1}{8}$.
12. $\frac{1}{4} - \frac{1}{8}$.
13. $\frac{4}{8} - \frac{1}{8}$.
14. $\frac{1}{2} - \frac{1}{8}$.
15. $\frac{7}{8} - \frac{2}{8}$.
16. $\frac{5}{6} - \frac{1}{6}$.
17. $\frac{5}{6} - \frac{2}{3}$.
18. $\frac{7}{8} - \frac{6}{8}$.
19. $\frac{7}{8} - \frac{3}{4}$.
20. $\frac{7}{8} - \frac{1}{4}$.

Write the following fractions as indicated:

21. $\frac{3}{4}$ as eighths.
22. $\frac{3}{4}$ as sixteenths.
23. $\frac{5}{6}$ as 24ths.
24. $\frac{3}{8}$ as sixteenths.

Subtract as indicated:

25. $\frac{3}{8} - \frac{1}{4}$.
26. $\frac{3}{4} - \frac{3}{8}$.
27. $\frac{5}{8} - \frac{1}{2}$.
28. $\frac{7}{8} - \frac{1}{2}$.
29. $1 - \frac{3}{8}$.
30. $\frac{5}{6} - \frac{1}{2}$.
31. $\frac{1}{2} - \frac{1}{6}$.
32. $\frac{1}{4} - \frac{1}{6}$.
33. $\frac{3}{4} - \frac{1}{6}$.
34. $\frac{5}{6} - \frac{3}{4}$.
35. $\frac{1}{2} - \frac{5}{12}$.
36. $\frac{1}{2} - \frac{1}{16}$.
37. $\frac{1}{2} - \frac{5}{16}$.
38. $\frac{9}{16} - \frac{1}{2}$.
39. $\frac{9}{16} - \frac{1}{4}$.
40. Subtract $3\frac{7}{8}$ from $11\frac{1}{4}$ and check your result.

Perform the following additions and subtractions:

41. $2\frac{7}{8} + 1\frac{3}{4} - \frac{5}{8}$.
42. $3\frac{5}{8} + 2\frac{1}{2} - 1\frac{3}{4}$.
43. $6\frac{1}{2} + 3\frac{5}{8} - 2\frac{1}{4}$.
44. $2\frac{3}{8} + 1\frac{1}{16} - 1\frac{3}{4}$.
45. $5\frac{2}{3} + 3\frac{1}{6} - 2\frac{1}{2}$.
46. $8\frac{7}{8} - 1\frac{1}{4} - 2\frac{1}{16}$.

PROBLEMS IN ADDITION AND SUBTRACTION

1. Before we try these problems let us review counting forward and backward by fractions. It is good fun and it will help us in problems with fractions. Count to 4 by $\frac{1}{2}$, beginning $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, and so on. Then count backward, beginning 4, $3\frac{1}{2}$, 3, $2\frac{1}{2}$, and so on.

As in Ex. 1, count forward to and backward from

2. 4 by $\frac{1}{4}$.

3. 4 by $\frac{1}{8}$.

4. 3 by $\frac{1}{16}$.

5. 3 by $\frac{1}{6}$.

6. 2 by $\frac{1}{12}$.

7. 2 by $\frac{1}{10}$.

8. A basket weighs $1\frac{3}{4}$ lb. When filled with cherries it weighs $29\frac{1}{4}$ lb. How much do the cherries weigh?

9. John needs a stick $34\frac{3}{8}$ in. long to hold up a window. A stick $36\frac{1}{2}$ in. long is how much too long?

10. Helen had a piece of ribbon $1\frac{1}{8}$ yd. long and used $\frac{5}{8}$ yd. of it. How much was left?

11. Ben dug $20\frac{1}{4}$ lb. of potatoes, and Tom dug $18\frac{1}{2}$ lb. How many pounds did they both dig?

12. Ben can jump over a rope $3\frac{3}{4}$ ft. above the ground, and Tom can jump over it when it is $3\frac{1}{3}$ ft. above the ground. How much higher can Ben jump than Tom?

13. The boys had a rope 40 ft. long. Ben cut off $8\frac{1}{2}$ ft., and Tom cut off $8\frac{1}{4}$ ft. How much rope was left?

14. Ben, Tom, and Tony ran in a relay race. Ben's time was $1\frac{2}{3}$ min., Tom's was $1\frac{1}{2}$ min., and Tony's was $1\frac{4}{5}$ min. How long did it take them to run the race?

15. State what must be added to each of these fractions to make it equal to $1\frac{1}{4}$: $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{12}$, $\frac{1}{16}$, $\frac{7}{8}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{3}{16}$, $\frac{7}{16}$, $\frac{3}{8}$.

DRILL CHART IN MULTIPLYING FRACTIONS

Numbers 1 to 21, oral

State rapidly the results of these multiplications:

1. $\frac{1}{2} \times \frac{1}{2}$.
2. $\frac{1}{2} \times \frac{1}{4}$.
3. $\frac{1}{2} \times \frac{3}{4}$.
4. $\frac{1}{2} \times \frac{1}{8}$.
5. $\frac{1}{2} \times \frac{3}{8}$.
6. $\frac{1}{2} \times \frac{5}{8}$.
7. $\frac{1}{2} \times \frac{7}{8}$.
8. $\frac{1}{2} \times \frac{1}{3}$.
9. $\frac{1}{2} \times \frac{1}{6}$.
10. $\frac{1}{2} \times \frac{5}{6}$.
11. $\frac{1}{3} \times \frac{1}{8}$.
12. $\frac{1}{3} \times \frac{5}{8}$.
13. $\frac{1}{3} \times \frac{7}{8}$.
14. $\frac{2}{3} \times \frac{5}{8}$.
15. $\frac{2}{3} \times \frac{7}{8}$.
16. $\frac{1}{4} \times \frac{1}{6}$.
17. $\frac{1}{4} \times \frac{5}{6}$.
18. $\frac{1}{4} \times \frac{1}{8}$.
19. $\frac{3}{4} \times \frac{1}{8}$.
20. $\frac{3}{4} \times \frac{7}{8}$.

21. Which is the larger, $2 \times 1\frac{7}{8}$ or $1 \times 2\frac{7}{8}$?

Perform the following multiplications:

22. $\frac{1}{2} \times \frac{2}{3}$.
23. $\frac{1}{3} \times \frac{3}{4}$.
24. $\frac{2}{3} \times \frac{3}{4}$.
25. $\frac{1}{2} \times \frac{4}{5}$.
26. $\frac{1}{3} \times \frac{3}{8}$.
27. $\frac{2}{3} \times \frac{3}{8}$.
28. $\frac{1}{6} \times \frac{3}{8}$.
29. $\frac{5}{6} \times \frac{3}{8}$.
30. $\frac{4}{5} \times \frac{7}{8}$.
31. $\frac{1}{3} \times \frac{3}{16}$.
32. $\frac{2}{3} \times \frac{3}{16}$.
33. $\frac{1}{6} \times \frac{3}{16}$.
34. $\frac{2}{5} \times \frac{5}{16}$.
35. $\frac{4}{5} \times \frac{5}{16}$.
36. $\frac{1}{2} \times \frac{1}{12}$.
37. $\frac{1}{2} \times \frac{5}{12}$.
38. $\frac{6}{7} \times \frac{7}{12}$.
39. $\frac{3}{10} \times \frac{5}{12}$.

40. Multiply $2\frac{1}{2}$ by $3\frac{2}{3}$ and $3\frac{1}{2}$ by $2\frac{2}{3}$.

Perform the following multiplications:

41. $1\frac{1}{2} \times 1\frac{1}{2}$.
42. $1\frac{1}{2} \times 2\frac{1}{4}$.
43. $2\frac{1}{2} \times 2\frac{3}{4}$.
44. $3\frac{1}{4} \times 2\frac{1}{8}$.
45. $3\frac{3}{4} \times 2\frac{1}{8}$.
46. $4\frac{1}{3} \times 1\frac{1}{2}$.
47. $4\frac{2}{3} \times 1\frac{1}{2}$.
48. $4\frac{2}{3} \times 5\frac{1}{2}$.
49. $5\frac{1}{8} \times 5\frac{1}{8}$.
50. $5\frac{1}{8} \times 5\frac{3}{8}$.
51. $1\frac{1}{3} \times 2\frac{1}{6}$.
52. $1\frac{2}{3} \times 2\frac{1}{6}$.
53. $1\frac{2}{3} \times 2\frac{5}{6}$.
54. $1\frac{1}{3} \times 1\frac{1}{8}$.
55. $1\frac{2}{3} \times 2\frac{3}{8}$.

PROBLEMS IN MULTIPLICATION

1. If Ben can walk $\frac{2}{3}$ of the way to town in 1 hr., what part of the distance can he walk at this rate in $\frac{1}{2}$ hr.?

2. If you buy $\frac{3}{4}$ lb. of candy and give half of it to a friend, how much do you have left?

3. Our motor boat traveled at the rate of $8\frac{1}{2}$ mi. an hour for $3\frac{1}{4}$ hr. How far did it go?

4. How much will $4\frac{1}{2}$ lb. of sugar cost at $6\frac{2}{3}$¢ a pound?

5. If Tony can weed $3\frac{1}{2}$ rows of onions in 1 hr., how many rows can he weed in $3\frac{1}{2}$ hr.?

6. If Tom's steps measure $2\frac{1}{2}$ ft. each, and he takes $18\frac{1}{2}$ steps to cross the yard, how wide is the yard?

7. On an automobile trip we averaged $24\frac{1}{2}$ mi. an hour for $5\frac{3}{4}$ hr. How far did we go?

8. If our school cafeteria uses an average of $57\frac{1}{2}$ lb. of butter per week, how much will it use in $2\frac{2}{5}$ wk.?

9. How much do $1\frac{5}{12}$ doz. cups cost at $4.80 per dozen?

10. If the pupils in our room drink $131\frac{1}{4}$ qt. of milk a week, how much shall we use in $1\frac{3}{5}$ wk.?

11. The gasoline tank on Mr. Green's car holds $18\frac{1}{2}$ gal. When it is $\frac{3}{4}$ full, how much gasoline has he?

12. The gasoline tank on a truck holds $36\frac{3}{4}$ gal. If it is only $\frac{1}{4}$ full, how much will it take to fill it?

13. The radiator on Mr. Brown's truck has a capacity of $3\frac{3}{4}$ gal. To keep it from freezing in the winter he fills it $\frac{1}{3}$ full of denatured alcohol and $\frac{2}{3}$ full of water. How much alcohol does he use?

DRILL CHART IN DIVIDING FRACTIONS

Numbers 1 to 20, oral

State rapidly the results of the following divisions:

1. $\frac{1}{2} \div \frac{1}{2}$.
2. $\frac{1}{4} \div \frac{1}{4}$.
3. $\frac{3}{4} \div \frac{1}{4}$.
4. $\frac{1}{4} \div \frac{3}{4}$.
5. $\frac{3}{4} \div \frac{3}{4}$.
6. $1 \div \frac{1}{2}$.
7. $1 \div \frac{1}{4}$.
8. $1 \div \frac{1}{5}$.
9. $2 \div \frac{1}{2}$.
10. $2 \div \frac{1}{4}$.
11. $3 \div \frac{1}{3}$.
12. $4 \div \frac{1}{4}$.
13. $5 \div \frac{1}{5}$.
14. $\frac{1}{3} \div 3$.
15. $\frac{1}{4} \div 4$.
16. $\frac{1}{8} \div \frac{1}{8}$.
17. $\frac{3}{8} \div \frac{1}{8}$.
18. $\frac{1}{8} \div \frac{3}{8}$.
19. $\frac{5}{8} \div \frac{3}{8}$.
20. $\frac{5}{8} \div \frac{7}{8}$.

21. Divide $\frac{7}{8}$ by $\frac{1}{8}$ and $\frac{1}{8}$ by $\frac{7}{8}$.

Perform the following divisions:

22. $\frac{1}{2} \div \frac{1}{4}$.
23. $\frac{1}{4} \div \frac{1}{2}$.
24. $\frac{1}{2} \div \frac{3}{4}$.
25. $\frac{3}{4} \div \frac{1}{2}$.
26. $\frac{1}{4} \div \frac{1}{8}$.
27. $\frac{1}{8} \div \frac{1}{4}$.
28. $\frac{3}{8} \div \frac{1}{4}$.
29. $\frac{3}{8} \div \frac{3}{4}$.
30. $\frac{3}{4} \div \frac{3}{8}$.
31. $\frac{3}{4} \div \frac{7}{8}$.
32. $\frac{7}{8} \div \frac{3}{4}$.
33. $\frac{5}{8} \div \frac{3}{4}$.
34. $\frac{1}{4} \div \frac{5}{8}$.
35. $\frac{1}{3} \div \frac{1}{2}$.
36. $\frac{1}{2} \div \frac{1}{3}$.
37. $\frac{1}{2} \div \frac{2}{3}$.
38. $\frac{2}{3} \div \frac{1}{2}$.
39. $\frac{1}{3} \div \frac{1}{4}$.
40. $\frac{1}{8} \div \frac{5}{8}$.
41. $\frac{2}{3} \div \frac{7}{8}$.

42. How much greater is $2\frac{3}{4} \div 3\frac{2}{3}$ than $22\frac{1}{2} \div 30$?

Perform the following divisions:

43. $2\frac{1}{2} \div 1\frac{1}{4}$.
44. $3\frac{1}{2} \div 1\frac{3}{4}$.
45. $3\frac{1}{2} \div 10\frac{1}{2}$.
46. $5\frac{1}{2} \div 3\frac{1}{4}$.
47. $3\frac{1}{8} \div 2\frac{3}{4}$.
48. $2\frac{3}{4} \div 3\frac{7}{8}$.
49. $3\frac{1}{8} \div \frac{7}{8}$.
50. $2\frac{7}{8} \div \frac{1}{3}$.
51. $3\frac{7}{8} \div \frac{2}{3}$.
52. $\frac{5}{8} \div 2\frac{1}{2}$.
53. $\frac{7}{8} \div 2\frac{3}{4}$.
54. $1\frac{7}{8} \div 2\frac{3}{4}$.
55. $3\frac{2}{3} \div \frac{1}{8}$.
56. $3\frac{2}{3} \div \frac{7}{8}$.
57. $3\frac{2}{3} \div 2\frac{7}{8}$.
58. $6\frac{3}{5} \div 2\frac{1}{5}$.
59. $6\frac{4}{5} \div 2\frac{2}{5}$.
60. $7\frac{4}{5} \div 3\frac{3}{8}$.

PROBLEMS IN DIVISION

1. Mr. Field had $7\frac{3}{8}$ A. of land on the shore of a lake. After he divided it into lots containing $\frac{1}{8}$ A. each, how many lots did he have?

2. We had a sheet of colored paper that was $18\frac{3}{4}$ in. wide. We wished to cut it into strips $\frac{3}{4}$ in. wide. How many strips could we make?

3. How many pieces $\frac{3}{4}$ yd. long can be cut from a roll of ribbon $15\frac{3}{4}$ yd. long?

4. A barrel usually holds $31\frac{1}{2}$ gal. How many barrels of water are there in a tank containing $409\frac{1}{2}$ gal.?

5. At the rate of $2\frac{3}{4}$ mi. an hour, how long will it take a troop of Boy Scouts to hike $13\frac{3}{4}$ mi.?

6. If a $1\frac{3}{4}$-acre field yields $185\frac{3}{4}$ bu. of potatoes, how many bushels does it yield per acre?

7. Helen's father has a car which averages $17\frac{1}{2}$ mi. to each gallon of gasoline. How many gallons will the car use in going $192\frac{1}{2}$ mi.?

8. How many strips of carpet $\frac{3}{4}$ yd. wide will it take to cover the floor of a room $5\frac{1}{4}$ yd. wide?

9. How many posts $7\frac{1}{2}$ ft. long can be made from an old flagpole that is $52\frac{1}{2}$ ft. long?

10. How many ice-cream bricks containing $1\frac{1}{2}$ qt. each can be made from $61\frac{1}{2}$ qt. of ice cream?

11. If an automobile jack raises the car $\frac{3}{8}$ in. each time Frank pushes down the handle, how many times must he push it down to raise the car $3\frac{3}{4}$ in.?

Fractional Relations. Suppose that you have 21 pieces of candy and eat $\frac{1}{3}$ of them. You evidently eat $\frac{1}{3}$ of 21 pieces, or 7 pieces.

To put it another way, suppose you eat 7 pieces of candy out of 21. You evidently eat $\frac{7}{21}$ of the pieces, or $\frac{1}{3}$ of them, which is correct because $\frac{1}{3}$ of 21 pieces is 7 pieces.

Similarly, if you eat 3 pieces of candy out of 10, you eat $\frac{3}{10}$ of the pieces. This is correct because $\frac{3}{10}$ of 10 pieces is 3 pieces.

As another simple illustration, suppose that you wish to find what part the number 2 is of 4. Evidently 2 is $\frac{2}{4}$ of 4, or $\frac{1}{2}$ of 4. As a check, it is easily seen that $\frac{1}{2}$ of 4 is 2.

To find what fraction the first of two numbers is of the second, divide the first number by the second and then express the result in lowest terms.

FRACTIONAL RELATIONS

1. If you eat 6 chestnuts out of 60, what fraction of them do you eat? Is this more or is it less than $\frac{1}{8}$ of them?

2. If your school team wins 8 of 12 games of baseball played, what fraction of the games does it win?

3. In Ex. 2, what fraction of the games does it lose?

4. If 7 of the 28 da. in February are stormy, what fraction of the days are stormy? are not stormy?

Find what part the first of these numbers is of the second:

5. 8, 16.
6. 8, 24.
7. 16, 24.
8. 24, 32.
9. 24, 28.
10. 25, 35.
11. 25, 75.
12. 50, 75.

Fractional Parts. Suppose that there are 40 words in a spelling test, and $\frac{1}{20}$ of them are names of cities. Evidently, $\frac{1}{20}$ of the 40 words, or 2 words, are names of cities.

Now suppose that 2 words in a spelling test are names of cities, and this is $\frac{1}{20}$ of the total number. Evidently, if 2 is $\frac{1}{20}$ of the number, the number of words in the test must be 20 × 2, or 40.

Again, suppose that you solve 28 of the examples in a test, and that this is $\frac{7}{8}$ of the total number. To find the total number in the test we have

$\frac{7}{8}$ of the number = 28,

and so $\frac{1}{8}$ of the number = $\frac{1}{7}$ of 28,

and the number = $8 \times \frac{1}{7} \times 28$, or 32.

FRACTIONAL PARTS

1. How much is $\frac{1}{4}$ of \$8? $\frac{3}{4}$ of \$8? $\frac{1}{8}$ of 16 ¢?

2. If 3 is $\frac{1}{4}$ of some number, what is the number?

3. If 6 is $\frac{3}{8}$ of some number, what is $\frac{1}{8}$ of the number? What is the number?

Find the number of which

4. 2 is $\frac{1}{2}$.
5. 2 is $\frac{1}{4}$.
6. 2 is $\frac{1}{8}$.
7. 3 is $\frac{1}{2}$.
8. 3 is $\frac{1}{3}$.
9. 3 is $\frac{1}{4}$.
10. 4 is $\frac{1}{8}$.
11. 12 is $\frac{1}{2}$.
12. 11 is $\frac{1}{3}$.
13. 10 is $\frac{1}{4}$.
14. 15 is $\frac{1}{5}$.
15. 12 is $\frac{1}{6}$.
16. 10 is $\frac{1}{7}$.
17. 25 is $\frac{1}{8}$.
18. 12 is $\frac{2}{3}$.
19. 12 is $\frac{3}{4}$.
20. 12 is $\frac{4}{5}$.
21. 15 is $\frac{3}{8}$.
22. 15 is $\frac{5}{8}$.
23. 14 is $\frac{7}{8}$.
24. 25 is $\frac{5}{8}$.
25. 8 is $\frac{1}{16}$.
26. 9 is $\frac{3}{16}$.
27. 10 is $\frac{5}{16}$.
28. 21 is $\frac{7}{16}$.
29. 45 is $\frac{9}{16}$.
30. 45 is $\frac{15}{16}$.
31. 10 is $\frac{5}{32}$.

A SILENT READING LESSON

In an airplane race the winning plane traveled 328 mi. in 2 hr. What was the average speed in miles per minute?

Since 2 hr. = 120 min., we divide by 120 and find the result to be $2\frac{88}{120}$.

This fraction has larger terms than are used in ordinary work. We can see better what such a fraction means if we reduce it to lowest terms.

$$\begin{array}{r} 2\frac{88}{120} \\ 120\overline{)328} \\ \underline{240} \\ 88 \end{array}$$

Since 88 is an even number, and 120 ends in zero, both terms of $\frac{88}{120}$ are divisible by 2; that is,

$$\frac{88}{120} = \frac{44}{60}.$$

Similarly, continuing to divide both terms by 2, we have

$$\frac{44}{60} = \frac{22}{30} = \frac{11}{15}.$$

That is, the average speed was $2\frac{11}{15}$ mi. per minute.

As another example, suppose that some boys measure a distance of 150 yd. for a race. After Tom has run 60 yd., what part of this distance has he run?

He has evidently run $\frac{60}{150}$ of the distance. Reducing $\frac{60}{150}$ to lowest terms by dividing both terms first by 10 and then by 3, we find that $\frac{60}{150} = \frac{6}{15} = \frac{2}{5}$.

That is, Tom has run $\frac{2}{5}$ of the distance.

In the case of small fractions like $\frac{4}{6}$, we can easily see at once by what number to divide both terms in order to reduce the fraction to lowest terms.

In the case of such fractions as $\frac{88}{120}$ and $\frac{60}{150}$, the best way is to continue to divide by the common factors that we see, until there are no more left.

PROBLEMS FOR GIRLS

1. A recipe for biscuits calls for 2 cups of flour, $1\frac{1}{2}$ tablespoonfuls of shortening, $\frac{3}{4}$ cup of milk, 4 teaspoonfuls of baking powder, and $\frac{1}{2}$ teaspoonful of salt. If you wish to make three times the amount called for by this recipe, how much of each should you take?

2. A recipe for molasses cookies calls for 1 cup of molasses, $\frac{1}{2}$ cup of boiling water, $2\frac{1}{2}$ cups of flour, 1 teaspoonful of soda, $1\frac{1}{2}$ teaspoonfuls of ginger, and 4 tablespoonfuls of butter. Write a recipe for five times this quantity.

3. A recipe for pie crust calls for 2 cups of flour, $\frac{3}{8}$ cup of lard, $\frac{1}{2}$ teaspoonful of baking powder, 1 teaspoonful of salt, and $\frac{1}{4}$ cup of cold water. Write a recipe for three times this amount of pie crust.

4. A recipe for rice pudding for three persons calls for $\frac{1}{8}$ teaspoonful of salt, $\frac{1}{4}$ cup of rice, 3 tablespoonfuls of sugar, 2 cups of hot milk, and a little nutmeg for seasoning. Write a recipe for four persons.

5. A cup of shredded codfish weighs 6 oz. This is what fraction of a pound? How many cups of codfish are there in 1 lb.? in $1\frac{3}{4}$ lb.? in $2\frac{3}{4}$ lb.?

6. A tablespoonful of olive oil weighs $\frac{2}{3}$ oz. How many tablespoonfuls will weigh 2 oz.? 1 oz.?

7. Housewives allow 16 tablespoonfuls of dry material to a cup, and 3 teaspoonfuls to a tablespoonful. How many teaspoonfuls of hominy are there to a cup? A teaspoonful of dry hominy is what part of a cup? How many cups will make 24 tablespoonfuls?

PROBLEMS FOR BOYS

1. Mr. Weston sells his sour cherries to Mr. Kane for 8¢ a pound on the trees. Mr. Weston pays Sam 1¢ a quart for weighing the cherries. Sam finds that a quart of cherries averages $1\frac{3}{4}$ lb., and the empty baskets weigh $2\frac{1}{4}$ lb. How much does Mr. Kane pay for a basket of cherries that weighs 46 lb.?

2. How much does Sam receive for the basket of cherries in Ex. 1? How much does Mr. Weston receive after paying Sam?

3. On Saturday Sam kept this record of the boys that Mr. Kane hired to pick the cherries. Fill in the blank spaces and find how much Sam and Mr. Weston received:

PICKER	QUARTS	POUNDS	DUE	SAM	MR. WESTON
Henry	21	$36\frac{3}{4}$	$2.94	21¢	$2.73
James	24				
Tom	36				
William	42				
Totals					

4. Mr. Weston pays Sam 4¢ a quart for picking sweet cherries, which he sells at his road-side market for 15¢ a pound. If the empty crate weighs $10\frac{1}{4}$ lb., and 1 qt. of cherries weighs $1\frac{3}{4}$ lb., how much does Sam get for picking a crate of cherries weighing $66\frac{1}{4}$ lb.?

5. How much does Mr. Weston make on the crate of cherries in Ex. 4 after paying Sam?

Review of the Principles. Before we leave our work with fractions, we should briefly review the important principles that we studied last year.

REVIEW OF THE PRINCIPLES

1. If we multiply the numerator of a fraction, what does this do to the value of the fraction? Show this by multiplying the numerator of $\frac{1}{3}$ by 2; by 3; by 6.

2. If we divide the numerator, what does this do to the value of the fraction? Show this by dividing the numerator of $\frac{3}{4}$ by 3, considering the special case of $\frac{3}{4}$ of a dollar.

3. If we multiply the denominator, what does this do to the value of the fraction? Show this by multiplying the denominator of $\frac{1}{2}$ by 2.

4. If we divide the denominator, what does this do to the value of the fraction? Show this by dividing the denominator of $\frac{1}{8}$ by 4.

5. If we multiply both terms by the same number, what does this do to the value of the fraction? Show this by multiplying both terms of $\frac{1}{2}$ by 2; by 3; by 4; by 5.

6. What happens to the value of a fraction like $\frac{8}{16}$ if we divide both terms by the same number?

7. If we add the same number to both terms of a proper fraction, what does this do to the value of the fraction? Show this by adding 1 to both terms of the fraction $\frac{1}{2}$; of the fraction $\frac{1}{3}$; of the fraction $\frac{3}{4}$.

8. What is the effect on the value of a proper fraction if we subtract the same number from both terms?

III. DECIMALS

A SILENT READING LESSON

We have already studied decimal fractions and know that such fractions are ordinarily called simply *decimals.* We know that 1.25 is only another way of writing $1\frac{25}{100}$, which is equal to $1\frac{1}{4}$. Other examples are

Decimal	Name	Fraction	Fraction Reduced
0.2	2 tenths	$\frac{2}{10}$	$\frac{1}{5}$
0.25	25 hundredths	$\frac{25}{100}$	$\frac{1}{4}$

We have learned that $0.4 = \frac{4}{10} = \frac{2}{5}$, and also that $0.40 = \frac{40}{100} = \frac{4}{10} = \frac{2}{5}$; that is, we may annex zeros to a decimal without changing its value. In the same way we may cut off zeros from the right of a decimal without changing its value; that is, 0.500 = 0.50 = 0.5.

We also know that $3.3\frac{1}{3} = 3\frac{1}{3}$, because $3\frac{1}{3} \div 10 = \frac{1}{3}$.

We know that $0.50 means $\frac{50}{100}$ of $1, or 50¢. Because we might easily erase the decimal point or not see it at all, it would be easy to mistake $.50 for $50. This is the reason why people who have much work to do with numbers usually write $0.50 instead of $.50.

We know that the first decimal place, counting to the right from the decimal point, is tenths, and the second is hundredths. Similarly,

0.001 is 1 thousandth,
0.3124 is 3124 ten-thousandths,
3.41267 is 3 and 41,267 hundred-thousandths,
7.001286 is 7 and 1286 millionths.

READING AND WRITING DECIMALS

Copy the following table, filling in the blank spaces:

	Decimal	Name	Fraction	Fraction Reduced
1.	0.5		$\frac{5}{10}$	$\frac{1}{2}$
2.		Four tenths		$\frac{2}{5}$
3.	0.275		$\frac{275}{1000}$	
4.		Eight hundredths		$\frac{2}{25}$
5.	$3.3\frac{1}{3}$			
6.	1.3750			
7.	0.000125			
8.		Eighteen millionths		
9.			$\frac{375}{1000}$	
10.	14.014		$14\frac{14}{1000}$	

Copy these decimals and after each write the name, the fraction, and the fraction reduced, as in the above table:

11. 0.2.
12. 0.6.
13. 0.8.
14. 0.15.
15. 0.25.
16. 0.35.
17. 0.45.
18. 0.75.
19. 0.4.
20. 0.40.
21. 0.400.
22. 0.440.
23. 0.444.
24. 0.500.
25. 0.550.
26. 0.555.
27. 12.5.
28. 1.25.
29. 17.5.
30. 1.75.
31. 1.55.
32. 15.5.
33. $1.16\frac{2}{3}$.
34. $11.6\frac{2}{3}$.
35. $0.3\frac{1}{3}$.
36. $0.33\frac{1}{3}$.
37. $0.6\frac{2}{3}$.
38. $0.66\frac{2}{3}$.
39. $5.12\frac{1}{2}$.
40. $51.2\frac{1}{2}$.
41. 51.25.
42. 5.125.

Addition of Decimals. When we use decimals in measuring, we always carry the measurement to the same number of decimal places; that is, in practical work, we never have to add 2.3 ft., 4.07 ft., and 0.1976 ft. If we can find one measurement to tenths, we can find all to tenths; if we can find one measurement to thousandths, we can find all to thousandths, and so on. Therefore, in practical work all decimals are usually given to the same number of decimal places. If we say that one length is 2.3 ft. and another is 4.07 ft., we mean that the first is 2.30 ft. and the second is 4.07 ft., both to the nearest hundredth of a foot (0.01 ft.).

If asked to add 2.3, 4.07, and 0.1976, the most convenient way is to write these numbers with the decimal points in a column and annex zeros until they all have the same number of decimal places.

```
2.3000
4.0700
 .1976
------
6.5676
```

We then add as with whole numbers and place the decimal point in the result below those in the numbers added.

In a column the zero before the decimal point is omitted except when the number comes at the top.

ADDITION OF DECIMALS

Perform the following additions:

1.	2.	3.	4.	5.
4.82	7.962	9.2036	0.12876	5.728398
.68	3.408	5.1279	.27394	.402609
3.82	.973	.3796	.00920	2.002300
1.76	1.720	6.3535	.00876	.000426
4.60	5.600	6.0003	.80096	.578699

In these examples, which represent cases for drill rather than practical cases, annex as many zeros as are necessary to give each of the following sets of numbers the same number of decimal places and then add:

6. 1.2, 38, 4.72, 6.115.
7. 37.962, 0.2, 4, 7.33.
8. 5.2, 0.7, 3.0009, 7.6.
9. 52.7864, 3.4, 0.82, 62.
10. 296.8, 31.04, 0.99678, 3.4.
11. 4.670939, 5.428, 0.98009.
12. 3.2975, 5.575, 0.2376.
13. 34.2, 0.0651, 32.78, 64.29.

Perform the following additions:

14.	**15.**	**16.**	**17.**
4.296	6.0302	0.98324	9.312786
5.175	.7624	1.32785	2.218566
3.872	8.7000	.00726	.352989
.625	9.2555	.00005	2.370000
5.000	5.2078	2.40300	6.828759
.803	9.0237	5.08969	.003625
6.710	.0096	8.12754	8.758432

18. In measuring a city lot a surveyor found that the four sides were 32.8 ft., 66.4 ft., 31.9 ft., and 62.1 ft. What was the perimeter of (the distance around) the field?

19. If a pupil should measure a school playground, which is in the form of a rectangle, and should report that it was 210.7 ft. long and 107.92 ft. wide, what would you say at once was wrong with his measurements? If he was measuring to the nearest 0.1 ft., how should the width be given? If he was measuring to the nearest 0.01 ft., how should the length be given? What is the perimeter of the playground to the nearest 0.1 ft.?

Subtraction of Decimals. In subtracting decimals we arrange the numbers in the same way as in addition. For example, we subtract 3.2980 from 4.0732 as here shown.

$$\boxed{\begin{array}{r}4.0732\\ \underline{3.2980}\\ 0.7752\end{array}}$$

We may check the work by adding the result to the number subtracted. If our work is correct, this result is the number from which we subtracted.

SUBTRACTION OF DECIMALS

Perform the following subtractions:

1. $\begin{array}{r}3.7286\\ \underline{.4939}\end{array}$

2. $\begin{array}{r}5.9268\\ \underline{3.7999}\end{array}$

3. $\begin{array}{r}8.0607\\ \underline{4.3278}\end{array}$

4. $\begin{array}{r}0.9020\\ \underline{.4782}\end{array}$

5. $\begin{array}{r}5.23842\\ \underline{.36984}\end{array}$

6. $\begin{array}{r}0.92307\\ \underline{.86439}\end{array}$

7. $\begin{array}{r}5.81082\\ \underline{2.39687}\end{array}$

8. $\begin{array}{r}6.33034\\ \underline{5.72687}\end{array}$

9. $\begin{array}{r}4.78236\\ \underline{2.92785}\end{array}$

10. $\begin{array}{r}3.62082\\ \underline{1.48987}\end{array}$

11. $\begin{array}{r}9.67802\\ \underline{4.76082}\end{array}$

12. $\begin{array}{r}33.4285\\ \underline{26.9287}\end{array}$

13. $\begin{array}{r}322.4731\\ \underline{29.6892}\end{array}$

14. $\begin{array}{r}129.3492\\ \underline{39.8896}\end{array}$

15. $\begin{array}{r}242.3481\\ \underline{129.6742}\end{array}$

16. $\begin{array}{r}304.0000\\ \underline{92.8709}\end{array}$

Subtract after copying in the proper form:

17. $2 - 0.7$.
18. $3.4 - 2$.
19. $47 - 0.8$.
20. $5.8 - 0.07$.
21. $0.1 - 0.09$.
22. $32.07 - 5.8$.
23. $6.003 - 0.9$.
24. $60.92 - 7.387$.
25. $4.2807 - 2.9$.
26. $5.02 - 3.7986$.
27. $7 - 0.29$.
28. $0.9 - 0.1276$.
29. $3.1 - 2.009$.
30. $2.01 - 0.346$.
31. $0.98 - 0.098$.

Reduction of Decimals to Fractions. As we saw on pages 125 and 213, we can always reduce a decimal to a fraction. For example, take 0.625. After writing it as $\frac{625}{1000}$, we can continue to cancel 5 until we reach $\frac{5}{8}$; that is,

$$0.625 = \frac{625}{1000} = \frac{125}{200} = \frac{25}{40} = \frac{5}{8}.$$

Sometimes we cannot reduce to lower terms the first fraction that we obtain; for example,

$$0.7 = \frac{7}{10}, \quad 0.63 = \frac{63}{100}, \quad \text{and} \quad 0.119 = \frac{119}{1000}.$$

In adding 0.625 to $0.333\frac{1}{3}$, it is easier to use decimals than to reduce to fractions; that is, than to add $\frac{5}{8}$ and $\frac{1}{3}$. In multiplying, it is usually better to use the fraction.

Important Fractional Parts. On page 147 we learned most of the following important relations:

$\frac{1}{2} = 0.50$
$\frac{1}{3} = .33\frac{1}{3}$
$\frac{2}{3} = .66\frac{2}{3}$
$\frac{1}{4} = .25$
$\frac{3}{4} = .75$

$\frac{1}{5} = 0.20$
$\frac{2}{5} = .40$
$\frac{3}{5} = .60$
$\frac{4}{5} = .80$
$\frac{1}{6} = .16\frac{2}{3}$

$\frac{5}{6} = 0.83\frac{1}{3}$
$\frac{1}{8} = .125$
$\frac{3}{8} = .375$
$\frac{5}{8} = .625$
$\frac{7}{8} = .875$

$\frac{1}{10} = 0.10$
$\frac{9}{10} = .90$
$\frac{1}{12} = .08\frac{1}{3}$
$\frac{1}{16} = .06\frac{1}{4}$
$\frac{1}{32} = .03\frac{1}{8}$

REDUCTION OF DECIMALS TO FRACTIONS

Reduce the following to fractions or mixed numbers:

1. 0.875. 3. 0.625. 5. 1.7. 7. 3.5. 9. 0.05625.
2. 0.375. 4. 1.875. 6. 2.6. 8. 7.9. 10. 0.00125.

Write the fraction by which it would be easier to multiply or divide than by

11. 0.75. 12. $0.16\frac{2}{3}$. 13. 0.625. 14. 0.875. 15. $0.333\frac{1}{3}$.

Multiplication of Decimals. If you multiply $\frac{3}{10}$ by $\frac{3}{10}$, what is the result? Is tenths times tenths equal to tenths? to hundredths? to thousandths? If you multiply 0.1 by 0.1, what is the result?

If you multiply $\frac{3}{100}$ by $\frac{7}{10}$, what is the result? Is tenths times hundredths equal to tenths? to hundredths? to thousandths? If you multiply 0.01 by 0.1, what is the result? Give the reason.

In multiplying by a decimal, multiply as with whole numbers, and then, beginning at the right, point off as many decimal places in the product (result) as there are decimal places in the two numbers together.

$$\boxed{\begin{array}{r}2.43\\ 13.5\\ \hline 1215\\ 729\\ 243\\ \hline 32.805\end{array}}$$

In the example shown, there are two decimal places in one number and one in the other. Hence there are three decimal places in the result.

MULTIPLICATION OF DECIMALS

Perform the following multiplications:

1. $\begin{array}{r}2.3\\ \underline{7}\end{array}$
2. $\begin{array}{r}2.3\\ \underline{0.7}\end{array}$
3. $\begin{array}{r}2.3\\ \underline{0.07}\end{array}$
4. $\begin{array}{r}0.23\\ \underline{0.7}\end{array}$
5. $\begin{array}{r}47.2\\ \underline{3}\end{array}$
6. $\begin{array}{r}47.2\\ \underline{0.30}\end{array}$
7. $\begin{array}{r}4.72\\ \underline{0.3}\end{array}$
8. $\begin{array}{r}47.2\\ \underline{0.03}\end{array}$
9. $\begin{array}{r}32.4\\ \underline{23}\end{array}$
10. $\begin{array}{r}32.4\\ \underline{2.3}\end{array}$
11. $\begin{array}{r}3.24\\ \underline{2.3}\end{array}$
12. $\begin{array}{r}32.4\\ \underline{0.23}\end{array}$
13. $\begin{array}{r}1.241\\ \underline{62}\end{array}$
14. $\begin{array}{r}1.241\\ \underline{6.2}\end{array}$
15. $\begin{array}{r}124.1\\ \underline{0.62}\end{array}$
16. $\begin{array}{r}12.41\\ \underline{6.2}\end{array}$
17. $\begin{array}{r}3428.3\\ \underline{121}\end{array}$
18. $\begin{array}{r}34.283\\ \underline{1.21}\end{array}$
19. $\begin{array}{r}3.4283\\ \underline{12.1}\end{array}$
20. $\begin{array}{r}34.283\\ \underline{0.121}\end{array}$

PROBLEMS IN MULTIPLICATION

1. If you are driving your car at a speed of 18.8 mi. per hour (often written 18.8 mi./hr.), how far will you go in 2 hr.? in 3 hr.? in 4 hr.?

2. If a freight train averages 28.9 mi./hr., how far will it go in 2 hr.? in 3 hr.? in 4 hr.? in 12 hr.?

3. If an airplane averages 92.8 mi./hr., how far will it go in 7 hr.? in 8 hr.? in 9 hr.?

4. If an express train averages 48.4 mi./hr., how far will it travel in $3\frac{1}{4}$ hr.? in $3\frac{3}{4}$ hr.? In such a case we multiply as if we were multiplying 484 by $3\frac{1}{4}$ and then notice that there are $1 + 0$ decimal places in the two numbers. How many places do you point off?

5. If you spend an average of 2.2¢ per mile for gasoline, to how much would this amount for an automobile trip of 162.8 mi.? Multiply in the usual way with decimals, and then write the result to the nearest cent.

6. If an engine has to pull a train of 38 freight cars, which have an average weight of 47.9 T. each, what is the total weight of the freight cars that it has to pull?

7. In multiplying 37.42 by 1.78, Charles gave the result as 666.0760, Kate gave it as 66.6076, and Fred as 66.0676. Which was right?

8. In multiplying 48.015 by 4.02, Richard gave the result as 19.30203, where there are five decimal places; Mary gave it as 193.02030, where there are also five; and Gertrude gave it as 19302.030, where there are five figures in the whole number. Which was right?

DRILL CHART IN MULTIPLYING DECIMALS

Numbers 1 to 15, oral

State rapidly the results of these multiplications:

1. 2 × 1.2.
2. 2 × 0.12.
3. 0.2 × 1.2.
4. 0.2 × 12.
5. 0.2 × 0.12.
6. 0.1 × 1.
7. 0.1 × 0.1.
8. 0.1 × 10.
9. 0.1 × 0.01.
10. 0.1 × 0.001.
11. 2 × 3.
12. 2 × 30.
13. 2 × 0.3.
14. 0.2 × 30.
15. 0.2 × 0.3.

Perform the following multiplications, seeing how many correct results you can get in 10 min.:

16. 24 × 36.
17. 2.4 × 36.
18. 24 × 3.6.
19. 2.4 × 3.6.
20. 2.4 × 0.36.
21. 0.24 × 3.6.
22. 0.25 × 3.6.
23. 25 × 0.36.
24. 1.25 × 36.
25. 12.5 × 3.6.
26. 6.8 × 7.9.
27. 0.79 × 6.8.
28. 0.68 × 7.9.
29. 75 × 124.
30. 7.5 × 124.
31. 75 × 12.4.
32. 75 × 1.24.
33. 7.5 × 1.24.
34. 7.5 × 12.4.
35. 85 × 1.25.
36. 8.5 × 12.5.
37. 85 × 12.5.
38. 8.5 × 1.25.
39. 1.9 × 16.8.
40. 19 × 1.68.
41. 0.27 × 8.5.
42. 125 × 144.
43. 12.5 × 144.
44. 1.25 × 144.
45. 125 × 14.4.
46. 125 × 1.44.
47. 12.5 × 14.4.
48. 1.25 × 1.44.
49. 0.125 × 144.
50. 0.125 × 14.4.
51. 0.125 × 1.44.
52. 5.75 × 9.28.
53. 57.5 × 928.
54. 0.575 × 92.8.

A SILENT READING LESSON

We have learned how to divide when one or both numbers are decimals, and we know where to place the decimal point in the result. Let us now review this work.

If we wish to divide 4 by 2, we may indicate this in the fractional form, $\frac{4}{2}$. We then know that

$$\tfrac{4}{2} = \tfrac{40}{20} = \tfrac{400}{200}, \text{ and so on.}$$

If we wish to divide 1.25 by 2.5, we see that

$$\frac{1.25}{2.5} = \frac{12.5}{25} = \frac{125}{250} = \frac{1250}{2500}, \text{ and so on.}$$

Any one of these divisions would give the required result. We may therefore take the easiest one, which is the one that has the smallest whole number for the divisor (the number by which we divide).

$$\boxed{\begin{array}{r} 0.5 \\ 25\overline{)12.5} \\ \underline{12\ 5} \end{array}}$$

In dividing by a decimal, make the divisor a whole number by multiplying both it and the number divided by 10, 100, 1000, or some other power of 10, and then divide as with whole numbers, placing the decimal point in the result above the decimal point in the number divided.

Always place the decimal point before you begin to divide, so that you will not forget it.

Always estimate the result in advance. This will keep you from giving foolish answers.

For example, in 12.5 ÷ 25, we see at a glance that 12 is about half of 25. Hence the result must be about 0.5, and could not possibly be 5, 50, or 0.05.

DRILL CHART IN DIVIDING DECIMALS

Numbers 1 to 20, oral

State rapidly the results of the following divisions:

1. $2\overline{)4}$
2. $2\overline{)0.4}$
3. $2\overline{)0.48}$
4. $2\overline{)0.04}$
5. $2\overline{)0.048}$
6. $3\overline{)6}$
7. $3\overline{)60}$
8. $3\overline{)0.6}$
9. $3\overline{)0.06}$
10. $3\overline{)0.66}$
11. $5\overline{)25}$
12. $5\overline{)2.5}$
13. $5\overline{)0.25}$
14. $5\overline{)0.025}$
15. $5\overline{)0.555}$
16. $8\overline{)8}$
17. $8\overline{)0.8}$
18. $8\overline{)80}$
19. $8\overline{)8.8}$
20. $8\overline{)0.88}$

Divide as indicated, keeping a record of your time:

21. $25\overline{)625}$
22. $2.5\overline{)625}$
23. $2.5\overline{)6.25}$
24. $25\overline{)62.5}$
25. $2.5\overline{)62.5}$
26. $37\overline{)444}$
27. $37\overline{)44.4}$
28. $37\overline{)4.44}$
29. $3.7\overline{)4.44}$
30. $3.7\overline{)44.4}$
31. $525\overline{)65{,}625}$
32. $5.25\overline{)65{,}625}$
33. $5.25\overline{)656.25}$
34. $5.25\overline{)6.5625}$
35. $0.525\overline{)6562.5}$
36. $752\overline{)175{,}968}$
37. $752\overline{)17.5968}$
38. $75.2\overline{)17.5968}$
39. $7.52\overline{)175.968}$
40. $0.752\overline{)1.75968}$
41. $873\overline{)297{,}693}$
42. $873\overline{)29.7693}$
43. $87.3\overline{)29.7693}$
44. $8.73\overline{)2.97693}$
45. $0.873\overline{)297.693}$
46. $573\overline{)328{,}329}$
47. $5.73\overline{)328{,}329}$
48. $57.3\overline{)32{,}832.9}$
49. $5.73\overline{)32.8329}$
50. $0.573\overline{)3.28329}$

A SILENT READING LESSON

When we divide 625 by 4, we may say that the result is 156 with a remainder of 1, or that it is $156\frac{1}{4}$. Now that we know how to use decimals we may divide 625.00 by 4 and obtain the result 156.25, as here shown:

$$\begin{array}{r} 156\text{, remainder } 1 \\ 4\overline{)625}\phantom{\text{, remainder } 1} \end{array} \qquad \begin{array}{r} 156\frac{1}{4} \\ 4\overline{)625\phantom{\frac{1}{4}}} \end{array} \qquad \begin{array}{r} 156.25 \\ 4\overline{)625.00} \end{array}$$

We need not speak of remainders any more, but may continue divisions as far as we wish. In some cases we need to give a result correct to three decimal places, or, as we may say, to give it to the nearest 0.001.

Let us study these three examples in long division:

$$\begin{array}{r}
12.7 \\
25\overline{)317.5} \\
\underline{25} \\
67 \\
\underline{50} \\
175 \\
\underline{175}
\end{array}
\qquad\qquad
\begin{array}{r}
5.48 \\
25\overline{)137.00} \\
\underline{125} \\
120 \\
\underline{100} \\
200 \\
\underline{200}
\end{array}
\qquad\qquad
\begin{array}{r}
5.708 \\
24\overline{)137.000} \\
\underline{120} \\
170 \\
\underline{168} \\
200 \\
\underline{192} \\
8
\end{array}$$

In the third case we may continue the division further, and find that the result is 5.708333+, the rest of the figures being 3's. We see, however, that the remainder 8 gives us $\frac{8}{24}$, or $\frac{1}{3}$, and so we may at once write the result as $5.708\frac{1}{3}$, or as 5.708 to the nearest 0.001.

DRILL CHART IN DIVIDING DECIMALS

Divide as many of these as you can in 5 min.:

1. $43\overline{)103.2}$
2. $22\overline{)23.76}$
3. $26\overline{)2.808}$
4. $53\overline{)60.42}$
5. $27\overline{)3.942}$
6. $2.1\overline{)42.21}$
7. $3.1\overline{)4.061}$
8. $7.3\overline{)0.1533}$
9. $0.37\overline{)6.364}$
10. $1.7\overline{)3131.4}$
11. $8.3\overline{)137.78}$
12. $0.42\overline{)15.624}$
13. $0.083\overline{)11.620}$
14. $0.031\overline{)7.0711}$
15. $1.88\overline{)16.168}$

Divide as follows, annexing as many zeros as may be necessary to carry the result in each case to ten-thousandths, so as to give the answer to the nearest 0.001:

16. $41\overline{)21}$
17. $41\overline{)2.1}$
18. $4.1\overline{)21}$
19. $5.1\overline{)2.1}$
20. $2.9\overline{)51.83}$
21. $6.1\overline{)5.663}$
22. $4.1\overline{)239.7}$
23. $0.73\overline{)8.49}$
24. $1.1\overline{)12.32}$
25. $0.81\overline{)0.8268}$
26. $0.94\overline{)63.4}$
27. $0.51\overline{)0.0211}$

Divide as follows, giving results to the nearest 0.01:

28. $5.9\overline{)3.2171}$
29. $3.2\overline{)70.448}$
30. $3.3\overline{)75.988}$
31. $3.2\overline{)0.1116}$
32. $620\overline{)403.93}$
33. $0.63\overline{)0.42294}$
34. $0.34\overline{)1.16371}$
35. $0.52\overline{)3.22998}$
36. $0.42\overline{)1.76841}$
37. $0.42\overline{)0.78267}$
38. $83\overline{)581.084}$
39. $0.31\overline{)7.06894}$

Multiplying by a Mixed Number. We have learned how to divide decimals, and we shall need this work in multiplying decimals by a mixed number.

For example, supposing that you can drive your car at a speed of 18.4 mi./hr., how far can you drive in $2\frac{3}{4}$ hr.?

$$\begin{array}{r} 18.4 \\ \underline{2\frac{3}{4}} \\ 138 \\ \underline{368} \\ 50.6 \end{array} \qquad \qquad \begin{array}{rr} & 18.4 \\ & \underline{2\frac{3}{4}} \\ \frac{3}{4} \text{ of } 18.4 = & 13.8 \\ 2 \times 18.4 = & \underline{36.8} \\ & 50.6 \end{array}$$

That is, you can drive your car 50.6 mi. in the $2\frac{3}{4}$ hr.

The first of the above solutions is the one that we use ordinarily. The second is given simply to show more clearly the reason for each step.

MULTIPLYING BY A MIXED NUMBER

Perform the following multiplications:

1. $\begin{array}{r} 18.4 \\ \underline{3\frac{1}{4}} \end{array}$

2. $\begin{array}{r} 1.84 \\ \underline{5\frac{3}{4}} \end{array}$

3. $\begin{array}{r} 0.184 \\ \underline{7\frac{3}{4}} \end{array}$

4. $\begin{array}{r} 0.184 \\ \underline{0.07\frac{1}{2}} \end{array}$

5. $\begin{array}{r} 0.184 \\ \underline{0.07\frac{1}{4}} \end{array}$

6. $\begin{array}{r} 8.184 \\ \underline{0.08\frac{3}{4}} \end{array}$

7. $\begin{array}{r} 4968 \\ \underline{5\frac{1}{8}} \end{array}$

8. $\begin{array}{r} 49.68 \\ \underline{5\frac{3}{8}} \end{array}$

9. $\begin{array}{r} 4.968 \\ \underline{0.05\frac{5}{8}} \end{array}$

10. $\begin{array}{r} 2745 \\ \underline{12\frac{1}{5}} \end{array}$

11. $\begin{array}{r} 27.45 \\ \underline{12\frac{3}{5}} \end{array}$

12. $\begin{array}{r} 2.745 \\ \underline{1.2\frac{4}{5}} \end{array}$

13. $\begin{array}{r} 3.008 \\ \underline{0.08\frac{1}{4}} \end{array}$

14. $\begin{array}{r} 0.0125 \\ \underline{35\frac{2}{5}} \end{array}$

15. $\begin{array}{r} 0.0375 \\ \underline{0.8\frac{3}{5}} \end{array}$

16. $\begin{array}{r} 0.0393 \\ \underline{825\frac{1}{3}} \end{array}$

Short Cuts. We shall now review the most important of the short cuts, and shall learn a few more.

1. *To multiply by 5, move the decimal point one place to the right, annexing zeros if necessary, and divide by 2.*

For example, $5 \times 6248 = \frac{1}{2}$ of $62{,}480 = 31{,}240$.

Similarly, to multiply by 0.5, which is merely $\frac{1}{2}$, it is easier to divide by 2.

For example, $0.5 \times 3044 = \frac{1}{2}$ of $3044 = 1522$.

2. *To divide by 5, multiply by 2 and divide by 10.*

For example, $\frac{1}{5}$ of $7205 = 2 \times 720.5 = 1441$.

Similarly, to divide by 0.5, which is $\frac{1}{2}$, multiply by 2.

For example, $307 \div 0.5 = 2 \times 307 = 614$.

3. *To multiply by 25, move the decimal point two places to the right, annexing zeros if necessary, and divide by 4.*

For example, $25 \times 723.6 = \frac{1}{4}$ of $72{,}360 = 18{,}090$.

The reason is that $25 = \frac{100}{4}$. Hence to multiply by 25 is to multiply by 100 and divide by 4.

4. *To divide by 25, move the decimal point two places to the left and multiply by 4.*

For example, $7825 \div 25 = 4 \times 78.25 = 313$.

The reason is that $\frac{1}{25} = \frac{4}{100}$. How do we find $\frac{4}{100}$ of 100?

5. *To multiply by 125, move the decimal point three places to the right, annexing zeros if necessary, and divide by 8.*

For example, $125 \times 4864 = \frac{1}{8}$ of $4{,}864{,}000 = 608{,}000$.

6. *To divide by 125, move the decimal point three places to the left and multiply by 8.*

For example, $78{,}245 \div 125 = 8 \times 78.245 = 625.96$.

PROBLEMS IN SHORT CUTS

1. How much must we pay for 1500 five-cent stamps?

2. If each blow of a hammer drives a spike 0.5 in. into the wood, how many blows will it take to drive it in 4.5 in.?

3. If a pile driver drives a pile an average of 0.5 ft. into the ground at each blow of the hammer, how far will 65 blows drive it into the ground?

4. In Ex. 3, how many blows will it take to drive it 4 ft. 6 in. into the ground?

5. What is the cost of 360 writing tablets at 25¢ each?

6. At 25¢ a gallon, what is the cost of 120 gal. of gasoline? of 148 gal.? of 265 gal.?

7. Find the cost of 372 newspapers at 2.5¢ each.

8. At 2.5¢ each, how many pencils can you buy for $5?

9. If a coal mine produced 1,640,000 T. in 2.5 yr., how much would be produced in 1 yr. at this rate?

10. Find the cost of 400 boxes of crayons at 25¢ a box.

11. The painter was paid 80¢ an hour for painting our schoolhouse last summer. He handed in a bill for 125 hr. of work. What was the amount due him?

12. A buyer for a large department store bought radio sets at $125 each. The bill amounted to $28,125. How many sets did he buy?

13. To fit out a Girl-Scout camp it was necessary to buy 75 tents at $125 each. How much did they cost?

14. Make up a rule for a short cut in dividing a number by 25 × 125.

IV. HOW TO SOLVE PROBLEMS

A SILENT READING LESSON

In the problems worked out in this book the computation is usually put at the right, and questions which we have to answer, or the explanations, are placed at the left.

Sometimes the teacher has put the number work on the board and has asked us to give orally an explanation like the one in the book. At other times we have written out a brief explanation.

Here is a problem and its solution:

If 7 yd. of cloth cost $4.20, how much will 12 yd. cost?

The number work:

$$\frac{12 \times \$\overset{0.60}{\cancel{4}.2\cancel{0}}}{7} \qquad \begin{array}{r} \$0.60 \\ 12 \\ \hline \$7.20 \end{array}$$

This work must be done quickly and accurately.

An oral explanation:

Since 7 yd. cost $4.20,
we see that 1 yd. costs $\frac{1}{7}$ of $4.20, or $0.60,
and so 12 yd. will cost 12 × $0.60, or $7.20.

A written explanation:

7 yd. cost $4.20.
1 yd. costs $\frac{1}{7}$ of $4.20.
12 yd. cost 12 × $\frac{1}{7}$ of $4.20, or $7.20.

We do not usually give all these three parts of the solution unless the teacher so directs.

HOW TO SOLVE PROBLEMS

In solving these problems, give the number work and either the oral or the written explanation, as the teacher directs:

1. If Mollie pays 72¢ for 6 lb. of hominy, how much does she pay for 8 lb.?

2. If Mollie buys 5 packages of cornstarch for 90¢, how much does she pay for 8 packages?

3. If 6 lb. of prunes cost $1.26, what do 7 lb. cost?

4. If 8 cans of salmon cost $2.32, what do 5 cans cost?

5. If 4 cans of cottonseed oil cost $3.52, how much do 7 cans cost? How much do 8 cans cost?

6. Mr. Ray's rent for 3 mo. was $192. How much would it be for 5 mo.? How much would it be for 9 mo.?

7. If 8 cans of cocoa cost $2.56, what do 7 cans cost?

8. A dealer pays $63.75 for 5 gross of toothbrushes. How much must he pay for 14 gross?

9. If 42 yd. of linoleum cost $94.50, what do 34 yd. cost? How much will 84 yd. cost?

10. If 16 yd. of flannel cost $10.88, what do 28 yd. cost?

11. If 8 yd. of sheeting cost $8.96, what do 22 yd. cost?

12. If 12 lb. of raisins cost $4.08, what do 8 lb. cost?

13. If 100 sheets of blotting paper cost $6.50, how much do 250 sheets cost? How much do 400 sheets cost? How much do 3750 sheets cost?

14. If 11 yd. of cotton scrim cost $3.63, how much do 18 yd. cost? How much do 44 yd. cost?

A Railroad Time-Table. Here is part of a railroad time-table showing the time that a certain train is scheduled to leave stations between New York City and Buffalo:

MILES	STATIONS	LEAVING
0.0	New York City	8 45
72.8	Poughkeepsie	10 42
142.2	Albany	12 10
159.1	Schenectady	12 55
236.8	Utica	2 28
250.6	Rome	2 50
289.6	Syracuse	3 48
369.9	Rochester	5 26
438.4	Buffalo . . . . . . (*Arrives*)	7 15

This time-table also gives the distance of each station from New York City.

PROBLEMS ABOUT THE TIME-TABLE

1. How far is it from New York to Buffalo? What is the fare at 3.6¢ per mile?

2. Find the distance from Poughkeepsie to Albany; to Schenectady; to Utica; to Syracuse; to Buffalo.

3. At 3.6¢ per mile, find the cost of a ticket from New York to Albany; to Schenectady; to Rochester.

4. At 3.6¢ per mile, find the fare from Albany to Syracuse; to Rochester; to Buffalo.

5. How long does it take to go by this train from New York to Albany? from Albany to Syracuse?

6. Make up five problems about this time-table.

A SILENT READING LESSON

We know that rapidity and accuracy are especially important in solving problems. Here are some suggestions that will help us to improve our work:

Work rapidly and accurately. The business man demands both rapidity and accuracy.

The most important thing is accuracy. Reasonable rapidity usually improves accuracy.

To insure accuracy, always check the work. One of the good checks is to estimate the answer in advance.

Make the figures neatly. Slovenly work is unbusinesslike and leads to inaccuracy.

Put the work in columns. The business man does this because it is quicker and easier and because he is less likely to make mistakes.

Work efficiently. In addition, we add upward because it is the most efficient way. When we are pointing with a pencil, we can see better above it than below it.

Analyze the problems. We think out the steps in advance, stating the reasons to ourselves. *Thinking* makes a business man successful and it makes a pupil a leader.

Use short methods. The greatest complaint that business men make is that the men and women they hire cannot add, subtract, multiply, and divide quickly and accurately. Short methods will help us to overcome this.

Do the work mentally when possible. We do not use pencil and paper to find the cost of $12\frac{1}{2}$ yd. at 64¢ a yard; we use a short method and find the cost mentally. We always watch for chances to do our work this way.

MISCELLANEOUS PROBLEMS

1. At 48¢ a pound, how many packages of tea, each containing 25 lb., can a merchant buy for $3000?

2. If 32 bricks are required to pave a square that is 1 yd. on a side (that is, one that contains 1 sq. yd.), how many bricks will be needed to pave $128\frac{3}{4}$ sq. yd.?

3. A yacht in one of the international races made 30 mi. in 3 hr. 12 min. 30 sec. Write this as minutes and a decimal. To find the average time per mile, by what must we divide this number? What was the average time per mile made by the yacht?

4. If a homing pigeon flies 1000 mi. in $57\frac{3}{4}$ hr., what is the average speed per hour?

5. A cubic foot of pure cast gold weighs 1204 lb., and a cubic foot of granite weighs 172 lb. Gold is how many times as heavy as granite?

6. In a Western irrigation project, 250 mi. of main ditches were put in at an average cost of $10,000 a mile, and 400 mi. of side ditches at an average cost of $1250 a mile. Find the total cost of the project.

7. A load of coal was sold for $17.76. The wagon and coal together weighed 4160 lb., and the wagon was known to weigh 1200 lb. What was the weight of the coal alone? At what price per ton was it sold?

8. Suppose that you wish to curtain four windows, each of which is 6 ft. high and requires two widths of material. If you allow 9 in. on each strip for hemming, how much will the material cost at $1.10 a yard?

A SILENT READING LESSON

Many problems are so easy that we can solve them without using pencil and paper. The following cases show how to think out a solution:

For example, suppose that a dealer pays \$10 for 8 books of the same kind. To find the cost of each book, we see that, since 8 books cost the dealer \$10,

1 book costs $\frac{1}{8}$ of \$10, or \$1.25.

Again, suppose that a stationer buys 100 sheets of blotting paper for \$6.50 and retails them at 10¢ a sheet, and that we wish to find how much more than the cost he receives. Since he sells 1 sheet for 10¢, he sells 100 sheets for 100 times 10¢, or \$10. Hence he receives

\$10 − \$6.50, or \$3.50, more than the cost.

Suppose that we wish to find how much a dealer, who pays \$48 for 120 doz. pencils, pays for each pencil. For 1 doz. he evidently pays $\frac{1}{120}$ of \$48 (which we think of as $\frac{1}{12}$ of \$4.80), or 40¢. Hence for

1 pencil he pays $\frac{1}{12}$ of 40¢, or $3\frac{1}{3}$¢.

A dealer pays \$7.20 for 1 doz. knives and sells them at 90¢ each. In finding how much more than the cost he receives, we see that he pays $\frac{1}{12}$ of \$7.20, or 60¢, for each knife. He then receives 90¢ − 60¢, or 30¢, more than the cost on each knife. Hence on

12 knives he receives 12 × 30¢, or \$3.60.

We could have taken 12 × 90¢, or \$10.80, and then subtracted \$7.20; but it is easier to keep the numbers small.

PURCHASES AT THE STORE

All work oral

Using short methods when possible, find the cost of

1. 25 yd. of felt @ \$2.40; @ \$2.48; @ \$2.20.
2. 12 screw drivers @ 30¢; @ 25¢; @ 40¢.
3. 5 yd. of dimity @ 34¢; @ 38¢; @ 46¢; @ 50¢.
4. 12½ lb. of feathers @ 96¢; @ \$1.20; @ \$1.60.
5. 32 yd. of cambric @ 40¢; @ 50¢; @ 60¢; @ 75¢.
6. 8 yd. of cashmere @ \$1.25; @ \$1.37½; @ \$1.50.
7. 16 yd. of gingham @ 40¢; @ 50¢; @ 75¢; @ 80¢.
8. 20 panes of glass @ 40¢; @ 50¢; @ 60¢; @ 75¢.
9. 25 pairs of shears @ 40¢; @ 64¢; @ 80¢.
10. 11 chisels @ 40¢; @ 50¢; @ 60¢; @ 70¢; @ 90¢.
11. 10 pieces of beading @ 33¢; @ 34¢; @ 36¢; @ 40¢.
12. 25 saws @ \$1.20; @ \$1.60; @ \$1.80.
13. 10 boxes of screws @ 30¢; @ 40¢; @ 38¢.
14. 22 ft. of rope @ 20¢; @ 40¢; @ 70¢; @ 80¢.
15. 12 yd. of linen lawn @ \$1.50; @ \$1.75; @ \$2.
16. 7 gross of screws @ 30¢; @ 31¢; @ 40¢.
17. 3 doz. bolts @ 80¢; @ 81¢; @ 82¢; @ 83¢.
18. 10 yd. of canvas @ 65¢; @ 78¢; @ 87½¢.
19. 7 lb. of solder @ 30¢; @ 31¢; @ 32¢.
20. 4 doz. spools of basting cotton @ 30¢; @ 32¢; @ 40¢.
21. 11 knives @ 43¢; @ 45¢; @ 55¢; @ 60¢.

Relations of Numbers. Observing the relations between the numbers in a problem often simplifies a solution.

1. If 6 T. of hay cost \$96, what is the cost of 18 T.?

Since 18 is 3 times 6, we see that the cost of 18 T. of hay is 3 × \$96, or \$288.

2. At \$18.50 a ton, what is the cost of 17,500 lb. of hay?

Since 2000 lb. cost \$18.50, 17,500 lb. will cost $\frac{17500}{2000}$ as much. Canceling, we have the result \$161.875, which we give as \$161.88.

$$\boxed{\frac{\overset{8.75}{\cancel{17500}} \times \$18.50}{\cancel{2000}} = \$161.875}$$

To find the cost of a given number of pounds at a given price per ton, point off three figures in the number of pounds, divide by 2, and multiply the price per ton by this result.

RELATIONS OF NUMBERS

1. If 8 T. of coal cost \$84, how much will 16 T. cost?
2. If 3 T. of hay cost \$45, how much will 27 T. cost?

At the following prices per ton, find the cost of

3. 22,500 lb. of coal @ \$13.20; @ \$15.50.
4. 8400 lb. of coal @ \$9; @ \$9.50; @ \$11.20.
5. 38,100 lb. of coal @ \$9.50; @ \$9.75; @ \$11.45.
6. 94,300 lb. of coal @ \$9.25; @ \$9.90; @ \$12.20.
7. 127,500 lb. of coal @ \$8.75; @ \$9.30; @ \$11.75.
8. 242,600 lb. of coal @ \$9.15; @ \$8.75; @ \$10.25.

Buying by the Hundred, Thousand, and Gross. In some kinds of business it is the custom to sell by the hundred (C), by the thousand (M), or by the gross (12 doz.), although smaller quantities are sometimes sold. Here are the ways in which some things are sold:

Envelopes, by the package of 25, by the hundred, or by the box of 500.

Typewriter paper, by the package of 500 sheets, or by the pound.

Buttons, by the gross or by the dozen.

Coal, by the ton of 2000 lb. or by the sack.

Lumber, by the thousand feet.

Bricks, by the thousand.

MAKING LARGE PURCHASES

All work oral

1. At $12 per M, how much will 12 M bricks cost?

2. At 60¢ per C, how much will 12 C envelopes cost?

3. If 7 M bricks cost $70, how much will 14 M cost?

4. At 55¢ per 500 sheets, how much will 2500 sheets of typewriter paper cost?

5. If 6 C sheets of blotting paper cost $20, what is the cost of 36 C sheets? of 42 C sheets? of 72 C sheets?

6. If 2 M feet of finest oak cost $300, what is the cost of 8 M feet? of 14 M feet? of 60 M feet?

7. At 80¢ a gross, what is the cost of 1½ gross of buttons? of 2½ gross? of 120 doz.?

BUYING IN LARGE UNITS

1. At \$9.75 per M, how much will 5½ M bricks cost? Remember that a dealer will often consider as a whole cent any fraction of a cent that is due him, but in this book the rule of counting ½¢ or more as 1¢, and rejecting fractions less than ½¢ is to be followed.

2. At \$1.25 a gross, find the cost of 22½ gross of buttons.

3. If 7 M feet of lumber cost \$630, how much will a dealer have to pay for 28 M feet? Solve by only one multiplication.

4. If the freight on 9 M pounds of goods is \$18.70, how much will it be on 27 M pounds at the same rate?

5. Find the cost of 4500 bricks (that is, of 4½ M) at \$9.80 per M.

6. A dealer bought 7500 envelopes at \$6.80 per M and sold them at 90¢ per C. How much more than the cost did he receive?

7. At \$11 per ton, find the cost of 5000 lb. of coal.

8. At \$12.20 per ton, find the cost of 8400 lb. of coal; of 12,500 lb.; of 12½ T.

9. If blotting paper is sold at \$34.50 per M sheets, how much must a dealer pay for 500 sheets?

10. At \$1.20 per C pounds, how much are the express charges on 1250 lb. of goods?

11. A man paid \$29.40 for 3½ M bricks. At the same rate, how much must he pay for 10½ M bricks?

12. A dealer can buy pasteboard boxes for his goods at \$32.50 per M. How much is this per C? per dozen?

V. REVIEW AND DRILL

MINIMUM ESSENTIALS

1. Write 1928 and 1935 in Roman numerals, and LXXIX and CLXXXVIII in common numerals.

Add, subtract (the second number from the first), multiply, and divide (the first number by the second) in each of the following pairs:

2. 6531, 2177.
3. 18,846, 3141.
4. 29,547, 4221.
5. 30,195, 6039.
6. 56,080, 7010.
7. 28,404, 7101.
8. 31,104, 3456.
9. 24,258, 4043.
10. 69,993, 7777.

Add, subtract (the second number from the first), multiply, and divide (both the first number by the second and the second by the first) in each of the following pairs:

11. $\frac{3}{4}$, $\frac{2}{3}$.
12. $\frac{3}{8}$, $\frac{1}{3}$.
13. $\frac{2}{3}$, $\frac{5}{8}$.
14. $\frac{2}{3}$, $\frac{1}{8}$.
15. $\frac{3}{4}$, $\frac{1}{8}$.
16. $\frac{3}{4}$, $\frac{5}{8}$.
17. $\frac{1}{2}$, $\frac{3}{16}$.
18. $\frac{5}{16}$, $\frac{1}{8}$.
19. $\frac{3}{8}$, $\frac{5}{16}$.
20. $\frac{5}{8}$, $\frac{3}{16}$.

Find the value in each of the following cases:

21. $3\frac{1}{2} + 2\frac{7}{8}$.
22. $15 - 2\frac{1}{2}$.
23. $17 - 3\frac{7}{8}$.
24. $5\frac{3}{4} \times 5\frac{1}{4}$.
25. $9\frac{1}{8} \times 2\frac{1}{2}$.
26. $3\frac{1}{3} \div \frac{1}{2}$.
27. $6\frac{2}{3} \div \frac{1}{2}$.
28. $6\frac{2}{3} \div 3\frac{1}{3}$.
29. 1.5×68.
30. 1.5×6.8.
31. $75 \div 25$.
32. $7.5 \div 2.5$.
33. $3.7 \div 0.2$.
34. $8 \div 0.0025$.
35. $1425 \div 2.5$.
36. 0.15×26.92.
37. 3.73×42.3.
38. 67.61×3.19.
39. 67.42×9.86.
40. $493.2 \div 27.4$.
41. $1.25 \div 0.025$.
42. $\frac{1}{2}$ of 2.5×6.4.
43. $\frac{3}{4}$ of 6.4×1.6.
44. $\frac{3}{8}$ of 6.8×9.8.

PROBLEMS WITHOUT NUMBERS

All work oral

1. If you are given a decimal, how do you express it as a fraction in lowest terms?

2. If you are to find a certain fractional part of a number, how do you express the part as a decimal?

3. If you know the cost of a certain number of things of the same value, how do you find the cost of one of them?

4. If you know the cost of a certain number of things of the same value, how do you find the cost of a certain other number of these things?

5. If you know what a given fraction of a number is, how do you find the number?

6. If you know how many bricks are needed for a certain wall, how do you find the number needed for another wall of the same thickness but a certain number of times as long and a certain other number of times as high?

7. If you wish to add two fractions having different denominators, how do you do it?

8. If you can buy a suit for a certain fraction "off" the regular price, how do you find how much you must pay for the suit?

9. If two numbers have been multiplied and you are given the number of decimal places in the result, and the number of decimal places in one of them, how can you find the number of decimal places in the other?

PROBLEMS FOR COMPLETION

1. The three sides of a triangle are respectively 3.7 in., 4.2 in., and 5.6 in. Complete the problem in any reasonable way and solve it.

2. On an automobile trip we went 28.1 mi. the first hour and 26.8 mi. the second hour. Complete the problem and solve it.

Complete and solve each of the following problems:

3. A cubic foot of water weighs 62.5 lb., and ice is 0.92 as heavy as water. A certain block of ice contains 8.3 cu. ft.

4. Clay is 1.2 times as heavy as water, and 1 cu. ft. of water weighs 62.5 lb.

5. Tin is 7.29 times as heavy as water, and 1 cu. in. of water weighs 0.58 oz.

6. A cubic foot of cast iron weighs 450 lb., and a cubic foot of water weighs 62.5 lb.

7. A circular race track is 190.4 ft. across, and the distance around it is $3\frac{1}{7}$ times the distance across.

8. A man used 72,000 cu. ft. of gas in 4 mo., and the gas company charged $1.10 per 1000 cu. ft.

9. Kate has $45 in the bank. Her father gives her $\frac{1}{5}$ as much more to put in the bank.

10. Richard buys $18.40 worth of books and pays $\frac{3}{4}$ of this amount, his mother paying the rest.

11. A mail plane flew from Washington to New York in 1 hr. 34 min. at an average speed of 138 mi. per hour.

12. John bought a suit marked $16.60 at "$\frac{1}{4}$ off."

MISCELLANEOUS PROBLEMS

1. If you are on an express train which is averaging 51.6 mi./hr., how far will you go in 2 hr.? in $2\frac{1}{2}$ hr.? in $2\frac{3}{4}$ hr.? in 3 hr. 20 min.?

2. If an airplane averages 82.4 mi./hr., how far will it go in 3 hr.? in $3\frac{1}{4}$ hr.? in $3\frac{3}{4}$ hr.? in 4 hr. 45 min.?

3. If the speedometer of your automobile registers 24.8 mi./hr., how far will you go in $4\frac{3}{4}$ hr. if this rate is kept up? How far will you go in $5\frac{1}{4}$ hr.? in $3\frac{1}{2}$ hr.? in 2 hr. 10 min.?

4. If you are buying velvet at $4.80 a yard, how much must you pay for $3\frac{3}{4}$ yd.? for $4\frac{1}{8}$ yd.? for $5\frac{7}{8}$ yd.? for 24 in.? for 48 in.?

5. How long will it take a steamer to go 48.8 mi. at the rate of a mile in 3.4 min.?

6. A freight train traveling at the rate of 24.8 mi./hr. travels how many miles in 3 hr. 45 min.?

7. At $3.20 a yard, how much must you pay for $5\frac{1}{2}$ yd. of goods? for $5\frac{3}{4}$ yd.? for $6\frac{5}{8}$ yd.?

8. If we multiply $2\frac{1}{2}$ by 3.4, we get the same result as if we multiply 3.4 by $2\frac{1}{2}$. What is the result?

9. At 3.6 ¢ a mile how much will a railroad ticket cost from New York to Albany, a distance of 142.5 mi.?

10. A cubic foot of water weighs 62.5 lb. What is the weight of $17\frac{1}{2}$ cu. ft. of water?

11. If you can walk at the rate of 1 mi. in 20 min., how long will it take you to walk $1\frac{3}{4}$ mi.?

THRIFT IN THE HOME

1. Mrs. Rowe can save at least 80¢ a week by going to the market and buying in person. Counting 52 wk. to the year, how much will she save in this way in a year?

2. Henry says that he will raise vegetables to help cut down the high cost of living. Allowing $1.80 for plowing, $3.25 for fertilizer, $1.50 for seed, and 85¢ each week for 22 wk. for wages, which his mother pays him, what is the entire cost of the vegetables for the season?

3. Henry makes his mother a fireless cooker by stuffing a box with excelsior and lining it with asbestos. By using the cooker Mrs. Rowe saves on her gas bills an average of $1.20 a month. How much does she save in a year?

4. At a bargain sale Henry buys a suit of clothes marked $30 at $\frac{1}{4}$ off, three shirts marked $2.50 at $\frac{1}{10}$ off, a pair of shoes marked $6.25 at $\frac{1}{5}$ off, a dozen 30-cent handkerchiefs at $\frac{1}{6}$ off, and a hat, a pair of gloves, and some ties amounting to $9 at $\frac{1}{5}$ off. How much does he save in all?

5. Mrs. Rowe can buy rolled oats in 20-ounce packages for 13¢ a package, or she can buy rolled oats of the same quality in bulk at 9¢ a pound. If she wishes to buy 20 lb., how much does she save if she buys it in bulk?

6. Mrs. Rowe can buy salad oil at 68¢ a quart or at 38¢ a pint. How much does she save if she buys 4 qt.?

7. Mrs. Rowe can buy sugar in packages at $9\frac{1}{2}$¢ a pound or in bulk at 9¢ a pound. How much does she save if she buys 10 lb. in bulk?

VI. LITTLE EXAMINATIONS

I. **1.** Reduce $37\frac{1}{2}$ to halves.
2. $\frac{2}{3} + \frac{3}{4} + \frac{1}{2} + \frac{5}{8} + 2\frac{1}{4}$.
3. $1675\frac{1}{2} - 156\frac{7}{8}$.
4. $27 \times \$231.66\frac{2}{3}$.
5. $\frac{3}{4}$ of \$164.48.
6. $3\frac{3}{4} \times 164$.
7. $8\frac{3}{4} \div 3\frac{1}{2}$.
8. $6 \times \$1.12\frac{1}{2}$.

II. **1.** 0.28×1247.6.
2. $5.527 - 0.378$.
3. $2897.28 \div 1.44$.
4. $0.16\frac{2}{3} \times \$248.46$.
5. $1.03 \times 84\frac{1}{2}$ ft.
6. $0.5 \times \$7876$.
7. $0.18 \times \$675$.
8. $4.12 \times \$990$.

III. **1.** $7.147 \div 0.05$.
2. 0.8×0.08.
3. $0.16\frac{2}{3} \times \$78.60$.
4. $7.147 + 0.968 + 1.736$.
5. $71.47 + 16.98 - 17.36$.
6. $0.03\frac{3}{4} \times 4$ ft.
7. $12\frac{7}{8} \times 8$ yd.
8. $0.8 \times \$85.50$.
9. $2.8 \times \$75$.
10. $2.75 \times \$800$.

IV. **1.** $5\frac{7}{8} =$ (?) eighths.
2. $3.5 =$ (?) fourths.
3. $7.00 - 0.77$.
4. $34.56 \div 0.72$.
5. 0.72×2434.56.
6. $29 \div 7\frac{1}{4}$.
7. 25×9664.
8. 125×65.68.
9. 3.75×65.68.
10. $1375 \div 1.25$.

V. **1.** How much is \$475 less 0.08 of itself?
2. How much is \$850 added to 0.16 of itself?
3. Add 2.478, 0.396, 2.073, and 7.136.
4. From $62\frac{7}{8} + 3\frac{3}{4}$ take $26\frac{1}{2} + 4\frac{5}{8}$.
5. Multiply 474.24 by 12.5, and check by division.

CHAPTER IV

I. BILLS, RECEIPTS, AND ACCOUNTS

A SILENT READING LESSON

If we buy anything at a store, we must pay for it unless we *have an account* there. This expression means that our record for paying promptly what we owe is so good that the dealer will let us *charge* what we buy and pay him at the end of the month or early in the next month. At that time he will send a *bill,* which shows what we owe.

There are different ways of making out a bill. For example, the *heading* of the bill may look like this model:

Holmes, Rogers & Company
Grocers
1927 North [redacted] St., [redacted]

SOLD TO Mrs. Robert Roberts
4289 East Superior St.
Feb. 1, [redacted]

The heading may also have a form something like this:

BUFFALO, Feb. 1, 1928

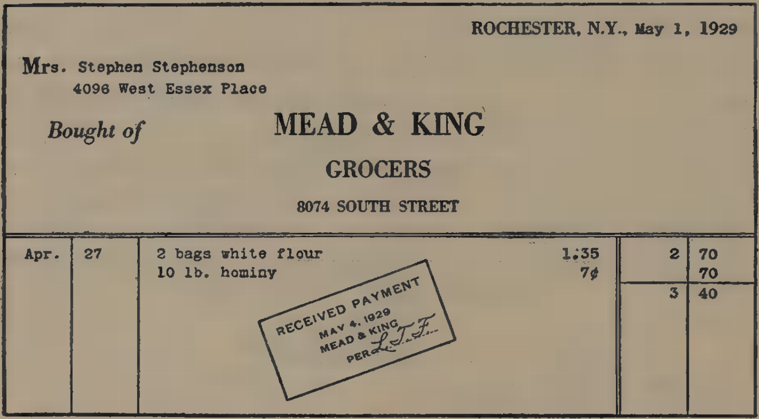
Mrs. Robert Roberts
4289 East Superior St.

BOUGHT OF **Holmes, Rogers & Company**
1927 North Essex St.

Bills. Bills are often ruled with only one column in which to put down the amount due for each purchase.

They may also be ruled with two columns, as shown on page 247. The amount of each purchase is then put in the column at the left, and any payments made or the value of goods returned are put in the one at the right.

Here is a bill showing the single column for purchases. It also shows the receipt stamp, signed by the cashier (with initials), which indicates that the bill has been paid.



ROCHESTER, N.Y., May 1, 1929

Mrs. Stephen Stephenson
4096 West Essex Place

Bought of MEAD & KING
GROCERS
8074 SOUTH STREET

Apr.	27	2 bags white flour	1.35	2	70
		10 lb. hominy	7¢		70
				3	40

RECEIVED PAYMENT
MAY 4, 1929
MEAD & KING
PER L.J.F.

Later on we shall find that bills are often paid by check, and that the check itself is a receipt. On this account few bills of any size are receipted if the buyer has a charge account.

Sometimes the symbol @, which means "at," is put before the price of each article. For example, "10 lb. hominy @ 7¢" means 10 lb. at 7¢ a pound, but this is usually understood without being written.

Bills with Double Rulings. The following bill shows the double ruling for purchases and *credits*, the latter including any payments made "on account" or goods returned:

THE MILLER MARSHALL CO.
7296 West Salina Ave.
SYRACUSE, N.Y.

June 1, 1929

Mrs. Richard X. Doe
5978 So. Onondaga Ave.

DATE		ITEM		PURCHASES		CREDITS	
May	16	24 yd. cretonne	1.10	26	40		
	18	2 pr. gloves	3.50	7	00		
		8 yd. velvet	4.20	33	60		
	19	2 pr. gloves	3.50			7	00
				67	00		
		Credit		7	00		
		Balance due		60	00		

The bill shows purchases amounting to $67. It also shows that the gloves were returned and credited the day after they were purchased. The balance due is $60.

MAKING OUT BILLS

Make out bills, ruled for purchases only, for the following items, and receipt each bill:

1. Feb. 7, 3 yd. satin @ $3.60, 7 yd. flannel @ $1.55.

2. Mar. 9, 2 doz. plates @ $10.40, ½ doz. cups and saucers @ $10.50; Mar. 12, ½ doz. tumblers @ $2.50.

3. Apr. 6, 4 doz. eggs @ 52¢, 10 lb. sugar @ 8½¢; Apr. 8, 3 lb. butter @ 35¢, 2 doz. oranges @ 60¢.

4. Jan. 4, 5 lb. beef @ 38¢; Jan. 5, 3 lb. pork @ 28¢; Jan. 6, 6 lb. chicken @ 36¢.

Make out bills, ruled for purchases and credits, for the following items, and receipt each bill:

5. Oct. 6, 4 ties @ $2.75; Oct. 15, 3 yd. crêpe @ $4.95, 15 yd. muslin @ 15¢, $1\frac{1}{4}$ yd. silk @ $1.60.

6. Nov. 1, 2 lb. fancy butter @ 55¢, 2 doz. eggs @ 68¢, 2 lb. powdered sugar @ 10¢; Nov. 8, 6 packages soap flakes @ 9¢, 2 boxes bacon @ 59¢, $\frac{1}{2}$ lb. cheese @ 40¢.

7. Apr. 7, 2 packages washing soda @ 10¢, 2 packages puffed rice @ 20¢, 3 packages cornflakes @ 12¢; Apr. 12, $\frac{1}{4}$ lb. citron @ 72¢, 5 lb. granulated sugar @ 8¢; Apr. 13, credit, 3 packages cornflakes @ 12¢, returned.

8. Mar. 7, $2\frac{1}{2}$ yd. silk @ $4.50; Mar. 9, 5 yd. silk @ $4.50; Mar. 12, 1 gown, $9.50; Mar. 13, credit, 1 gown, $9.50, returned.

9. Mar. 1, due from Feb. 1, $12.75; Mar. 2, $1\frac{3}{4}$ lb. salmon @ 80¢; Mar. 4, $6\frac{1}{4}$ lb. roast @ 40¢; Mar. 5, credit, cash, $12.75; Mar. 8, $5\frac{1}{2}$ lb. chicken @ 32¢.

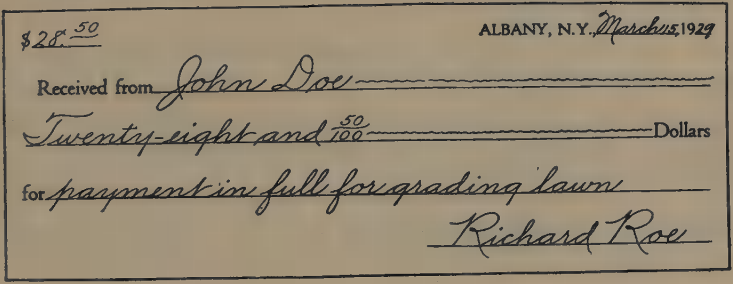
10. Apr. 2, 1 double boiler, $1.50; Apr. 4, 2 aluminum sauce pans @ $2.75; Apr. 6, 2 wire strainers @ 12¢; Apr. 6, credit, 1 saucepan, $2.75, returned; Apr. 9, $\frac{1}{2}$ doz. dish cloths @ $1.64; Apr. 15, $\frac{7}{12}$ doz. tumblers @ $8.40.

11. Apr. 12, 4 chairs @ $7.75; Apr. 13, 2 rocking chairs @ $15.25; 1 dining table, $38.75; Apr. 14, credit, 1 rocking chair, $15.25, returned.

12. May 8, $3\frac{1}{2}$ yd. canvas @ 90¢; May 9, 1 scarf, $3.50; May 10, $6\frac{1}{2}$ yd. satin @ $4.60; May 10, credit, 1 scarf, $3.50, returned; May 14, $2\frac{3}{8}$ yd. velvet @ $3.20.

Receipts. Sometimes a separate receipt is given instead of one stamped or written upon a bill. For example, if you employ a person to do some work for you, you might ask him to give you a receipt when you pay him.

The following model shows a simple form of receipt:



$28.50 ALBANY, N.Y., March 15, 1929

Received from John Doe

Twenty-eight and $\frac{50}{100}$ Dollars

for payment in full for grading lawn

Richard Roe

WRITING RECEIPTS

Write the receipts, dated today at the place where you live, which Richard Roe would give to John Doe for

1. $75.50, in full for painting a garage.
2. $8.75, in full for lumber furnished.
3. $250, for the balance due on an automobile.
4. $32.50, for building and installing a radio set.

Write receipts for the following amounts paid by you to different persons for work done, dating the receipts today at the place where you live and using play names:

5. $2.
6. $3.
7. $1.50.
8. $2.75.
9. $3.25.
10. $5.00.
11. $6.50.
12. $10.00.
13. $15.75.
14. $20.00.

PROBLEMS IN THRIFT

1. Play that you have an allowance of $5 a month for buying things that you need in school and for other expenses. Make out a budget for next month showing how you would like to spend your money. Try to save a little for opening a bank account.

2. Play that you are to manage a house next month, where you will have to buy food for a family of four and pay the rent, gas bill, and other necessary expenses, except clothing. Play that you have $100 for the month, and make out a budget showing how the money is to be spent. Do not be extravagant, and try to see how people have to plan in order to get along on a fixed amount.

3. Suppose that in Ex. 2 you allow $15 for food for a week. Inquire about food prices where you live, and make an account, the left side showing the amount allowed ($15) and the right the dates and nature of the probable expenses. Keep within the $15; in a few years you may have to do something of this kind in earnest.

4. From newspapers or from notices in store windows, make up and bring to class next time a few statements about the reductions that stores often make at sales. Try to show how you can save money by being careful about buying things at the right time, and make up an example about it.

5. Tell the class what you mean by saying that you returned some goods to a store and had them "credited to your account."

II. DENOMINATE NUMBERS

A SILENT READING LESSON

Last year we learned the common tables of measure. We shall now review these tables and shall learn a few new ones. Here is the table of length:

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet = 1 rod (rd.)
320 rods, or 5280 feet = 1 mile (mi.)

We know how to *reduce* from one unit to another; for example in reducing $2\frac{1}{2}$ ft. to inches, we have

$$1 \text{ ft.} = 12 \text{ in.,}$$

and so $$2\tfrac{1}{2} \text{ ft.} = 2\tfrac{1}{2} \times 12 \text{ in., or } 30 \text{ in.}$$

Similarly, in reducing 40 in. to feet., we have

$$1 \text{ in.} = \tfrac{1}{12} \text{ ft.,}$$

and so $$40 \text{ in.} = 40 \times \tfrac{1}{12} \text{ ft., or } 3\tfrac{1}{3} \text{ ft.}$$

In reducing 4 yd. 2 ft. 7 in. to inches, we may first change both yards and feet to inches, as follows:

$$\begin{aligned} 4 \times 36 \text{ in.} &= 144 \text{ in.} \\ 2 \times 12 \text{ in.} &= 24 \text{ in.} \\ & \underline{7 \text{ in.}} \end{aligned}$$

Adding, we have 175 in.

Some prefer to reduce first to feet, and then to inches, as shown at the right.

$$\boxed{\begin{array}{lll}
4 \text{ (yd.)} & 2 \text{ (ft.)} & 7 \text{ in.} \\
\underline{3\phantom{\text{ (yd.)}}} & & \\
 & \underline{12 \text{ (ft.)}} & \\
 & 14 \text{ (ft.)} & \\
 & \underline{12\phantom{\text{ (ft.)}}} & \\
 & & \underline{168 \text{ (in.)}} \\
 & & 175 \text{ in.}
\end{array}}$$

TABLE OF LENGTH

1. Mollie needs 60 in. of braid for a dress. Express this in yards and a fraction, as she would do in buying it.

2. Kate has $2\frac{3}{4}$ yd. of lace. She needs to use 40 in. on a waist. How many inches will she have left? how many yards; that is, how many yards and a fraction?

3. When Fred won the 100-yard dash, his sister asked how many rods he had to run. How many was it?

4. In a 3-mile hike, how many yards do the Boy Scouts go? How many feet? For practical purposes, no one would ever measure this distance in inches, but it is interesting to know what it is. How many inches is it?

Make the following reductions:

5. 72 in. to feet.
6. 72 in. to yards.
7. 72 ft. to inches.
8. 72 yd. to feet.
9. 72 rd. to yards.
10. 72 rd. to feet.
11. 2 ft. 3 in. to inches.
12. 2 ft. 3 in. to feet.
13. 7 yd. 8 in. to inches.
14. 7 yd. 8 in. to feet.
15. 16 yd. 18 in. to rods.
16. 16 yd. 18 in. to feet.

If your teacher so directs, make these reductions:

17. 7 yd. 1 ft. 20 in. to inches; to feet and inches.
18. 2 rd. 2 yd. 2 ft. to feet; to yards; to yards and feet.
19. 1 mi. 75 rd. 2 yd. to yards; to rods.
20. 2 mi. 160 rd. $8\frac{1}{4}$ ft. to feet; to rods.
21. 175 in. to yards; to yards and feet.

Square Measure. Read and learn the following table:

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards = 1 square rod (sq. rd.)
160 square rods = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)

Cubic Measure. Read and learn the following table:

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)

SQUARE AND CUBIC MEASURE

1. Find the length and width of your schoolroom, each to the nearest foot. If you multiply the number of feet in the length by the number of feet in the width, you will have the number of square feet in the area of the ceiling. Find this number.

2. In Ex. 1 find the area in square yards.

3. It may interest you to know the area in Ex. 1 in square inches. Express it in this way.

4. Find or estimate the height of your schoolroom to the nearest foot. From this and the measurements taken in Ex. 1, find the number of square feet in each wall.

5. In Ex. 4 find the number of square yards.

Make the following reductions:

6. 3 sq. ft. 72 sq. in. to square inches.

7. 1728 sq. in. to square feet; to square yards.

8. 540 cu. ft. to cubic yards; 540 cu. yd. to cubic feet.

Table of Weight. Read and review this table of weight:

16 ounces (oz.) = 1 pound (lb.)
100 pounds = 1 hundredweight (cwt.)
2000 pounds = 1 ton (T.)

TABLE OF WEIGHT

1. What is your own weight? It may interest you to know this in ounces. Express it in this way.

2. You never thought of your weight in tons; in fact, no one would ever speak of a person's weight in this way. It may interest you, however, to find what fraction of a ton you weigh. Express it in this way.

3. It is not unusual, however, to express ounces as a fraction of a pound. Do this in the case of 8 oz.; of 4 oz.; of 2 oz.; of 10 oz.; of 12 oz.

4. If you buy $2\frac{1}{2}$ lb. of meat, you buy how many ounces more than 2 lb.? If you buy 1 lb. 12 oz. of cheese, how many ounces do you buy?

Make the following reductions:

5. 32 oz. to pounds.
6. 32 lb. to ounces.
7. 32 T. to pounds.
8. $2\frac{3}{4}$ T. to pounds.
9. $2\frac{3}{4}$ lb. to ounces.
10. $2\frac{3}{4}$ oz. to pounds.

11. In Ex. 10 you found, as in other similar cases, that you had to express $2\frac{3}{4}$ oz. as a fraction of a pound. Now express it as a decimal part of a pound.

12. Express 17,500 lb. as tons; as hundredweight.

13. Express 365,000 lb. as tons; 365 T. as pounds.

Liquid Measure. Read and review the following table:

2 pints (pt.) = 1 quart (qt.)
4 quarts = 1 gallon (gal.)

Dry Measure. Read and review the following table:

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

LIQUID AND DRY MEASURE

1. If you drink 1 pt. of milk each day, how many quarts do you drink in 5 da.?

2. If you drink 1 qt. of milk each day, how many pints do you drink in 5 da.?

Make these reductions of liquid measures:

3. 15 qt. to pints.
4. 17 pt. to quarts.
5. 3 gal. 2 qt. to quarts.
6. 55 qt. 1 pt. to pints.

7. If you buy $1\frac{1}{2}$ bu. of potatoes, how many pecks do you buy? If you buy $1\frac{1}{2}$ pk. of green peas, how many quarts do you buy?

Make these reductions of dry measures:

8. 19 pt. to quarts.
9. 19 pt. to pecks.
10. 1 bu. 2 pk. to pecks.
11. 16 pk. 4 qt. to quarts.

If the teacher so directs, make these reductions:

12. 2 bu. 3 pk. 5 qt. to quarts; to pecks; to pints.

13. 2 gal. 3 qt. 1 pt. to pints; to quarts.

Time. Read and review the following table of time:

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
12 months (mo.) = 1 year (yr.)

Money. Read and review the following table of money:

10 mills = 1 cent (ct., c., or ¢)
10 cents = 1 dime (d.)
10 dimes = 1 dollar ($)
100 cents = 1 dollar

TABLES OF TIME AND MONEY

1. How many months and days is it from your last birthday to today? Write your age in years, months, and days.

2. You may be interested to know how many months old you are. Write your age in months and days. Then give it in days, which is even more interesting.

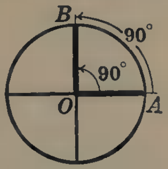
Make the following reductions:

3. $2\frac{1}{2}$ hr. to minutes.
4. $2\frac{1}{2}$ min. to seconds.
5. $2\frac{1}{2}$ wk. to days.
6. $2\frac{1}{2}$ yr. to months.
7. $2.50 to dimes.
8. $2.50 to cents.
9. 370¢ to dimes.
10. 480¢ to dollars.

11. How many days are there in a common year? in a leap year? When is the next leap year?

12. Reduce 1 wk. 2 da. 15 hr. to days; to hours.

Angle Measure. The length of a circle (that is, the distance around the space inclosed by the curve line) is called the *circumference*, and any part of the circle, as from A to B in this figure, is called an *arc*.



In measuring arcs, the unit used is the *degree* (written 1°), and this unit is $\frac{1}{360}$ of the circumference. In the figure it is evident that the size of the angle at O between the lines OA and OB is related to the size of the arc between A and B. Accordingly, we measure angles in degrees, and say that an angle, like the one between OA and OB, which cuts off a fourth of a circumference is a *right angle* and contains $\frac{1}{4}$ of 360°, or 90°.

Similarly, if a wagon wheel makes $\frac{1}{6}$ of a revolution, each spoke turns through an angle of $\frac{1}{6}$ of 360°, or 60°; if it makes $\frac{1}{2}$ of a revolution, each spoke turns through an angle of 180°; and if it makes $1\frac{1}{2}$ revolutions, each spoke turns through an angle of $1\frac{1}{2} \times 360°$, or 540°.

Read and learn the following table of angle measure:

60 seconds (″) = 1 minute (′)
60 minutes = 1 degree (°)

ANGLE MEASURE

Make the following reductions:

1. 540″ to minutes.
2. 540′ to seconds.
3. 540′ to degrees.
4. 285′ to degrees.
5. 2° 10′ to minutes.
6. 2′ 10″ to seconds.
7. 3° 3′ 3″ to seconds.
8. 5° 10′ 20″ to seconds.

Counting Paper. Read and learn this counting table:

[redacted] sheets, or 25 sheets = 1 quire
20 quires = 1 [redacted]

The quire of 24 sheets (480 sheets to the ream) is used in counting stationery and folded paper. The quire of 25 sheets (500 sheets to the ream) is used for unfolded paper. Paper is sold also by the pound.

PROBLEMS IN MEASURES

1. At 50¢ a quire, how much do 16 quires of paper cost?

2. At $16\frac{2}{3}$¢ a quire, what is the cost of 24 quires?

3. From a piece of tin containing $7\frac{1}{2}$ sq. ft. Rob cuts 725 sq. in. How many square inches are left?

4. How many square rods are there in a field which contains $\frac{3}{4}$ A.? in one which contains $1\frac{7}{8}$ A.?

5. A strip of carpet is 38 yd. 2 ft. long. What is the length of the strip in feet?

6. In excavating a cellar 540 cu. yd. of earth are removed. How many cubic feet of earth are removed?

7. A machinist is welding together two pieces of steel in such a way that they make an angle equal to $\frac{3}{4}$ of a right angle. How many degrees are there in this angle?

8. In sailing, a point is $\frac{1}{8}$ of a right angle. A boat's course was changed 6 points. How many degrees was this?

9. The sum of all the angles in a figure of five sides is 540°. If the angles are all equal, how many degrees are there in each one? How many minutes are there?

Adding Compound Numbers. Suppose we know that a schoolhouse stands 44 ft. 8 in. from the street and that the street is 40 ft. 10 in. wide. If we wish to find the distance from the schoolhouse to the opposite side of the street, we might measure it, but it is easier to add the given numbers.

Adding the inches first, we have 18 in., or 1 ft. 6 in. We therefore write 6 under inches and add 1 to feet. The result is 85 ft. 6 in., which is the distance that we wished to find.

ft.	in.
44	8
40	10
85	6

That is, we add compound numbers in much the same way as we add ordinary whole numbers. In ordinary numbers ten units make one unit of the next higher order, but in compound numbers the number of units required to make the next higher one varies with the measure used.

ADDING COMPOUND NUMBERS

Add as follows:

1.

ft.	in.
32	11
16	8
14	4

3.

sq. ft.	sq. in.
36	40
74	75
31	62

5.

yd.	in.
34	12
10	24
26	17

7.

lb.	oz.
236	12
364	8
228	6

2.

ft.	in.
37	10
26	8
45	3
8	7
13	10

4.

sq. ft.	sq. in.
29	28
64	96
9	
45	94
27	48

6.

yd.	in.
38	10
7	8
26	8
44	9
22	19

8.

lb.	oz.
486	14
324	13
42	7
238	11
112	8

Subtracting Compound Numbers. Suppose that in a room 18 ft. 5 in. square you have put up picture molding on three sides and have a piece 9 ft. 7 in. long for the fourth side. How much more do you need for this side of the room?

In subtracting 9 ft. 7 in. from 18 ft. 5 in., we proceed as we would in the case of subtracting $9\frac{7}{12}$ from $18\frac{5}{12}$; that is, we think of 18 ft. 5 in. as 17 ft. 17 in.

First subtracting the inches and then the feet, the result is 8 ft. 10 in.; that is, you need 8 ft. 10 in. of molding.

ft.	in.
18	5
9	7
8	10

SUBTRACTING COMPOUND NUMBERS

1. A grocer has a piece of cheese weighing 7 lb. 9 oz. If he sells a piece weighing 1 lb. 12 oz., how much has he left?

2. If Bertha has a roll of ribbon 18 yd. 8 in. long and cuts off 9 yd. 28 in., how much ribbon is left in the roll? How much less than 10 yd. is left?

Subtract as follows:

	lb.	oz.		bu.	qt.		ft.	in.		A.	sq. rd.
3.	48	14	**6.**	46	10	**9.**	62	10	**12.**	76	105
	22	8		29	16		28	4		38	118
	bu.	pk.		qt.	pt.		lb.	oz.		sq. ft.	sq. in.
4.	40	1	**7.**	96		**10.**	50	10	**13.**	98	102
	28	2		43	1		36	14		49	128
	gal.	qt.		ft.	in.		yd.	in.		cu. ft.	cu. in.
5.	22	2	**8.**	30	$5\frac{1}{2}$	**11.**	58	22	**14.**	76	272
	15	3		18	$9\frac{1}{4}$		46	28		48	564

Multiplying Compound Numbers. Suppose that a wholesaler ships walnuts in cases of $\frac{1}{2}$ doz. boxes, each box holding 4 lb. 9 oz. of walnuts. How many pounds are there in a case?

Multiplying the ounces first, we have 54 oz., or 3 lb. 6 oz. We therefore write 6 under ounces. Then $6 \times 4 + 3 = 27$; that is, as here shown, there are 27 lb. 6 oz. of walnuts in each case.

lb.	oz.
4	9
	6
27	6

In multiplying 243 ft. 7 in. by 27, however, it is easier not to reduce while multiplying, as we did above. That is, we first find 6561 ft. 189 in. as the result and then change to 6576 ft. 9 in., as shown at the right.

A clerk in a store would not multiply 5 lb. 4 oz. by 8, but would take

$$8 \times 5\tfrac{1}{4} \text{ lb.},$$

and similarly in most cases of this kind.

ft.	in.
243	7
	27
6561	189
6576	9

MULTIPLYING COMPOUND NUMBERS

1. Water weighs 62 lb. 8 oz. per cubic foot. What is the weight of water in a tank containing 375 cu. ft. of water?

2. The average height of the stories in an 8-story building is 11 ft. 9 in. What is the height of the building?

3. On a 6-day automobile trip a man uses an average of 9 gal. 3 qt. of gasoline a day. How much does he use in all?

4. Find the weight of cereal in a dozen packages, each of which contains 1 lb. 10 oz.

Dividing Compound Numbers. If a board 7 ft. 6 in. long is cut into two equal lengths to make a top for a workbench, how long is each part?

First dividing the feet, the result is 3 with a remainder of 1 ft., or 12 in. Then adding this 12 in. to the 6 in. and dividing, the complete result is 3 ft. 9 in.

$$\begin{array}{rrr} & \text{ft.} & \text{in.} \\ 2) & 7 & 6 \\ \cline{2-3} & 3 & 9 \end{array}$$

That is, each part is 3 ft. 9 in. in length.

A practical case in which we need to divide one compound number by another is rare. If, for example, we have to divide 41 ft. 3 in. by 2 ft. 9 in., we reduce both to feet ($41\frac{1}{4}$ ft. ÷ $2\frac{3}{4}$ ft. = 15) or to inches (495 in. ÷ 33 in. = 15).

DIVIDING COMPOUND NUMBERS

1. The perimeter of an equilateral (equal-sided) triangle is 17 ft. $9\frac{1}{2}$ in. Find the length of each side.

2. The perimeter of a square flower bed is 7 ft. 8 in. Find the length of each side.

3. An angle of 66° 44′ is to be divided into 7 equal parts. How many degrees and minutes should each part contain?

4. If the school day is 5 hr. 45 min., how many hours, minutes, and seconds are there in half a school day?

5. At the rate of a mile in 4 min., how far will an automobile go in 1 hr.? in $2\frac{1}{2}$ hr.? in $4\frac{1}{4}$ hr.?

6. Traveling at the rate of a mile in 12 min., how far will a horse go in $2\frac{1}{2}$ hr.?

7. If the circumference of a wagon wheel is 10 ft. 6 in., how many times will the wheel turn in going 25 mi.?

Athletic Records. The table below compares various world records in amateur track and field events with performances in these events in recent meets:

Event	World Record	Recent Meets
Walking, 3 mi.	20 min. 25.8 sec.	21 min. 50.1 sec.
Running, 100 yd.	9.6 sec.	10 sec.
Running, 220 yd.	20.8 sec.	22.1 sec.
Running, 1 mi.	4 min. 10.4 sec.	4 min. 17 sec.
Relay race, 1 mi.	3 min. 16.4 sec.	3 min. 21 sec.
Standing broad jump	11 ft. $4\frac{7}{8}$ in.	9 ft. $5\frac{1}{4}$ in.
Running broad jump	25 ft. 3 in.	24 ft. 6 in.
Hammer throw	189 ft. $6\frac{1}{2}$ in.	158 ft. $9\frac{3}{4}$ in.

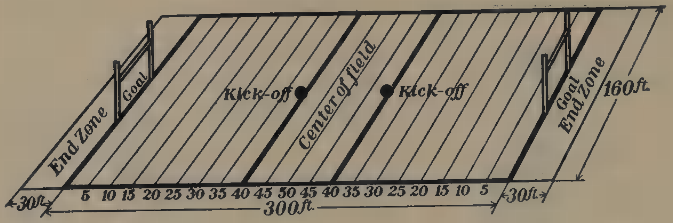
Many world records are held by American athletes.

ATHLETIC RECORDS

1. Find the difference between the world record and the time in recent meets for the 3-mile walking race.

2. As in Ex. 1, find the difference for each running race.

3. As in Ex. 1, find the difference in distance for each of the jumping events and for the hammer throw.



ATHLETIC FIELDS

1. Here is a plan of a football field. Not counting the 10-yard end zones, what is the length of the field in yards?

2. What is the width of the field in yards?

3. How many yards is it from each kick-off to the center of the field? to the nearer goal? to the other goal?

4. From its own 15-yard line a team advanced the ball in eight plays to the opponent's 10-yard line. What was the average distance gained in each play?

5. Standing on his own 20-yard line, a player made a kick of 45 yd. The ball was caught by the quarter back of the other team. On what line was he standing?

6. Not counting the two 10-yard end zones, what is the perimeter of the field in yards?

7. How many square yards are there in an acre?

8. If a football field contains 48,000 sq. ft., which has the greater number of square feet, the field or an acre, and what is the difference between them?

9. A double tennis court is 36 ft. wide and 78 ft. long. Draw a plan to the scale of 1 in. to 6 yd.

10. How many yards are there in the perimeter of the tennis court described in Ex. 9?

REVIEW OF MEASURES

Numbers 1 to 20, oral

Read the following, supplying the missing numbers:

1. 1 ft. = —— in.
2. $\frac{1}{2}$ ft. = —— in.
3. 3 ft. = —— in.
4. 3 ft. = —— yd.
5. 9 ft. = —— yd.
6. 1 rd. = —— ft.
7. 1 rd. = —— yd.
8. 2 yd. = —— ft.
9. 2 rd. = —— yd.
10. 2 rd. = —— ft.

11. —— yd. = 36 in.
12. —— yd. = 72 in.
13. —— yd. = 18 in.
14. —— yd. = 9 in.
15. 45 in. = —— yd.
16. 1 ft. 2 in. = —— in.
17. 1 yd. 2 in. = —— in.
18. 3 ft. 4 in. = —— in.
19. 1 yd. 4 in. = —— in.
20. 2 yd. 3 in. = —— in.

21. Express 1 sq. yd. as square inches.

22. A square is 2 ft. 3 in. on a side. How many feet are there in the perimeter? How many yards?

Copy, supplying the missing numbers:

23. $\frac{1}{4}$ lb. = —— oz.
24. $1\frac{1}{4}$ lb. = —— oz.
25. $\frac{1}{2}$ sq. yd. = —— sq. ft.
26. 3 sq. yd. = —— sq. ft.
27. 10 A. = —— sq. rd.
28. 1 sq. mi. = —— A.
29. 1 sq. ft. = —— sq. in.
30. 4 sq. yd. = —— sq. ft.

31. 9 T. = —— lb.
32. 9000 lb. = —— T.
33. 9 qt. = —— pt.
34. 9 gal. = —— qt.
35. 9 pk. = —— qt.
36. 9 bu. = —— pk.
37. 9 wk. = —— da.
38. 9 doz. = —— units.

WRITTEN REVIEW OF MEASURES

1. A schoolroom is 28 ft. long and 23 ft. wide. To decorate the room for the closing exercises, the class has to know the perimeter in yards. What is it?

2. Our school opens at 8 45 A.M. and has a recess at 10 30 A.M. How long is it from the opening to recess?

3. If a train leaves at 9 37 A.M. and the next stop is at 11 10 A.M., how long does it take to make the run?

4. If you need $44\frac{1}{2}$ in. of picture molding for your class picture and have a piece 4 ft. long, have you enough? Have you too much? How many inches more than enough or less than enough have you?

5. If the work shown at the right is an example in addition, what is the result?

6. If it is an example in subtraction, what is the result?

ft.	in.
13	8
	9

7. If it is an example in multiplication, what is the result? Check by division.

8. What is the perimeter of a square that is 2 ft. 6 in. on a side? of one that is twice as long on a side?

9. If each side of a triangle is 48 in., how many yards are there in the perimeter? how many feet? You may find the perimeter in inches and express the result in yards or feet, or you may express 48 in. as yards or feet and then find the perimeter. Which of these methods is the shorter?

10. Jack's birthday is March 8 and Mabel's is April 8. How many days are there from one date to the other?

SPEED TEST CHART IN MEASURES

Add as follows, recording the number of minutes:

	ft.	in.		yd.	in.		gal.	qt.		bu.	pk.
1.	2	3	**3.**	2	16	**5.**	12	3	**7.**	36	2
	4	11		3	9		16	2		27	

	ft.	in.		yd.	in.		gal.	qt.		hr.	min.
2.	3	9	**4.**	4	27	**6.**	16	3	**8.**	2	37
	5	2		3	32		12			3	12
	2	8		5	8		8			1	48

Subtract as follows, recording the number of minutes:

	ft.	in.		yd.	in.		gal.	qt.		bu.	pk.
9.	8	2	**11.**	4	2	**13.**	21	1	**15.**	12	2
	3	11		2	26		16	3		8	3

	ft.	in.		yd.	in.		gal.	qt.		hr.	min.
10.	6	0	**12.**	5	18	**14.**	30	0	**16.**	9	40
	2	8		1	32		12	2		2	10

Multiply as follows, recording the number of minutes:

17.	2 lb. 4 oz.	**18.**	2 ft. 4 in.	**19.**	2 hr. 4 min.
	5		5		5

Divide as follows, recording the number of minutes:

20. 3 ft. 4 in. by 2.
21. 3 yd. 4 in. by 2.
22. 3 hr. 4 min. by 2.
23. 2 lb. 1 oz. by 3.
24. 2 pk. 1 qt. by 3.
25. 2 gal. 1 qt. by 3.

MISCELLANEOUS PROBLEMS

1. A dealer marked down some collars from 30¢ each to 25¢ each. The lower price is what part of the higher price?

2. The amount of the reduction in Ex. 1 is what part of the higher price? what part of the lower price?

3. A dealer marked down some neckties from 3 for $2 to 60¢ each. The lower price is what part of the higher price?

4. After a man's salary was raised $\frac{1}{8}$, he received $2700 a year; that is, he received $\frac{9}{8}$ of what he received before. How much was his salary before it was raised?

5. The daily pay roll for 3600 men, who work in a certain factory, is $21,700. After deducting the amount paid to 100 men, who receive $7 each per day, find the average daily wage for the other men employed by the factory.

6. On a certain railroad, 2850 conductors received an increase in wages amounting to $4275 a day. What was the average daily increase per man?

7. A farmer shipped 800 barrels of apples to a city dealer. The dealer sold the apples for $4.40 a barrel, and charged the farmer 22¢ a barrel for selling, 8¢ a barrel for storage, $39.75 for freight, and $38.50 for cartage. How much did the farmer receive from the dealer?

8. When cotton was selling for 31¢ a pound, a dealer sold 175 bales, which weighed on the average 500 lb. each. Find the amount that the dealer received for the lot.

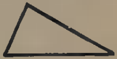
9. A man earns $7.25 a day, and works 6 da. a week. If during one week he pays $8.75 for groceries and $9.50 for rent, how much does he have left for other purposes?

III. PRACTICAL MEASUREMENTS

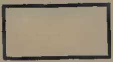
A SILENT READING LESSON

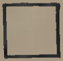
The denominate numbers, which we have been studying, do not mean much unless we know how to use them. We use such numbers in measuring. We know how to measure length, but we have not studied much about area.

We know that a *triangle* is a figure which has three sides. In problems we have often had to find the *perimeter;* that is, the distance around a triangle. We have also had to find the length of one side when we were given the perimeter and the lengths of the other two sides.

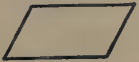

Triangle

We know about two kinds of four-sided figures, one of which is the *rectangle,* and the other the *square.* The four angles of a rectangle are all right angles. A square is a rectangle with all four sides equal.

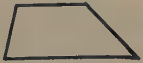

Rectangle


Square

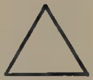
Another kind of four-sided figure is the *parallelogram.* It is formed by two pairs of parallel lines, which are lines that never meet. A rectangle is a kind of parallelogram; that is, it is one in which the angles are right angles.


Parallelogram

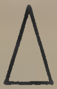
Still another kind of four-sided figure is called a *trapezoid.* In such a figure one pair of sides are parallel lines. In cities and towns not all the streets meet each other at right angles, and many house lots have the shape of a parallelogram or of a trapezoid. There are probably more, however, that have the shape of a rectangle or of a square.


Trapezoid

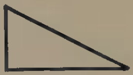
Triangles. The following figures show the kinds of common triangles about which we need to know:



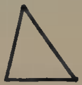
Equilateral



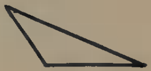
Isosceles



Right



Acute



Obtuse

Practice drawing these triangles on the board and on paper until you know them well. Ask your teacher to explain the names if you cannot tell what they mean from the figures.

A LITTLE WRITING LESSON

Copy and complete the following statements:

1. A triangle has —— sides and —— angles.

2. An *equilateral* triangle has all its sides ——.

3. An *isosceles* triangle has —— of its sides equal.

4. A *right* triangle has one —— angle.

5. An *acute* triangle has all its —— acute.

6. An *obtuse* triangle has one of its —— obtuse.

7. If you add all the —— of a figure, the result is called the perimeter of the ——.

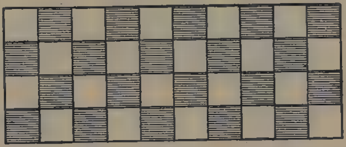
8. If one side of an equilateral triangle is 2 in., the —— of the triangle is 6 in.

9. If one side of an isosceles triangle is 2 in. and another is 3 in., the third —— is either —— in. or —— in.

10. A right triangle can also be an isosceles ——, but it cannot be an —— triangle, an —— triangle, or an —— triangle.

Base and Height. The line on which a figure appears to stand is called the *base*. The distance from the highest point to the base is called the *height* or *altitude*.

Area of a Rectangle. A school building has an entrance hall 10 ft. long and 4 ft. wide. The floor is made of marble squares 1 ft. on each side, as shown in this figure.



How many squares are there in each row? How many rows are there? We thus see that there are 4 × 10, or 40, squares in the floor. Since each square has an area of 1 sq. ft., we see that the area of the floor is 40 sq. ft. That is,

The area of a rectangle is the product of the base and height.

In using this rule, we do not multiply 10 ft. by 4 ft., but say that 4 × 10 = 40, the *number* of square feet, and write 4 × 10 sq. ft. = 40 sq. ft.

AREA OF A RECTANGLE

Using short methods, find the area of a rectangle which is

1. 3 in. by $6\frac{1}{2}$ in.
2. 5 in. by $7\frac{1}{4}$ in.
3. 8 in. by $9\frac{7}{8}$ in.
4. 9 in. by $9\frac{1}{4}$ in.
5. 38 ft. by 46 ft.
6. $33\frac{1}{3}$ yd. by 60 yd.
7. 75 ft. by 96 ft.
8. $66\frac{2}{3}$ yd. by 66 yd.
9. 65 ft. by 75 ft.
10. 25 ft. by 46 ft.
11. 62 ft. by 84 ft.
12. $87\frac{1}{2}$ in. by 88 in.
13. Measure your blackboards and find the area of each.

SCHOOL-AND-HOME PROJECTS

1. The school-and-home project in one school district was gardening; that is, each pupil cultivated a piece of land, kept track of the expenses, and reported the profit. A garden that is 8 rd. long and 2 rd. wide contains what part of an acre?

2. A pupil who cultivated a piece of land 10 rd. long and 4 rd. wide was cultivating what part of an acre?

3. Frank cultivated a garden 5 rd. long and $2\frac{1}{2}$ rd. wide. He put a fence around it, using four wires nailed to posts. If a spool of 330 yd. of wire cost $4, find the cost of the wire used in building the fence.

4. Richard Heath measured his piece of land, and found it to be 44 ft. wide and 66 ft. long. He said that it contained a little over $10\frac{1}{2}$ sq. rd. Was he right? How much over $10\frac{1}{2}$ sq. rd. was there in the piece of land?

5. On the piece of land described in Ex. 4, Richard raised 36 pk. of potatoes. Find to the nearest peck the amount of potatoes that he raised per square rod.

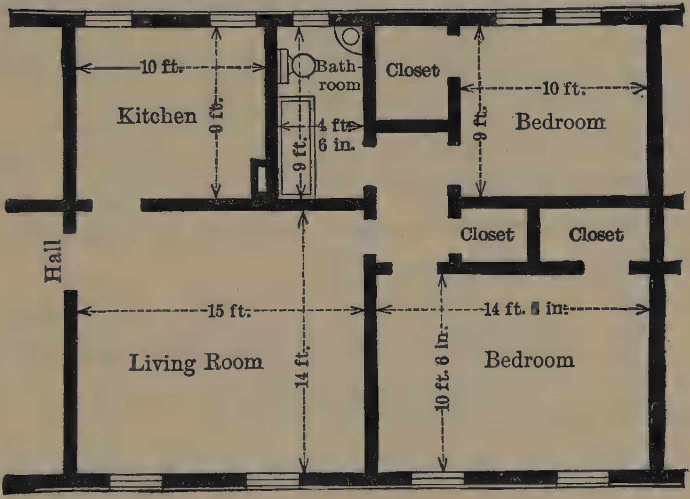
6. At the number of pecks per square rod found in Ex. 5, how many bushels should Richard raise per acre?

7. Richard sold the potatoes mentioned in Ex. 5 at $1.30 a bushel. How much money did he receive?

8. The storekeeper to whom Richard sold his potatoes sold them at 5¢ a pound. There are 60 lb. of potatoes in a bushel. How much did the storekeeper receive for 1 bu.? for all the potatoes? Disregarding the expenses of running his store, how much did he make in all?

USING SCALE DRAWINGS

1. The figure below shows the plan of a small apartment, drawn to the scale of 1 in. to 10 ft. You have already learned that the scale of 1 in. to 10 ft. means that we use 1 in. to represent 10 ft. The scale $\frac{1}{12}$ means that 1 in. represents 12 in. Express the scale of this plan as a fraction.



2. What is the width of the windows in the living room?

3. What is the area of the floor of the living room?

4. Measure the plan and find the size of the inner hall into which the bedrooms and the living room open.

5. What is the area of the floor of the kitchen?

6. What is the area of the floor of each bedroom?

7. What is the total floor space in the apartment, not counting the closets and bathroom?

DRAWING TO SCALE

1. The boys are making a book cover which is to be 6 in. by 9 in. Using the scale $\frac{1}{2}$, draw a plan of the cover.

2. A school corridor is 8 ft. by 80 ft. Draw a floor plan of the corridor, using the scale of $\frac{1}{16}$ in. to 1 ft.

3. A playground is 64 ft. by 96 ft. Draw a plan of the playground, using the scale of $\frac{1}{16}$ in. to 1 ft.

4. A schoolroom is 30 ft. long and 25 ft. wide. Draw a plan of the floor, using the scale of $\frac{1}{8}$ in. to 1 ft.

5. Draw a plan of a floor 24 ft. long and 16 ft. wide, using the scale of $\frac{1}{8}$ in. to 1 ft.

6. Draw a plan of a school garden 64 ft. long by 40 ft. wide to the scale of $\frac{1}{8}$ in. to 1 ft.

7. Draw a plan of a mat 16 in. by 28 in. to the scale $\frac{1}{4}$.

8. A plan for a rectangular playhouse is drawn to the scale of 1 in. to 2.5 ft. The plan is 6.4 in. long and 4 in. wide. Find the area of the playhouse floor.

9. The class has made a floor plan of a rectangular room. The drawing has an area of 20 sq. in. If a scale twice as large were used, what would be the area of the drawing? Draw two rectangles which show that your answer is correct.

10. Measure the top of your desk to the nearest inch. Make a drawing of the top, as you look at it from above, using the scale $\frac{1}{8}$.

11. Make two plans of the blackboard, one to the scale of $\frac{1}{4}$ in. to 1 ft. and the other to the scale of $\frac{1}{8}$ in. to 1 ft.

SIDES AND PERIMETERS

1. What is the perimeter of an equilateral triangle that is 19 ft. 7 in. on a side? Draw neat figures in all the work on this page, but you need not make them to scale.

2. A triangle has a perimeter of 114 ft. 2 in. One side is 30 ft., and another is half as long again. How long is the third side of the triangle?

3. The base of an isosceles triangle is 25 ft., and each of the equal sides is $\frac{1}{5}$ longer. How long is the perimeter?

4. Draw a triangle, cut off the three angles, and place them on a line, the angles meeting at one point. Do you find that they equal two right angles? Try this with several triangles of different shapes.

Find the sides of equilateral triangles with perimeters of

5. 113 ft. 3 in. **6.** 127 yd. **7.** 88 ft. 3 in.

Find the sides of squares with perimeters of

8. 110 ft. 4 in. **10.** 61 yd. 2 ft. **12.** 118 ft. 8 in.

9. 42 ft. $3\frac{1}{2}$ in. **11.** 2 ft. 4 in. **13.** 227 mi. 40 rd.

14. The perimeter of an isosceles triangle is 100 ft., and the unequal side is 33 ft. long. Find the length of one of the equal sides.

15. The perimeter of an isosceles triangle is 88 ft., and the unequal side is $\frac{1}{5}$ of the perimeter. Find the sides.

16. The perimeter of a triangle is 24 ft. One side is $\frac{1}{3}$ of the perimeter, and another is $\frac{3}{8}$ of the perimeter. How many feet are there in each of the three sides?

MEASUREMENTS IN GARDENING

1. In order to earn money for their summer-camp expenses, some boys rented a piece of land 40 rd. long and 20 rd. wide. At $10 an acre, how much was the rent?

2. The length of the field in Ex. 1 ran north and south. Starting from the northwest corner, the boys planted early potatoes in a strip 4 rd. wide the entire length of the field. How many acres of early potatoes did they plant?

3. Adjoining the strip in Ex. 2, they laid out, lengthwise of the field, a strip 6 rd. wide. They planted early sweet corn in $\frac{1}{4}$ of it. What part of an acre did they plant to early sweet corn?

4. They planted the rest of the strip in Ex. 3 to a later variety of corn. How many acres of this did they plant?

5. Make a plan of the field to the scale of $\frac{1}{8}$ in. to 1 rd., putting in all the measurements and the areas of the various pieces, and show on the plan what was planted in each one of the strips.

6. They planted the strip of early corn in Ex. 3 in rows 3 ft. 2 in. apart, the rows running north and south. The first and last rows were each 2 ft. from the edges of the strip. They planted the corn in hills 2 ft. 9 in. apart, leaving a space equal to the distance between the hills at each end of the strip. How many hills did they plant?

7. The hills in the strip of early corn produced an average of three ears per hill. If the boys sold it at 20¢ per dozen, how much did they receive for this corn?

8. The boys planted late potatoes in a strip 115½ ft. wide the entire length of the field, and planted sugar beets on the rest of the piece of land. How many acres did they plant to each of these crops? Complete the plan in Ex. 5, and check the areas of the different strips by adding them together and comparing the result with the number of acres which you found in Ex. 1.

9. It takes 7 bu. of seed potatoes to plant an acre. If seed potatoes cost $3.20 per 100 lb., and 1 bu. weighs 60 lb., how much did the boys pay for the seed potatoes in Ex. 8?

10. The strip of late potatoes in Ex. 8 yielded 3 pk. to each square rod of land, and the boys sold the potatoes for $1.50 per 100 lb. Taking the weight of 1 bu. as 60 lb., how much did they receive for their crop of potatoes?

11. Land such as was used for the sugar beets in Ex. 8 yields 18 T. per acre, and the crop is worth $10 per ton. The expenses per acre of raising sugar beets are as follows: rent, $10; seed, 12 lb. at 70¢ per pound; labor, $21. Find the profit the boys should make on their crop of beets.

12. The farmer whose land adjoined the field the boys rented was careless. He planted 82½ A. of corn without giving the seed an ear-to-ear test, the result being that four ears out of every bushel of seed corn failed to grow. If we estimate 100 ears of seed corn to a bushel, and the yield at 50 bu. to the acre, what was the farmer's loss, at 75¢ per bushel, from failing to test the seed?

13. If the work of testing the seed corn in Ex. 12 would have taken 5 da., what wages per day would the increased yield of corn have paid the farmer?

THE ATHLETIC FIELD

1. The infield of a baseball diamond is a square 90 ft. on a side. Draw a plan to the scale of 1 in. to 20 ft.

2. In the infield, the pitcher's box is $60\frac{1}{2}$ ft. from home plate, measured on a line between home plate and second base. Draw another plan of the infield, using the scale of $\frac{1}{16}$ in. to 1 ft., and mark the position of the pitcher's box.

3. There is a batter's box on each side of home plate. The boxes, which are 29 in. apart, are rectangles 6 ft. long and 4 ft. wide, with the center of the long side almost opposite the center of home plate. Show the batters' boxes on the plan of Ex. 2.

4. A school rents a piece of land 30 rd. wide and 40 rd. long for a baseball and football field. How many acres are there in the field?

5. Draw a plan of the field of Ex. 4 to the scale of 1 in. to 100 ft. and place the baseball diamond, drawn to the same scale, in what you think is the best place.

OUTDOOR MEASUREMENTS

1. Near the school some surveyors were laying out house lots on a tract of land. One of the lots was 70 ft. 6 in. along the front and 106 ft. 6 in. in depth. The class found that the area of this lot was —— sq. ft.

2. The class found that it would take —— ft. of fence to inclose the lot in Ex. 1.

3. The boys paced off 36 steps of 27 in. each along the front of the next lot. The frontage of this lot was —— ft.

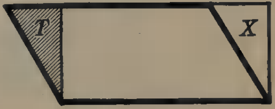
4. The class measured the distances between the stakes for a cellar and found that it was to be 22 ft. 6 in. by 34 ft. 6 in. The area of the cellar was to be —— sq. ft.

5. From the corner of the cellar nearest to a fire hydrant the boys paced off 256 steps of 27 in. each. The house was to be —— ft. from the hydrant.

6. Measure the length of your ordinary step and learn to pace short distances.

Area of a Parallelogram. If we cut off the triangle *T* from this parallelogram and place it in the position shown by *X*, what kind of figure do we have then?

How does this figure tell us that the area of a parallelogram is equal to the area of a rectangle with the same base and the same height? We thus have the following rule:



The area of a parallelogram is the product of the base and height.

AREA OF A PARALLELOGRAM

1. On paper draw and cut out three parallelograms with different bases and heights. Show that each parallelogram can be made into a rectangle with the same area.

2. To the scale $\frac{1}{4}$, draw a parallelogram in which the base is 12 in. and the height is 6 in.

Find the areas of parallelograms or of rectangles which have the following bases and heights:

3. 14 in., 12 in.
4. 28 ft., 18 ft.
5. 24 ft., $18\frac{1}{2}$ ft.
6. $4\frac{1}{4}$ in., $3\frac{1}{2}$ in.
7. 52 ft., $32\frac{1}{2}$ ft.
8. 8 yd., $14\frac{1}{2}$ yd.

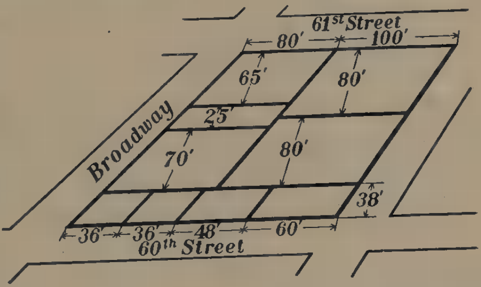
9. A woman cuts a piece of velvet on the bias, thus cutting off a parallelogram with a base of 28 in. and a height of 9 in. How many square inches are cut off?

Find the areas of parallelograms or of rectangles which have the following bases and heights:

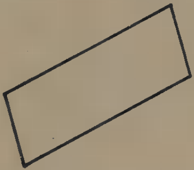
10. 8.7 in., 7.9 in.
11. 32 ft., $27\frac{1}{2}$ ft.
12. 27 ft. 9 in., 1 yd.
13. 8 ft. 6 in., 7 ft. 8 in.

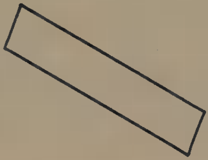
14. This figure shows a plan of the lots in a block on Broadway, between 60th St. and 61st St. Each lot has the form of a parallelogram. Find the area of each of the nine lots.

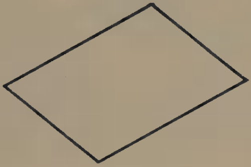
15. Check your work in Ex. 14 by finding the area of the entire block and showing that this result is the same as the total area of the nine lots.



16. What is the area of a parallelogram with a base of 180 ft. and a height of 38 ft.? Show that this result equals that of adding the areas of the lots on 60th St. in Ex. 14.







17. These three figures are drawn to the scale of $\frac{1}{8}$ in. to 10 ft. Find in square feet the area of the parallelogram represented by each figure. In measuring the heights, place the ruler at right angles to the longer sides.

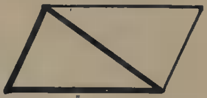
18. A school playground in the shape of a parallelogram is 180 ft. along the long sides and 75 ft. straight across between them. Draw a plan to scale, and find the area of the playground.

AREA OF A TRIANGLE

All work oral

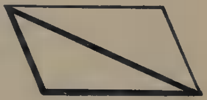
1. How is the area of a parallelogram found?

2. In this parallelogram, how does the area of the triangle indicated by the heavy lines compare with that of the other triangle? A triangle is what part of a parallelogram with the same base and the same height?

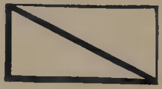


3. If the base in Ex. 2 is 6 in., and the height is 3 in., what is the area of the parallelogram? of each triangle?

4. In this parallelogram, how do the areas of the two triangles compare? Each of the triangles is what part of the parallelogram?



5. In the rectangle here shown, how does the area of the triangle indicated by the heavy lines compare with the area of the other triangle? A triangle is what part of a rectangle of the same base and the same height?



6. If the rectangle in Ex. 5 is 10 ft. wide and 5 ft. high, what is its area? What is the area of each of the triangles?

7. If you wish to find the area of some triangular piece of ground, what measurements shall you take?

Find the areas of triangles with bases and heights as follows:

8. 5 in., 4 in.

9. 7 in., 6 in.

10. 6 in., 4 in.

11. 8 in., 6 in.

12. 9 in., 10 in.

13. 10 ft., 20 ft.

Area of a Triangle. From page 282 we have this rule:

The area of a triangle is half the product of the base and height.

Thus, in finding the area of a triangle with a base of 18 in. and a height of 7 in., we have

$\frac{1}{2}$ of 18 × 7 sq. in.

We arrange the work as shown at the right, cancel, and find the result to be 63; that is, the area is 63 sq. in.

$$\boxed{\frac{\overset{9}{\cancel{18}} \times 7}{\cancel{2}} = 63}$$

AREA OF A TRIANGLE

Numbers 1 to 6, oral

State the areas of triangles with bases and heights as follows:

1. 40 in., 25 in.
2. 160 ft., 125 ft.
3. 90 in., $33\frac{1}{3}$ in.
4. 160 in., 25 in.
5. 64 in., 50 in.
6. 60 in., $16\frac{2}{3}$ in.

Find the areas of triangles with bases and heights as follows:

7. 27.9 in., 18.7 in.
8. 28.7 in., 36.3 in.
9. 25.2 in., 19.4 in.
10. $36\frac{1}{4}$ ft., $44\frac{1}{2}$ ft.
11. 4 ft. 8 in., 3 ft. 6 in.
12. 3 ft. $4\frac{1}{2}$ in., 2 ft. $6\frac{1}{2}$ in.

13. What is the area of an equilateral triangular flower bed with a perimeter of 36 ft. and a height of 10.4 ft.?

14. What is the area of a triangular lot which has a height of 6.4 rd. and a base of 17 rd.?

15. In a field in the shape of a right triangle the sides of the right angle are 30 rd. and 40 rd. Find the area.

HOME, SCHOOL, AND OUTDOOR WORK

1. Bring to class tomorrow three examples of measurements which you have made or which have been made in your home or your school when buying or making articles. If you are doing work in domestic science or in manual training, you may find some good examples there.

2. Tell the class tomorrow about the shape of one of the best-kept lots near the school. Tell how you would go to work to measure in order to find the area of the sidewalk, the area of the lot, and the area covered by the house. Tell why it is one of the best-looking lots near the school.

3. How should you find the area of the school lot? of your schoolroom? of the ground covered by the school building? If the teacher so directs, make the actual measurements to the nearest foot.

4. Draw the schoolroom floor to scale, using $\frac{1}{4}$ in. to 1 ft. Divide it into square feet, like the figure on page 271. See how much easier it is to find the number of squares by multiplying than by counting, and give the area of the floor in square feet.

5. Draw the school lot to scale, using $\frac{1}{8}$ in. to 10 ft. Find the area in square feet; in square yards.

6. Making your measurements only to the nearest inch, draw the top of your desk to scale, using 1 in. to 8 in. What is the area of the top of the desk?

7. A line from one corner of your desk to the opposite corner would divide the top into two triangles. What would be the area of each triangle?

IV. INTRODUCTION TO PERCENTAGE

A SILENT READING LESSON

The captain of the sixth-grade baseball team found that he could buy a catcher's mitt for $2. The clerk told him that if he could wait until the next day, these mitts would be marked down 10 per cent. The captain had to ask the clerk what this meant. From this we see that we ought to know what "per cent" means.

A radio set was advertised before the Christmas holidays at $35. After the holiday business, it could be bought at "20 per cent off." If we were buying a radio set, it would be worth something to us to know what is meant by the expression "20 per cent off."

In the newspaper Tom's mother saw an advertisement of a sale of boys' suits. The regular price of the suits was $15, but they were being marked down 20 per cent. If she buys Tom a suit, she needs to know what the advertisement means.

In making a report of the number of pupils tardy or absent last week, the teacher said that 5 per cent of the pupils had been tardy and that 2 per cent had been absent. We ought to know what this statement means.

A man wished to buy an automobile. He could buy a new one for $1160, but one nearly new was offered him for 25 per cent less than this. We ought to know how to find the price of the second car, but to do so we must know what "25 per cent less" means.

Per cents are frequently used, and we ought to know what they mean.

Per Cent. Another name for "hundredths" is *per cent.* Thus, instead of saying "ten hundredths" we may, if we wish, say "ten per cent." The two expressions have the same meaning.

When the captain of the baseball team found that catchers' mitts which cost \$2 were to be marked down 10 per cent, it meant that a mitt could be bought for $\frac{10}{100}$, or $\frac{1}{10}$, less than \$2; that is, for \$2 – \$0.20, or \$1.80.

We use the symbol % instead of the words "per cent." For example, we write **40%** for "40 per cent," or "0.40."

Hundredths as Per Cents. Because hundredths and per cents are the same, any fraction with the denominator 100 may be written with the symbol %. For example,

$$\frac{45}{100} = 0.45 = 45\% \quad \text{and} \quad \frac{235}{100} = 2.35 = 235\%$$

READING PER CENTS

All work oral

Read the following numbers as per cents:

1. $\frac{3}{100}$.
2. $\frac{5}{100}$.
3. $\frac{6}{100}$.
4. $\frac{8}{100}$.
5. $\frac{10}{100}$.
6. $\frac{18}{100}$.
7. $\frac{28}{100}$.
8. $\frac{37}{100}$.
9. $\frac{44}{100}$.
10. 0.50.
11. 0.65.
12. 0.75.
13. 0.82.
14. 0.95.
15. 1.00.

Read the following per cents as hundredths:

16. 10%.
17. 30%.
18. 38%.
19. 45%.
20. 60%.
21. 75%.
22. 82%.
23. 98%.
24. 200%.
25. 300%.

26. Read as per cents: $\frac{12\frac{1}{2}}{100}, \frac{62\frac{1}{2}}{100}, \frac{\frac{1}{4}}{100}, \frac{33\frac{1}{3}}{100}, \frac{\frac{1}{8}}{100}, \frac{66\frac{2}{3}}{100}$.

Halves and Fourths as Per Cents. Mr. Sinclair sold an automobile for 50% of its cost. What does this mean? He sold the car for how many hundredths of what it had cost him? Express this as a fraction in lowest terms.

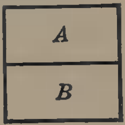
If a baseball team played 48 games and lost 25% of them, how many games were lost? How many hundredths of the games were lost? How many fourths?

We thus see that 50% = $\frac{50}{100} = \frac{1}{2}$, and 25% = $\frac{25}{100} = \frac{1}{4}$.

HALVES AND FOURTHS AS PER CENTS

All work oral

1. If this square represents a floor containing 100 sq. ft., how many square feet are there in the part marked *B*? Then *B* is how many hundredths of the square? What per cent of the square is *B*? Then *A* is what per cent of the square?

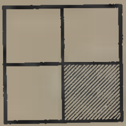


2. If asked to find 50% of a number, what fraction of the number may you find instead? Give an example to illustrate your answer.

Find 50% of each of the following numbers:

3. 30. **4.** 36. **5.** 48. **6.** 60. **7.** 300.

8. If this square represents a box cover containing 100 sq. in., how many fourths are shaded? How many hundredths are shaded? What per cent is shaded?



Find 25% of each of the following numbers:

9. 16. **11.** 40. **13.** 48. **15.** 72. **17.** 400. **19.** 4000.
10. 36. **12.** 44. **14.** 60. **16.** 80. **18.** 800. **20.** 8000.

Important Per Cents. If the sixth-grade team plays 8 games and loses 50% of them, how many does it lose?

If it loses 25%, how many games does it lose?

How much is $\frac{1}{2}$ of 25? $\frac{1}{2}$ of 25%? If the team loses $12\frac{1}{2}$% of 8 games, how many games does it lose?

It is important that we should know that

$\frac{1}{2} = 0.50 = 50\%$ $\frac{1}{4} = 0.25 = 25\%$ $\frac{1}{8} = 0.12\frac{1}{2} = 12\frac{1}{2}\%$

HALVES, FOURTHS, AND EIGHTHS

1. A school has 284 pupils, and 50% of them are girls. How many girls are there in the school?

2. A boy had 68 marbles and lost 25% of them. How many did he lose, and how many did he have left?

3. Fred made 48 runs in this year's games, and Jack made $12\frac{1}{2}$% more. How many runs did Jack make?

4. A school has 488 pupils, and $12\frac{1}{2}$% were tardy last week. How many were tardy?

5. A farmer sold $12\frac{1}{2}$% of the 104 sheep which he owned. How many sheep were sold?

Find the value in each of the following cases:

6. 50% of 2078.
7. 25% of 3572.
8. 50% of 16,390.
9. 25% of $4416.72.
10. 50% of 1276 bu.
11. 25% of 84 ft. 4 in.
12. $12\frac{1}{2}$% of $99.28.
13. $12\frac{1}{2}$% of $97.76.

14. A man owned 128 A. of land and sold $12\frac{1}{2}$% of it at $125 an acre. How many acres did he sell? How much did he receive for what he sold?

Other Important Per Cents. When handkerchiefs are marked down $33\frac{1}{3}\%$ at a bargain sale, what fraction are they marked down?

How much is $\frac{1}{3}$ of 100? How much is $\frac{2}{3}$ of 100?

How much is $\frac{1}{2}$ of $\frac{1}{3}$? How much is $\frac{1}{6}$ of 100?

It is important that we should know that

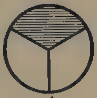
$$\frac{1}{6} = 0.16\frac{2}{3} = 16\frac{2}{3}\% \qquad \frac{1}{3} = 0.33\frac{1}{3} = 33\frac{1}{3}\%$$

$$\frac{2}{3} = 0.66\frac{2}{3} = 66\frac{2}{3}\%$$

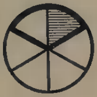
THIRDS AND SIXTHS

1. If circle *A* is called 100, how much is the shaded part? The shaded part is what per cent of the circle?

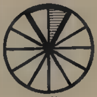
2. Read $0.33\frac{1}{3}$, using the words "per cent"; using the word "hundredths." How many thirds are there in 1? How many times is $33\frac{1}{3}\%$ contained in 1?


A

3. If circle *B* is called 100, the shaded part is what part of $33\frac{1}{3}$? How much is this? Then the shaded part is what per cent of the circle?


B

4. If circle *C* is called 100, the shaded part is what part of $16\frac{2}{3}$? How much is this? Then the shaded part of the circle is what per cent of the whole circle? Then $\frac{1}{12}$ is what per cent?


C

Find the value in each of the following cases:

5. $33\frac{1}{3}\%$ of 894.

6. $66\frac{2}{3}\%$ of \$3714.30.

7. $16\frac{2}{3}\%$ of \$1234.56.

8. $66\frac{2}{3}\%$ of 1047 yd.

9. $16\frac{2}{3}\%$ of 7386 bu.

10. $33\frac{1}{3}\%$ of 6528 mi.

Other Important Per Cents. We should also know that

$20\% = \frac{1}{5}$ $60\% = \frac{3}{5}$ $75\% = \frac{3}{4}$ $62\frac{1}{2}\% = \frac{5}{8}$

$40\% = \frac{2}{5}$ $80\% = \frac{4}{5}$ $37\frac{1}{2}\% = \frac{3}{8}$ $87\frac{1}{2}\% = \frac{7}{8}$

Study all these until you know them perfectly.

IMPORTANT PER CENTS

Numbers 1 to 9, oral

1. In a school of 465 pupils 20% of them are in the sixth grade. How many are in this grade?

2. After you have completed $37\frac{1}{2}\%$ of an automobile trip of 320 mi., how far have you gone?

3. In a school of 500 pupils 60% are in the first four grades. How many pupils are in those grades?

Find the value in each of the following cases:

4. 20% of 45.
5. 40% of 35.
6. 60% of 50.
7. 80% of 50.
8. 75% of 400.
9. $87\frac{1}{2}\%$ of 1600.

10. After finishing $37\frac{1}{2}\%$ of a railroad journey of 344 mi., how many miles have you gone?

11. A contractor has completed $62\frac{1}{2}\%$ of a job for which he is to receive $2720. If he is paid for the work completed, how much does he receive?

Find the value in each of the following cases:

12. $12\frac{1}{2}\%$ of 320 ft.
13. $37\frac{1}{2}\%$ of 248 mi.
14. $62\frac{1}{2}\%$ of 360 ft.
15. 10% of 10%.
16. 20% of $62\frac{1}{2}\%$.
17. 40% of $87\frac{1}{2}\%$.

ORAL DRILL CHART IN PER CENTS

State the value rapidly in each of the following cases:

1. $\frac{1}{4}$ of 20.
2. 25% of 20.
3. 25% of 32.
4. $\frac{1}{8}$ of 32.
5. $12\frac{1}{2}\%$ of 32.
6. $12\frac{1}{2}\%$ of 48.
7. $37\frac{1}{2}\%$ of 48.
8. $87\frac{1}{2}\%$ of 80.
9. $12\frac{1}{2}\%$ of 16.
10. 25% of 16.
11. $37\frac{1}{2}\%$ of 16.
12. 50% of 16.
13. $62\frac{1}{2}\%$ of 16.
14. $87\frac{1}{2}\%$ of 16.
15. $66\frac{2}{3}\%$ of 30.
16. $\frac{1}{5}$ of 150.
17. 20% of 150.
18. $\frac{2}{5}$ of 50.
19. 40% of 50.
20. 60% of 50.
21. 80% of 50.

Read the following, supplying the missing numbers:

22. $\frac{1}{4}$ of 16 = ——.
23. $\frac{1}{4}$ of —— = 10.
24. —— of 16 = 4.
25. —— of 16 = 2.
26. 25% of 40 = ——.
27. ——% of 40 = 4.
28. ——% of 44 = 11.
29. ——% of 40 = 20.
30. $\frac{1}{2}$ of 24 = ——.
31. 50% of —— = 12.
32. $\frac{1}{2}$ of 100 = ——.
33. $\frac{1}{2}$ of —— = 100.
34. 50% of 200 = ——.
35. $\frac{1}{8}$ of 160 = ——.
36. $\frac{1}{8}$ of —— = 10.
37. 50% of —— = 50.
38. ——% of 400 = 200.
39. ——% of 400 = 100.
40. $12\frac{1}{2}\%$ of 24 = ——.
41. $12\frac{1}{2}\%$ of —— = 5.
42. $\frac{3}{4}$ of 40 = ——.
43. 75% of 40 = ——.
44. ——% of 80 = 20.
45. ——% of 80 = 60.
46. 100% of 37 = ——.
47. 100% of —— = 29.

WRITTEN REVIEW OF PER CENTS

Find the value in each of the following cases:

1. $\frac{1}{2}$ of $4800.

2. 50% of $4800.

3. $\frac{1}{4}$ of $120.

4. 25% of $120.

5. 25% of $125.

6. 25% of $125.40.

7. 75% of $125.40.

8. $\frac{1}{3}$ of $4200.

9. $33\frac{1}{3}$% of $4500.

10. $\frac{1}{8}$ of $1680.

11. $12\frac{1}{2}$% of $1688.16.

12. $37\frac{1}{2}$% of $1688.16.

13. $62\frac{1}{2}$% of $3208.64.

14. $87\frac{1}{2}$% of $3208.64.

15. 25% of $172.80 is $4.32, $43.20, $4320, or $432. Which one is it?

16. 25% of $3364.40 is $1345.76, $1682.20, $841.10, or $168.22. Which one is it?

17. Is $33\frac{1}{3}$% of $1575.30 equal to $51.41, $50.51, or $52.51? If it is not equal to any of these, what change in the decimal point would make one of them correct, and which one is it?

18. A dealer told me that I could buy a coat for 75% of the regular price. If the regular price was $36, what would the coat cost me?

19. If a dealer told me that I could buy a coat for $15, and that this price was 50% of the regular price, what was the regular price of the coat?

20. If you buy a baseball from Ben for 50¢, which is 50% of what he paid for it, how much did Ben pay? How much less does Ben receive than he paid?

Per Cents and Fractions. Since per cent means hundredths, we may write

$$62\tfrac{1}{2}\% = \frac{62\frac{1}{2}}{100} = \frac{\frac{125}{2}}{100} = \frac{125}{200} = \frac{5}{8}.$$

To express a per cent as a fraction, write the number indicating the per cent as the numerator and 100 as the denominator, and reduce to lowest terms.

In multiplying by 13%, it is easier to use 0.13; but in multiplying by $33\frac{1}{3}\%$, it is easier to use $\frac{1}{3}$.

We know that $\frac{2}{3}$ may be reduced to hundredths by multiplying each term by $33\frac{1}{3}$. We then have

$$\frac{2}{3} = \frac{66\frac{2}{3}}{100} = 66\tfrac{2}{3}\%.$$

To express a fraction as per cent, change to hundredths and then write the numerator and the symbol %.

We may also divide the numerator by the denominator, as in

$$\tfrac{2}{3} = 2.00 \div 3 = 0.66\tfrac{2}{3} = 66\tfrac{2}{3}\%.$$

PER CENTS AND FRACTIONS

Express as fractions in lowest terms:

1. 36%.
2. 64%.
3. 25%.
4. $6\frac{1}{4}\%$.
5. 30%.
6. 125%.
7. 18%.
8. 375%.
9. $82\frac{1}{2}\%$.
10. $33.3\frac{1}{3}\%$.

Express each of the following numbers as per cents:

11. $\frac{3}{8}$.
12. $\frac{5}{8}$.
13. $\frac{7}{8}$.
14. $\frac{1}{5}$.
15. $\frac{2}{5}$.
16. $\frac{3}{5}$.
17. $\frac{4}{5}$.
18. $\frac{1}{16}$.
19. $\frac{3}{16}$.
20. $\frac{17}{16}$.
21. 1.
22. $3\frac{1}{2}$.

FRACTIONS, DECIMALS, AND PER CENTS

1. If you are asked to find 25% of a number, should you multiply by 0.25, or should you divide by 4? Why will the two results be the same? Which is the easier method?

2. If you are asked to find 50% of a number, what is the easiest way of proceeding? Illustrate your answer by finding 50% of $480.

3. If you are asked to find 75% of a number, how should you proceed? Is this the easiest way? Illustrate your answer by finding 75% of $240.

Copy the following tables, filling all the blank spaces:

	Fraction	Decimal	Per Cent
4.	$\frac{1}{2}$	0.50	50%
5.	$\frac{1}{4}$		
6.		0.75	
7.			$12\frac{1}{2}\%$
8.	$\frac{3}{8}$		
9.		$0.62\frac{1}{2}$	
10.			$87\frac{1}{2}\%$

	Fraction	Decimal	Per Cent
11.	$\frac{1}{3}$		
12.		$0.66\frac{2}{3}$	
13.			20%
14.		0.30	
15.	$\frac{2}{5}$		
16.		$0.16\frac{2}{3}$	
17.			$83\frac{1}{3}\%$

18. What is the shortest way of finding $\frac{1}{100}$ of a number? 1% of a number? 0.01 of a number? Illustrate by taking these parts of 125.

19. What is the shortest way of finding $\frac{1}{100}$ of 112? of finding $\frac{2}{100}$ of 112? of finding 3% of 112?

A SILENT READING LESSON

That part of arithmetic which relates to per cents is called *percentage.* In school, the word "percentage" is sometimes used to mean a per cent of a number.

For example, 10% of \$200 is \$20, and the \$20 is sometimes called the percentage, but not by business men.

Suppose that the regular price of a dress that Mary wishes to buy is \$16, but tomorrow it is to be marked down 15%. To find how much the dress will cost Mary, we must find a per cent of a number.

Since 15% = 0.15, we multiply as here shown, and find that the dress is to be marked down \$2.40.

Subtracting \$2.40 from \$16, we have \$13.60, the cost of the dress.

$$\begin{array}{r} \$16 \\ \underline{0.15} \\ 80 \\ \underline{16} \\ \$2.40 \end{array} \qquad \begin{array}{r} \$16.00 \\ \underline{2.40} \\ \$13.60 \end{array}$$

In finding a per cent of a number, express the per cent as a decimal and multiply the number by this decimal.

John finds that a camping suit, originally costing \$12, has been marked down $12\frac{1}{2}\%$. In finding $12\frac{1}{2}\%$ of \$12, John sees that it is easier to take $\frac{1}{8}$ of \$12 than $0.12\frac{1}{2} \times \$12$.

That is, the suit has been marked down \$1.50 and will therefore cost John

$$\$12.00 - \$1.50, \text{ or } \$10.50.$$

$$\begin{array}{r} \$1.50 \\ 8\overline{)\$12.00} \end{array} \qquad \begin{array}{r} \$12.00 \\ \underline{1.50} \\ \$10.50 \end{array}$$

If we wish to find 50%, 25%, $12\frac{1}{2}\%$, $16\frac{2}{3}\%$, $33\frac{1}{3}\%$, or $66\frac{2}{3}\%$ of a number, it is easier to use a fraction. By so doing we can frequently do the work mentally.

FINDING PER CENTS

Numbers 1 to 6, oral

1. If you have 80 words in a spelling test and are marked 90%, how many words did you spell correctly?

2. If you have 20 problems in arithmetic and are marked 90%, how many answers were correct?

3. If you have 40 questions in geography and are marked 75%, how many answers were correct?

4. If a dealer pays 4¢ apiece for pencils and sells them at 50% more than he paid, what must you pay for one?

5. If a milk dealer pays 10¢ a quart for milk and sells it at 20% more than it costs him, how much per quart does he receive for the milk?

6. If a butcher buys meat at 24¢ a pound and sells it at $33\frac{1}{3}$% more than it costs him, at what price per pound does he sell the meat?

7. In the world's championship series in a certain year the players' share was $135,000, of which the Giants received 60%. How much money did the Giants receive?

8. The players on the St. Louis team were at bat 4812 times in one season, and the base hits were 25% of the times at bat. How many base hits did they make?

9. A dealer in coats and suits advertised a reduction of 20% on overcoats. Find the selling price of a coat that, before the reduction, had sold for $100.

In Ex. 9 find the price of a coat that had sold for

10. $75. **11.** $65. **12.** $60. **13.** $50. **14.** $40. **15.** $30.

DRILL CHART IN FINDING PER CENTS

Find the following per cents:

1. 5% of \$48.
2. 2% of \$45.
3. 7% of \$140.
4. 6% of \$130.
5. 5% of \$1750.
6. 4% of \$2500.
7. 3% of \$6000.
8. $2\frac{1}{2}\%$ of \$8000.
9. $3\frac{1}{2}\%$ of \$5000.
10. 10% of \$7000.
11. 15% of \$48,000.
12. 22% of \$64,000.
13. 30% of \$1200.
14. 33% of \$1000.
15. $33\frac{1}{3}\%$ of \$1800.
16. $66\frac{2}{3}\%$ of \$1800.
17. $16\frac{2}{3}\%$ of \$4800.
18. $1\frac{1}{2}\%$ of \$2500.
19. $1\frac{1}{4}\%$ of \$3600.
20. $2\frac{1}{8}\%$ of \$4800.
21. $2\frac{3}{8}\%$ of \$72,800.
22. $5\frac{7}{8}\%$ of \$65,600.
23. 2.8% of \$25,000.
24. 3.7% of \$360,000.

25. Find $\frac{1}{2}$ of 100; $\frac{1}{2}\%$ of 100; 50% of 100.
26. Find $\frac{1}{2}$ of 400; $\frac{1}{2}\%$ of 400; 50% of 400.
27. Find 6% of \$100; 0.6% of \$100; 60% of \$100.

Find the value in each of the following cases:

28. 5% of \$200; 50% of \$200; $\frac{1}{2}$ of \$200.
29. 25% of \$1000; 2.5% of \$1000; $\frac{1}{4}\%$ of \$1000.
30. 1.25 × 200; $\frac{125}{100}$ of 200; 125% of 200.
31. 250 × 800; 250% of 800; $2\frac{1}{2} \times 400$.
32. 375 × 8000; 3.75 × 8000; 375% of 8000.

PROBLEMS ABOUT GEOGRAPHY

1. When the population of Texas was 4,500,000, it is said that 25% of the people lived within 100 mi. of Dallas. How many lived within that distance at that time?

2. The population of Alabama at one time was 2,140,000, and ten years later it had increased 9.3%. Find the population of the state then.

3. The receipts at the New Orleans docks were 125 carloads of freight on one day and 20% less the next day. What were the receipts on the second day?

4. Of 575 T. of ore shipped to Seattle in one day, 15% was sent by steamer. How much was sent by steamer?

5. If coal can be found under about 63% of the land in Illinois, which has an area of about 56,600 sq. mi., how many square miles of coal lands are there?

6. The area of the United States is 3,600,000 sq. mi., approximately, of which about $7\frac{1}{2}$% is in Texas. Find the number of square miles in Texas.

7. The total farm land of the United States contains about 880,000,000 A., of which 13% is in Texas. Find the number of acres of farm land in Texas.

8. Mt. McKinley, in Alaska, is 20,300 ft. high, and Blue Mountain, the highest elevation in Arkansas, is about 13.8% as high. Find the height of Blue Mountain to the nearest 100 ft.

9. North Truchas Peak in New Mexico is 13,306 ft. high, and East Peak in Nevada is about 1.2% lower. Find the height of East Peak to the nearest foot.

PROBLEMS ABOUT SCHOOLS

1. Of the 350 pupils enrolled in a certain school, 8% are in the sixth grade and 12% are in the fifth grade. How many pupils are there in each of these grades?

2. Of the 440 pupils in a certain school, 5% have been either tardy or absent this month. How many pupils have been either tardy or absent?

3. In a school which had 390 pupils last year, the increase in the enrollment this year is $3\frac{1}{3}$%. How many pupils are there in the school this year?

4. If it is expected that there will be an increase of 4% in enrollment next year, how many pupils should be planned for in a school which now has 250 pupils?

5. In a school of 300 pupils $4\frac{2}{3}$% are absent today. How many pupils are absent?

6. If 15% of the 240 pupils of a school are new pupils this year, how many new pupils are there?

7. The class in cooking finds that food which costs $8.84 at one store can be bought for 12% less at another. How much can be saved by buying at the second store?

8. If there are 25 pupils in a class, and 40% are boys, what fractional part of the class are boys? How many boys are there? If 60% are girls, how many girls are there?

9. A class in industrial arts visiting a factory learned from the foreman that there was a shrinkage of $\frac{1}{2}$% in washing a certain kind of cloth. How much would 50 yd. of the cloth shrink? What would be the length of the cloth after it was washed?

One Number as a Per Cent of Another. For example, suppose that you spell correctly 95 words out of 100. Evidently, you have spelled correctly $\frac{95}{100}$, or 0.95, of the words. But 0.95 = 95%; that is, 95% of them are correct.

Similarly, if you spell correctly 9 words out of 10, you have $\frac{9}{10}$ correct. But $\frac{9}{10}$ = 0.9 = 0.90; that is, 90% of the words are correctly spelled.

Again, if you have 100 examples to solve, and you solve them all correctly, you have solved correctly $\frac{100}{100}$ of the examples; that is, you have solved correctly 100% of them.

From these three examples we can see that

To find what per cent one number is of another divide the first by the second.

Thus, to find what per cent 95 is of 100, divide 95 by 100. The result is $\frac{95}{100}$, or 0.95; that is, it is 95%.

To find what per cent 4 is of 5, divide 4 by 5. The result is $\frac{4}{5}$, or 0.80; that is, it is 80%.

The whole of anything is $\frac{100}{100}$, or 100%, of it.

ONE NUMBER AS A PER CENT OF ANOTHER

Numbers 1 to 20, oral

State what per cent the first number is of the second:

1. 5, 100.
2. 8, 100.
3. 15, 100.
4. 25, 100.
5. 100, 100.
6. 10, 50.
7. 20, 100.
8. 125, 100.
9. 200, 100.
10. 250, 100.
11. 1, 10.
12. 2, 10.
13. 3, 10.
14. 1, 5.
15. 10, 10.
16. 5, 5.
17. 1, 3.
18. 1, 8.
19. $12\frac{1}{2}$, 100.
20. $33\frac{1}{3}$, 100.

21. If you miss 3 words out of 50 in a spelling test, what fraction of the words do you miss? How many hundredths do you miss? What per cent do you miss?

22. If you spell correctly 45 words out of 50, what per cent of the words do you spell correctly? What per cent do you miss? What per cent of the words is the sum of the number missed and the number spelled correctly?

23. If 2 out of a class of 40 are absent, what per cent are absent? What per cent are present? What per cent is equal to the sum of these two per cents?

Find what per cent the first number is of the second:

24. 25, 50.
25. 12, 24.
26. 35, 70.
27. 6, 12.
28. 10, 20.
29. $37\frac{1}{2}$, 50.
30. $62\frac{1}{2}$, 50.
31. 50, $62\frac{1}{2}$.
32. 50, $37\frac{1}{2}$.
33. 37, 74.
34. 12, 10.
35. 10, 12.
36. 15, 10.
37. 10, 15.
38. 20, 10.
39. 275, 100.
40. 100, 300.
41. 300, 100.
42. 150, 300.
43. 200, 400.

44. After you have gone 37.5 mi. on a trip of 75 mi., what per cent of the distance have you gone?

45. If 15 boys out of a school of 150 boys are selected for the baseball team, what per cent are selected?

46. Only 9 boys can play at a time on the team in Ex. 45. What per cent of the 15 boys play in any inning?

47. If your club wins 18 games out of 24, what per cent does it win? What per cent does it not win?

48. If the girls in a school of 350 number 182, what per cent of the pupils are girls? What per cent are boys? If you add the two per cents, what is the result?

RUNNING A STORE

1. If, in your store, you buy hunting knives at 90¢ each and sell them at $1.35 each, how much more than the cost do you receive? This is what per cent of the cost?

2. Suppose that the expenses of carrying on your business in Ex. 1 (such as rent, hiring clerks, light, and other items) amount to 40% of the selling price, how much should you charge up as expenses in selling the knife?

3. Comparing the gain on the knife as found in Ex. 1 with the cost of doing business as found in Ex. 2, do you make money or lose money on the sale, and how much? This illustrates a very important point in business. One reason why merchants often fail to succeed is that they do not consider the cost of doing business. The estimate of 40% given in Ex. 2 is about what is often allowed.

4. If, in your store, you buy skates for $1.60 a pair and sell them at $2.80, you gain what per cent of the cost? If it costs you 40% of the selling price to do business, do you gain or lose on the skates, and how much per pair?

5. If you buy skates at $1.60 and sell them at twice that price, it looks as if you were making a profit of 100%. But if you deduct 40% of the selling price for expenses, what amount per pair do you receive after making this allowance? This amount is how much more than the cost ($1.60)? Your profit is what per cent of the cost?

6. If, after deducting for expenses, you receive $4.86 when you sell a tennis racket that costs you $4.50, what is your per cent of profit on the cost?

V. USES OF PER CENTS

A SILENT READING LESSON

If a person buys or sells property for another person, he acts as an *agent* and is entitled to be paid for his work. This payment is usually a certain per cent of the amount involved in the sale.

Suppose that you sell a house for Mr. X for $12,000. If your *commission* is 2%, you are paid 2% of $12,000, or $240 for your part in the transaction. In this case you are called a *real-estate agent* or a *realtor*.

Such agents find purchasers that the person wishing to sell the property cannot easily find. If he could find them himself, he would save the commission; but it might easily cost him as much in the end as he would pay the agent.

Suppose that your business is receiving farm produce from farmers and selling it for them. In this case you are called a *commission merchant*. If you sell some produce for $350 and receive 5% as your commission, you receive 5% of $350, or $17.50. You then send to the farmer $350 – $17.50, or $332.50, which is called the *proceeds* or *net amount* of the sale.

There is no great difference between a real-estate agent and a commission merchant in the matter of payment. The payment in each case depends upon the amount of money involved in the sale.

Besides their regular wages or salaries, the clerks in large stores are often paid a commission on the amount of the sales that they make to customers.

PROBLEMS IN COMMISSION

1. Suppose that you are a real-estate agent and that you sell a house for me for $8000. If you charge a commission of 2%, how much commission do I pay you?

2. If, as a real-estate agent, you sell my farm for $12,500 and charge me a commission of $2\frac{1}{4}$%, how much commission do you receive?

3. Suppose that you have an old automobile that you wish to sell, and that I can find a purchaser who is willing to pay $550 for it. If I charge you 5% for my services, how much will my commission be? What will be the proceeds that you receive from the sale?

In each of the following, which show the amounts of sales at different rates of commission, find the amount of commission paid:

4. $1000, 2%.
5. $7500, $2\frac{1}{2}$%.
6. $8750, 3%.
7. $9875, 4%.
8. $12,500, $2\frac{1}{4}$%.
9. $16,750, $3\frac{1}{2}$%.
10. $15,000, $3\frac{3}{4}$%.
11. $18,600, $4\frac{1}{4}$%.
12. $125.50, 2%.
13. $275.50, 4%.
14. $9840, $4\frac{1}{2}$%.
15. $7820, $6\frac{1}{2}$%.

In each of the following, which show the amounts of sales at different rates of commission, find the commission to the nearest cent and then find the net amount of the sale:

16. $280, $2\frac{1}{2}$%.
17. $460, 3%.
18. $540, $3\frac{1}{2}$%.
19. $680, $4\frac{1}{4}$%.
20. $720, $3\frac{3}{4}$%.
21. $820, $2\frac{1}{4}$%.
22. $580, $2\frac{3}{4}$%.
23. $1675, 4%.
24. $175.50, 2%.
25. $325.75, $2\frac{1}{4}$%.
26. $428.60, $2\frac{3}{4}$%.
27. $129.72, $3\frac{1}{4}$%.

A SILENT READING LESSON

If during a year a merchant receives more money from his sales than it costs him to buy goods and to pay the running expenses of his store, the business makes a profit; if he receives less money, the business is run at a loss.

Expenses like rent, heat, light, insurance, salaries, repairs, and so on are often called *overhead charges*, or simply *overhead*. We shall call all of these the "expenses of doing business," or simply "expenses."

Some merchants express profits and losses as per cents of what their goods cost them.

Suppose that a merchant, whose expenses average 32% of the cost of goods, buys 960 yd. of linen lawn at 87½¢ a yard and sells it at $1.25 a yard. This is the way he solves the problem of finding his profit on the transaction:

$$\frac{7}{\cancel{8}} \times \overset{120}{\cancel{960}} = 840 \text{ (cost)}$$

$$\begin{array}{r} 840 \\ \underline{1.32} \\ 1680 \\ 2520 \\ \underline{840} \\ 1108.80 \text{ (cost + expenses)} \end{array} \qquad \begin{array}{r} 960 \\ \underline{1\tfrac{1}{4}} \\ 240 \\ \underline{960} \\ 1200 \text{ (sales)} \\ \underline{1108.80} \\ 91.20 \text{ (profit)} \end{array}$$

The merchant did not use the dollar signs, and in finding 960 × 87½¢, he took $\frac{7}{8}$ of $960. Since the expenses were 32% of the cost of goods, he saw that to find the amount of the cost plus expenses, he could multiply by 1.32, instead of multiplying by 0.32 and then adding $840.

PROBLEMS IN PROFIT OR LOSS

1. On some goods which cost $4920 a merchant wishes to make a profit of 10% of the cost. Find the amount of the profit that he wishes to make.

2. The merchant in Ex. 1 knows that his expenses average 20% of the cost of the goods he sells. How much must he allow for expenses in Ex. 1? For how much should he sell the goods if he is to clear his expenses and make the desired profit?

3. A man sold a house for which he had paid $11,520 at a loss of 4%. How much was his loss? If he spent $300 for repairs and $180 for other expenses before selling the house, how much was his total loss?

4. A dealer pays $4.32 a gross for penholders and sells them at a price which is $66\frac{2}{3}$% more than the cost. Find the price at which he sells each penholder.

5. A hardware dealer bought a lot of tools for $1937.50. His expenses of doing business average 18% of the cost of goods. If on the lot he wishes to make a profit of 12% of the cost, for how much should he sell the tools?

6. A dealer buys 80 sewing machines at $72 apiece and sells them all at $96 apiece. He pays $36 freight on the lot, and his other expenses are $9.40 per machine. Find his profit on the lot.

7. A merchant buys 12 gross of spools of silk at $9.76 a gross. He sells the entire lot of spools at $1.20 a dozen. Allowing 20% of the cost for expenses, how much is the profit that he makes?

8. A merchant buys 840 yd. of muslin at 28¢ a yard and sells it at the rate of a dozen yards for $4.25. His expenses are 3¾¢ per yard. How much is his profit or loss?

9. A dealer buys 740 yd. of scrim for $200, and sells 75% of it at 45¢ a yard and the rest at 30¢ a yard. Allowing 18% of the cost of the goods for expenses, how much is his profit or loss?

10. A dealer buys 380 reams of paper at $2.25. He sells 90 reams, which were damaged, at 85¢, and the rest at $3.25. Allowing 40¢ a ream for expenses, find his profit or loss on the lot.

11. A dealer buys 720 yd. of cloth at $1.36 a yard and wishes to make a profit of about $100 on the lot. Allowing 22% of the cost for expenses, find to the next higher cent the price per yard which he should ask for the cloth.

12. A dealer buys 1600 yd. of Victoria lawn for $600. If on this lot he estimates that the proper share of the expenses of doing business is $108, for how much per yard must he sell the lawn to make a profit of 3¾¢ a yard? He sees that this is an awkward price and so sells the cloth at 50¢ a yard. Find his total profit on the lot.

13. A dry-goods dealer buys 840 yd. of percaline at 38¢ a yard and estimates that his expenses are 5.7¢ a yard. He sells 500 yd. at 50¢ a yard, 120 yd. at 48¢ a yard, and the rest at 28¢ a yard. How much is his profit or loss?

14. A merchant's expenses average 28% of the cost of goods. If he wishes to make a profit of 12% of the cost, for how much per yard should he sell silk for which he pays $3.50 per yard?

Interest for a Year. Imagine that you are going into business and wish to buy a store and its stock of goods. You should have money enough to pay for at least part of the store, and should have experience enough to be reasonably certain of success. You can then borrow the money to pay for the rest of the store.

When people borrow money, they pay a certain per cent for its use. If you borrow $100 for 1 yr., you may be asked to pay 6% of this amount, or $6, for the year. This $6 is called the *interest* on the $100.

If you borrow $200 for 1 yr. at 6%, the interest for the year is 6% of $200, or $12.

If you borrow $300 for 1 yr. at 5%, the interest for the year is 5% of $300, or $15.

INTEREST FOR A YEAR

Find the interest for 1 yr. on each of the following amounts at the rate given:

1. $100, 5%.
2. $100, 7%.
3. $100, 4%.
4. $200, 5%.
5. $200, 6%.
6. $200, 4%.
7. $300, 4%.
8. $300, 5%.
9. $300, 6%.
10. $300, 7%.
11. $50, 6%.
12. $500, 6%.
13. $5000, 6%.
14. $75, 5%.
15. $750, 5%.
16. $7500, 5%.
17. $25, 4%.
18. $250, 4%.
19. $2500, 4%.
20. $2500, 5%.
21. $150, 4%.
22. $150, $4\frac{1}{2}$%.
23. $150, $5\frac{1}{2}$%.
24. $80, 6%.
25. $80, $5\frac{1}{2}$%.
26. $800, $5\frac{1}{4}$%.
27. $8000, $5\frac{3}{4}$%.
28. $5000, $5\frac{1}{4}$%.
29. $6000, $5\frac{3}{4}$%.
30. $6000, $7\frac{1}{2}$%.

Interest for Other Lengths of Time. If you borrow money for 2 yr., the interest is twice as much as for 1 yr.; if you borrow for 6 mo., or ½ yr., the interest is half as much as for 1 yr.; if you borrow for 2½ yr., the interest is 2½ times the interest for 1 yr.

For example, suppose that you wish to borrow $150 for 1 yr. 6 mo.; that is, for 1½ yr.

The interest for 1 yr. will be 0.06 × $150, or $9, and so the interest for 1½ yr. will be 1½ × $9, or $13.50.

FINDING INTEREST

1. Mr. Jackson borrows $300 to help start a gasoline filling station, and keeps it for 2 yr. If he has to pay interest at 6%, how much interest does he pay?

Find the interest for 2 yr. on each of the following amounts at the rate given:

2. $200, 5%.
3. $300, 5%.
4. $500, 4%.
5. $500, 6%.
6. $150, 6%.
7. $150, 5%.
8. $250, 4%.
9. $350, 6%.
10. $150, 5½%.
11. $250, 5½%.
12. $3750, 6%.
13. $5250, 5%.

Find the interest on each of the following amounts for the given time and rate:

14. $100, ½ yr., 6%.
15. $100, 1½ yr., 6%.
16. $100, 2 yr., 6%.
17. $100, 2½ yr., 6%.
18. $500, 2½ yr., 6%.
19. $260, ½ yr., 5%.
20. $480, 1½ yr., 5%.
21. $800, 2½ yr., 5%.
22. $4000, ½ yr., 4%.
23. $6000, 1½ yr., 6%.

A SILENT READING LESSON

Sometimes merchants have bargain sales at which they try to dispose of odd pieces or of goods that are going out of style. At such times they often sell goods at a certain "per cent off" the *marked price,* which is the price marked on the goods.

For example, an article marked \$3.50 and sold at 10% off is sold at \$3.50 less 10% of \$3.50; that is, we have

$$\$3.50 - \$0.35 = \$3.15.$$

A reduction from a price or amount is called a *discount.* If you offer to sell for \$2 a tennis racket that costs \$3, you are offering to sell it at a discount of \$1, or at a discount of $\frac{1}{3}$, or at a discount of $33\frac{1}{3}\%$. The last expression, which states the discount as a per cent, is the common form used in business.

The price of an article after the discount has been taken off is called the *net price* or *selling price* of the article; and, similarly, the amount of a bill after the discount has been taken off is called the *net amount* of the bill.

Business men who sell at wholesale (that is, to dealers) often sell at a discount from the price stated in their catalogues, or from the *list price,* as it is called.

The work at the right shows how we find the net price when a library table listed in a catalogue at \$245 (the list price) is sold at a discount of 32%. We first find the amount of the discount, \$78.40, and subtract it from the list price.

$$\begin{array}{r} \$245 \\ 0.32 \\ \hline \$78.40 \end{array}$$

$$\begin{array}{r} \$245.00 \\ 78.40 \\ \hline \$166.60 \end{array}$$

FINDING DISCOUNTS

Numbers 1 to 9, oral

1. If a baseball mitt that is marked $2.80 can be bought at 10% off this price, what is the amount of the discount? By subtracting this from $2.80, find the net price.

2. If a tennis racket that is marked $4.50 is offered at a discount of 20%, what is the discount? As in Ex. 1, what is the net price?

3. Solve Ex. 2 another way by first taking 20% from 100%, leaving 80%. Then take 80% of $4.50. Tell why this gives the same result as was found in Ex. 2.

Find, as in Ex. 2 and also as in Ex. 3, the net prices when the marked prices and the discounts are as follows:

4. $8, 10%.
5. $12, 25%.
6. $60, $33\frac{1}{3}\%$.
7. $80, 25%.
8. $300, 10%.
9. $500, 20%.

10. On an automobile which sells regularly at $1520 a man is offered a discount of 12%. Find the net price.

11. A suit of clothes marked $33 is advertised to be sold at $33\frac{1}{3}\%$ off. What is the net price?

12. A dealer offers a discount of $33\frac{1}{3}\%$ on Boy-Scout suits marked $17.85 each. Find the net price.

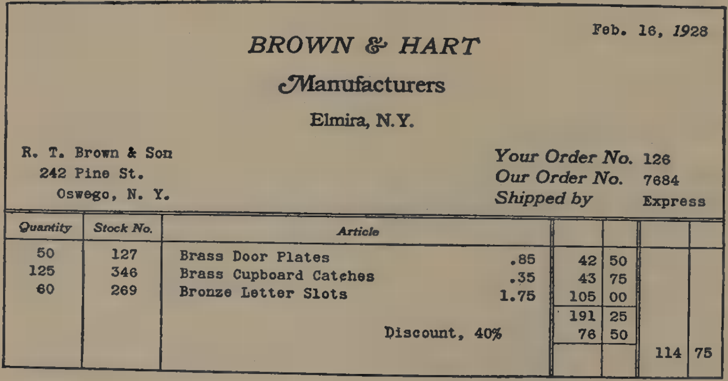
13. If a suit of clothes marked $28 is purchased at a discount of 15%, how much is saved on the marked price?

14. If the cost of the books, pencils, and paper which you use in a year is $8.50 at list prices, how much do they cost at a discount of 8%? Check by taking 92% of $8.50.

A SILENT READING LESSON

When a merchant wishes to order goods for his store, he may do so in several ways. He may give his order to the salesman who visits his store, he may order by telephone, or he may write to the wholesaler or manufacturer. In ordering goods, business houses generally use printed forms, which are filled in to show the quantity, style, prices, and other details of the order.

When the wholesaler or manufacturer ships the goods that the merchant ordered, an *invoice* (which is only another name for a bill) showing all the details of the shipment is mailed to the merchant. The following model shows a simple form of the invoice covering a shipment of goods on which the merchant is allowed a discount of 40% from the list prices in the manufacturer's catalogue:



Feb. 16, *1928*

BROWN & HART

Manufacturers

Elmira, N.Y.

R. T. Brown & Son
242 Pine St.
Oswego, N. Y.

Your Order No. 126
Our Order No. 7684
Shipped by Express

Quantity	*Stock No.*	*Article*					
50	127	Brass Door Plates	.85	42	50		
125	346	Brass Cupboard Catches	.35	43	75		
60	269	Bronze Letter Slots	1.75	105	00		
				191	25		
		Discount, 40%		76	50		
						114	75

In computing discounts, we follow the business practice in regard to fractions of a cent; that is, we count $\frac{1}{2}$¢ or more as a full cent and disregard anything less than $\frac{1}{2}$¢.

FINDING DISCOUNTS

Write invoices for the following orders for goods:

1. 8 chairs @ $6.50; 12 armchairs @ $9.25; 8 rocking-chairs @ $14.50; 24 tables @ $42.75. Discount, $33\frac{1}{3}\%$.

2. 420 yd. satin @ $3.50; 340 yd. flannel @ $1.60; 400 yd. canvas @ 90¢. Discount, $37\frac{1}{2}\%$.

3. 750 yd. lawn @ 90¢; 450 yd. fancy percale @ 60¢; 380 yd. satin @ $4.50. Discount, 15%.

4. 250 yd. serge @ $3.25; 300 yd. cheviot @ $2.75; 175 yd. velvet @ $3.40. Discount, 20%.

5. 75 doz. plates @ $9.25 (per dozen); 60 doz. cups and saucers @ $11.00; 40 doz. tumblers @ $2.40; 12 doz. individual butter plates @ $1.75. Discount, 35%.

Find the amounts of the following discounts:

6. 15% on $4000.
7. 20% on $2400.
8. 10% on $5750.
9. 12% on $7850.
10. 18% on $2235.
11. 15% on $32,350.
12. $33\frac{1}{3}\%$ on $6336.
13. $37\frac{1}{2}\%$ on $7328.
14. 25% on $472.64.
15. $16\frac{2}{3}\%$ on $94.54.

On orders for goods to the following amounts find the net amounts at the discounts given:

16. $375, 20%.
17. $475, 5%.
18. $663, 3%.
19. $850, $12\frac{1}{2}\%$.
20. $886, 16%.
21. $925, 10%.
22. $620, 14%.
23. $32.75, 20%.
24. $4202, 14%.
25. $5672, $12\frac{1}{2}\%$.
26. $58.84, $16\frac{2}{3}\%$.
27. $76.32, 50%.

REVIEW DRILL CHART IN PER CENTS

Find the value in each of the following cases, using fractions or short methods whenever it is easier:

1. 40% of 280.
2. 50% of 380.
3. 60% of 450.
4. 70% of 920.
5. 50% of $480.
6. 30% of $260.
7. 80% of $22.
8. 5% of 320.
9. 8% of 730.
10. 9% of 925.
11. $4\frac{1}{2}$% of 735.
12. 3% of $628.
13. 8% of $448.
14. 9% of $868.
15. $\frac{3}{4}$ of 1236.
16. 75% of 1236.
17. 0.75×1236.
18. $\frac{1}{5}$ of 295.
19. 20% of 295.
20. $\frac{3}{4}$ of 964.
21. 75% of 964.

On orders for goods to the following amounts find the net amounts at the discounts given:

22. $750, 20%.
23. $475, 15%.
24. $880, 25%.
25. $975, 8%.
26. $1640, $12\frac{1}{2}$%.
27. $1775, 24%.
28. $1560, $16\frac{2}{3}$%.
29. $9768, $12\frac{1}{2}$%.
30. $4578, $33\frac{1}{3}$%.

Find the value in each of the following cases:

31. $12\frac{1}{2}$% of $9860.
32. $16\frac{2}{3}$% of $9860.
33. 25% of $16,480.
34. $\frac{1}{2}$% of $12,400.
35. 50% of $12,800.
36. $33\frac{1}{3}$% of $6888.
37. $37\frac{1}{2}$% of $16,888.
38. 50% of $24,770.
39. 23.8% of $42,869.
40. $3\frac{1}{8}$% of $7200.
41. $62\frac{1}{2}$% of $16,896.
42. $5\frac{1}{2}$% of $1200.

VI. REVIEW AND DRILL

MINIMUM ESSENTIALS

1. Write in our common numerals the number sixty million three hundred thousand two hundred nine.

2. Write in our common numerals the number ninety-two and forty-four thousandths.

Add as follows:

3.	**4.**	**5.**	**6.**	**7.**
$7.98	$4.62	$8.43	$3.89	$9.48
.85	3.46	4.84	4.94	8.21
4.79	.34	7.06	7.43	.32
5.68	.62	8.06	8.52	5.87
.52	9.75	9.54	9.48	6.78
.48	8.25	8.46	4.87	.22
8.72	3.38	9.50	5.96	8.95
6.55	4.42	8.72	8.96	2.05

8. Add $98.86 and $74.69 and then subtract $109.57.

9. Add $95.42 and $87.69 and then subtract $129.78.

10. Multiply 48.756 by 0.078 and then add 0.723682.

11. Divide 37.84 by 6.93, finding the result correct to two decimal places; that is, to the nearest hundredth.

Perform the following operations:

12. $\frac{7}{8} + \frac{3}{4}$.

13. $\frac{7}{8} - \frac{1}{4}$.

14. $\frac{1}{2} - \frac{1}{3}$.

15. $\frac{3}{16} + \frac{1}{2}$.

16. $\frac{7}{16} \div \frac{1}{2}$.

17. $\frac{9}{16} \div \frac{3}{4}$.

18. $1\frac{1}{2} + 2\frac{7}{8}$.

19. $3\frac{1}{3} + 2\frac{1}{2}$.

20. 0.7×1.2.

21. 1.7% of 380.

22. 2.3% of 9.25.

23. $1.44 \div 1.2$.

PROBLEMS WITHOUT NUMBERS

All work oral

1. How do you find a fraction of a fraction?

2. How do you find the number of square feet of flooring needed to cover the floor of your schoolroom?

3. How do you find the cost of a sidewalk to be laid in front of the schoolhouse?

4. If you are asked to find the number of square yards of concrete that will be needed for a garage floor, how do you find it?

5. I have measured the length and width of a rectangular room in feet. How do I find the number of square yards of linoleum needed to cover the floor?

6. I know the length, width, and height of a rectangular room in feet. How do I find the number of square yards of plastering needed for the walls and ceiling, making no allowance for the doors and windows?

7. If you know what an automobile dealer pays for a car, and the per cent above cost at which he sells it, how do you find the selling price?

8. If you know the per cent of discount that a dealer is allowed in buying from a wholesaler, how do you find the net amount of the invoice covering an order for goods costing a certain amount?

9. If you know the rate of commission paid a real-estate agent, and the price at which he sells a house, how do you find the amount of the agent's commission?

PROBLEMS FOR COMPLETION

1. Mr. Brown bought at a reduction of 20% an automobile that had been marked $1985. The dealer also allowed him $325 for his old car. Complete the problem in any reasonable way and then solve it.

Complete and solve the following problems:

2. In a recent year the average consumption of meat for each person in this country was as follows: beef, 82.7 lb.; mutton, 7 lb.; pork, 103.5 lb.; other kinds of meat, 0.3 lb.

3. A woman who used gas for cooking bought a fireless cooker, and found that her gas bills, which had been $3.85, $3.80, $3.75, $3.20, and $2.90 for the first 5 mo. of last year, were now $3.10, $3, $2.90, $2.40, and $2.10 for the first 5 mo. of this year.

4. In his garden Fred raised 150 lb. of tomatoes, one third of which he sold to the neighbors at 8¢ a pound, and the rest at 6¢ a pound.

5. John bought a pig for $5.75. He fattened the pig, spending $11.75 for feed. When the pig weighed 240 lb., John sold it at 9¢ a pound.

6. A man who works in a machine shop earns $36 a week. He pays $10 a week for his board and room. His other expenses amount to 20% of his earnings.

7. Mary's recipe for making molasses candy called for 2 cups of molasses, 1 tablespoon of vinegar, 2 tablespoons of butter, ½ teaspoon of baking powder, 1 teaspoon of ginger extract. This was enough to make 1 lb. of candy. She wished to make 3 lb. for a church sale.

Testing Your Alertness. In the three lines below, a certain relation exists between the first two words, and a line is drawn under the word in parentheses that has the nearest similar relation to the third word in the line:

1. Square, side; cube, (area, edge, weight).
2. Foot, inch; pound, (ounce, ton, quart).
3. Bushel, peck; one, (six, sixth, fourth).

ALERTNESS TEST

After you understand numbers 1, 2, and 3, above, see if you can copy the lines given below, underlining the right words in each case, all in 10 min.:

1. Foot, inch; year, (area, month, rod).
2. Mile, rod; year, (day, foot, length).
3. Degree, minute; minute, (second, earth, clock).
4. Inch, foot; one, (tenth, seven, twelve).
5. Day, week; two, (seven, seventh, fourteen).
6. Second, minute; minute, (time, hour, week).
7. Triangle, rectangle; three, (five, seven, four).
8. Quart, pint; six, (ten, twelve, three).
9. One, half; four, (fourth, two, eight).
10. Year, month; foot, (rod, inch, mile).
11. Half, six; three, (ten, thirty-six, forty-eight).
12. Zero, ten; one, (twelve, eleven, ten).
13. Hundredth, one; fiftieth, (half, two, fourth).
14. Hour, week; minute, (day, area, ten).
15. Sale, commission; money, (interest, dollar, cent).
16. Per cent, hundredths; result, (even, odd, answer).

RAISING POULTRY

1. Tom Wallace set 260 eggs and hatched 85% of them. How many chickens hatched?

2. Tom sold 134 of his chickens for a total of $57.62. Find the average price received per chicken.

3. Tom's hens laid 699 eggs in April, 544 in May, 522 in June, 425 in July, 290 in August, and 112 in September. How many dozen eggs did his hens lay in these months?

4. In Ex. 3, Tom sold the eggs to a storekeeper at an average price of 48¢ a dozen. How much did he receive from the sale of his eggs?

5. Tom bought 3 geese at $2 each and paid $3.50 for feed. He raised 13 young geese, which he sold to the butcher at an average price of $1.50 each. How much did Tom make above his expenses?

6. In School District No. 6 there was a poultry club which reported that its members had sold 1345 eggs in March, 1209 in April, and 1142 in May, and that the average price received for the eggs was $42\frac{1}{2}$¢ a dozen. How much did the club members receive from the sale of eggs in these months?

7. In Ex. 6, if there were 77 hens laying eggs, what was the average number of eggs laid per hen during this period?

8. In School District No. 2 the poultry club's records showed that 1276 eggs were sold in March, 1140 in April, and 1070 in May, and that the average price received was $44\frac{1}{2}$¢ a dozen. How much was received from the sale of eggs in these months?

A SILENT READING LESSON WITH PROBLEMS

Last year Tom decided to join the garden club. He bought 25 lb. of onion seed and rented 69 sq. rd. of land from his grandfather. His uncle showed him how to sow the seed. His grandfather told him all about raising onion sets and showed him some of the latest and best methods. Tom followed his grandfather's advice, and the crop was very satisfactory. He gave his mother the money left over after paying the expenses, and she used it in paying for music lessons for Tom and his sister.

The itemized account of Tom's receipts and expenditures appears on page 321. The items are all for this year except the first two expenditures, which were made in November of last year.

If any of us should undertake work like this, we should need to know how to keep an account of the receipts and expenses. There are some interesting questions that can be asked about Tom's work and his account. Let us consider a few of them:

1. Tom had 69 sq. rd. of land. How many pounds of onion sets did he raise per square rod?

2. Considering all Tom's expenses, what was the cost per square rod?

3. At the rate found in Ex. 2, what would be Tom's expenses for an acre?

4. How much per square rod did Tom find that he made above his expenses?

5. At the rate found in Ex. 4, how much could Tom make if he planted an acre?

Copy the following account, completing it by filling in the places marked by the stars:

Receipts

Sept. 24,	6480 lb. onion sets at $5\frac{1}{2}$¢	$ ***
Oct. 1,	580 lb. overruns at $2\frac{1}{4}$¢	$13.20
	Total	***

Expenditures

Nov. 2,	Fertilizer, including hauling	$9.00
Nov. 4,	Plowing	1.80
Feb. 17,	25 lb. onion seed at 75¢	***
Apr. 10,	Preparing land for seeding	1.00
Apr. 10,	Sowing onion seed	.65
May 14,	Wheel-hoeing, $4\frac{1}{2}$ hr. at 15¢	***
May 18,	Weeding, 20 hr. at 15¢	***
May 28,	Wheel-hoeing, $4\frac{1}{2}$ hr. at 15¢	***
June 7,	Weeding, 20 hr. at 15¢	***
Aug. 6,	Loosening sets with harvester	.75
Aug. 6,	Harvesting 485 baskets at 5¢	***
Aug. 6,	5 men for harvesting, stacking, and covering crates, 25 hr. at 40¢	***
Aug. 6,	Tar paper to cover stacks	.59
Aug. 6,	26 lunches at 25¢	***
Aug. 6,	Rent of 114 onion crates at 7¢	***
Sept. 19,	Grading 192 bu. onion sets at 3¢	***
Sept. 19,	Marketing	7.00
Oct. 1,	Rent of land	7.50
	Total	***
	Balance	***

EARNING MONEY

1. Ben planted a piece of land to corn at an expense of \$31. An early frost damaged the crop so that it sold for only \$46.50, or half of what he expected. Ben wanted to put \$100 in the bank. If he had secured a full crop, how much would he have needed to make up the \$100?

2. Horace planted potatoes on 35½ sq. rd. of land, which he rented from his father for 20¢ a square rod. His father gave him the seed potatoes and charged him \$1.25 for plowing. Horace did the rest of the work himself. If he raised 37 bu. of potatoes and sold them at \$1.30 a bushel, how much did he make above expenses?

3. In January, Arnold paid \$5 for some hens and by August had raised 41 chickens that his hens had hatched from their own eggs. He sold the chickens at \$1.50 each. Before he set the hens, he sold 10 doz. eggs at an average price of 60¢ a dozen. He found that the total expenses for care and feed were \$25.50. How much did Arnold make above his expenses?

4. Oscar planted onions in his home garden, paid out \$4.15 for expenses, and sold the entire crop for \$52.25. He then bought a bicycle for \$20 and 2 pigs at \$8 each. How much money did he have left?

5. Harry had a plot of ground 22 ft. wide and 275 ft. long. On it he raised 20 bu. of potatoes at a total expense of \$3.85. He sold the potatoes at \$1.75 a bushel. How much did Harry make above expenses? At this rate, how much might he expect to make from an acre of land?

MISCELLANEOUS PROBLEMS

1. A man makes a profit of $12\frac{1}{2}\%$ in selling property which cost him \$3200. How much is the profit?

2. A typewriter was marked \$90, but was sold at a discount of 15%. What was the net price?

3. An automobile regularly selling for \$1275 was sold at a discount of 12%. What was the net price?

4. A discount of $16\frac{2}{3}\%$ on \$5438.40 is \$90.64, \$9.06, \$9064, or \$906.40. Which is it?

5. A man buys a house for \$8750 and pays 30% down. How much does he pay down? How much does he owe?

6. A man agrees to buy a building lot for \$1575 and pays $33\frac{1}{3}\%$ down. How much has he still to pay?

7. A man bought a house for \$8400 and sold it for 125% of what he had paid for it. How much did he gain?

8. The diameter of a circle is about 31.8% as long as the circumference. Find to the nearest 0.01 ft. the diameter of a circle with a circumference of 200 ft.

9. A meter is about 9.3% longer than a yard. How much does this differ from 39.37 in., which is the equivalent length legally established in this country?

10. The nautical mile, which is used in measuring distances at sea, is about 15.2% longer than the common mile. About how many feet are there in a nautical mile?

11. A merchant who bought goods amounting to \$5062.50 at list prices was allowed a discount of 20%. What was the net amount of the invoice?

VII. LITTLE EXAMINATIONS

I. 1. $62\frac{1}{2}$ ft. = (?) in.
2. $7\frac{1}{4}$ pk. = (?) qt.
3. $42\frac{1}{2}$ gal. = (?) qt.
4. $3\frac{1}{2}$ hr. = (?) min.
5. \$329.75 ÷ 25.
6. 1340 × \$5276.45.
7. 4876 × \$328.72.
8. 875 × \$8642.75.

II. 1. 66 ft. = (?) rd.
2. 63 sq. ft. = (?) sq. yd.
3. 800 sq. rd. = (?) A.
4. 10,560 ft. = (?) mi.
5. 38 × $42\frac{1}{2}$ sq. ft.
6. $9\frac{1}{2}$ × $15\frac{1}{2}$ sq. ft.
7. 5 × 9 ft. 8 in.
8. 9 × 5 lb. 8 oz.

III. 1. 1280 A. = (?) sq. mi.
2. 390 min. = (?) hr.
3. 8 ft. 7 in. + 3 ft. 2 in.
4. 9 ft. 9 in. − 2 ft. 11 in.
5. 11 yd. − 5 yd. 9 in.
6. 8 × 9 ft. 3 in.
7. 12 × 8 yd. 6 in.
8. 16 × $24\frac{1}{2}$ sq. rd.
9. $\frac{1}{2}$ × 8 × 4 sq. ft.
10. $\frac{1}{2}$ × 6 × $8\frac{1}{2}$ sq. ft.

IV. 1. $61\frac{2}{3}$ in. + $3\frac{1}{2}$ in.
2. $7\frac{3}{8}$ in. − $5\frac{3}{4}$ in.
3. $\frac{1}{2}$ of $8\frac{3}{4}$ × $9\frac{1}{2}$ in.
4. $7\frac{7}{8}$ ft. ÷ $4\frac{7}{16}$ ft.
5. 5.75 ÷ $2\frac{1}{2}$.
6. \$5.25 ÷ $1\frac{1}{4}$.
7. 38% of \$9.60.
8. $\frac{2}{3}$ of $13\frac{1}{2}$ ft.
9. $\frac{7}{8}$ of 17 ft. 4 in.
10. 75% of $14\frac{1}{4}$ lb.

V. 1. $16\frac{2}{3}$% of \$8.46.
2. $12\frac{1}{2}$% of \$96.40.
3. $37\frac{1}{2}$% of \$97.60.
4. $62\frac{1}{2}$% of \$41.60.
5. $87\frac{1}{2}$% of \$43.20.
6. $2\frac{1}{2}$% of \$650.
7. $3\frac{1}{3}$% of \$4.20.
8. 0.09% of \$84.00.
9. $2.3\frac{1}{3}$% of \$75.
10. 250% of 180 ft.

NEW YORK STATE ARITHMETICS

ADVANCED BOOK

CHAPTER I

I. REVIEW OF THE FUNDAMENTALS

A SILENT READING LESSON

In our earlier work in arithmetic we studied all the common operations. We know how to read and write numbers as large as we shall probably need to use, and we know how to add, subtract, multiply, and divide *whole numbers*, *fractions*, and *decimals*. We also know that decimals are only fractions written in another way, and have learned that *per cents* are only decimals written in another way; that is,

$$75\% = 0.75 = \tfrac{3}{4}$$

and

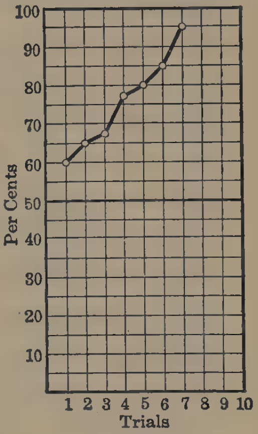
$$250\% = 2.50 = 2\tfrac{1}{2}.$$

We do not yet know how all this is used in everyday business; for example, we do not know about investing money, about insurance, about taxation, and about many other ways in which arithmetic is used. These we shall learn in the work of the next two years.

After our vacation we are a little "rusty" in handling numbers rapidly. We shall therefore take first a short review of the kind of work that we shall need when we begin to use arithmetic in daily life.

A SILENT READING LESSON (CONTINUED)

Our written drill work will mean much more to us if we keep a record of our standings. This is best done by means of a *graph* like the one here shown. To make such a graph, we first rule a page of our notebook with quarter-inch squares so that it looks like the figure at the right.



Suppose that Tom gets 60% on the first trial of a test like the one on page 3. In recording his score, he places a dot at the point where the vertical line marked 1 meets the horizontal line marked 60 (per cent). When the same test is taken a few days later, he records his standing in the same way. Thus, if his marks are 60%, 65%, 68%, 77%, 80%, 85%, and 95% for seven trials, his graph will look like this one.

We need not draw the squares below 50% if we are sure that we shall have no mark below 50%.

In all our drill work we should try to improve upon our previous scores, so that the line showing our progress will continue to rise steadily.

We can keep this kind of record for all the review drills on pages 3–8 by using a separate page for each graph.

We must remember that each answer is either right or wrong. There is no halfway point between right and wrong in arithmetic or anything else. There is no credit to be given for an answer that is not absolutely correct.

After we have read a silent reading lesson, like this one on graphs, if we do not understand it thoroughly, we talk it over for a few minutes in class.

REVIEW DRILL CHART IN ADDITION

Do as many of the following additions as you can in 5 min., counting each correct answer as 4 % in the two top rows and as 6 % in the two bottom rows:

1. 247 46 356 918 705 329	**5.** 726 835 104 53 689 745	**9.** 135 246 790 879 68 568	**13.** 239 75 145 687 529 380	**17.** 712 384 509 678 89 808
2. 142 768 59 905 827 356	**6.** 476 72 989 302 165 768	**10.** 892 374 607 123 458 36	**14.** 359 88 207 848 796 165	**18.** 176 829 193 903 87 465
3. 6782 148 3954 1066 5092 7768	**7.** 8012 3466 5098 273 7312 6076	**11.** 5791 2648 3097 8268 609 4017	**15.** 9465 93 7830 6972 1586 5070	**19.** 4328 2697 5401 2376 749 8080
4. 6843 87 6955 4432 380 1202	**8.** 2013 5746 475 9789 6054 3071	**12.** 7707 8088 9909 6660 820 1555	**16.** 2576 3985 4072 680 1879 6048	**20.** 3942 8076 5979 8213 8019 3091

REVIEW DRILL CHART IN SUBTRACTION

Do as many of these subtractions as you can in 5 min., counting each correct answer as 2%:

1. $\begin{array}{r}9072\\ \underline{46}\end{array}$

2. $\begin{array}{r}5079\\ \underline{298}\end{array}$

3. $\begin{array}{r}6341\\ \underline{2870}\end{array}$

4. $\begin{array}{r}8021\\ \underline{7231}\end{array}$

5. $\begin{array}{r}6643\\ \underline{4873}\end{array}$

6. $\begin{array}{r}6232\\ \underline{4252}\end{array}$

7. $\begin{array}{r}9872\\ \underline{1999}\end{array}$

8. $\begin{array}{r}8000\\ \underline{175}\end{array}$

9. $\begin{array}{r}6010\\ \underline{96}\end{array}$

10. $\begin{array}{r}8320\\ \underline{407}\end{array}$

11. $\begin{array}{r}8129\\ \underline{649}\end{array}$

12. $\begin{array}{r}7311\\ \underline{5876}\end{array}$

13. $\begin{array}{r}5700\\ \underline{3880}\end{array}$

14. $\begin{array}{r}6802\\ \underline{4902}\end{array}$

15. $\begin{array}{r}6294\\ \underline{1919}\end{array}$

16. $\begin{array}{r}8721\\ \underline{7812}\end{array}$

17. $\begin{array}{r}3704\\ \underline{1924}\end{array}$

18. $\begin{array}{r}4127\\ \underline{1396}\end{array}$

19. $\begin{array}{r}3070\\ \underline{480}\end{array}$

20. $\begin{array}{r}6121\\ \underline{5980}\end{array}$

21. $\begin{array}{r}3842\\ \underline{987}\end{array}$

22. $\begin{array}{r}8124\\ \underline{5248}\end{array}$

23. $\begin{array}{r}2232\\ \underline{1987}\end{array}$

24. $\begin{array}{r}7485\\ \underline{4626}\end{array}$

25. $\begin{array}{r}3340\\ \underline{1896}\end{array}$

26. $\begin{array}{r}4083\\ \underline{804}\end{array}$

27. $\begin{array}{r}7352\\ \underline{6199}\end{array}$

28. $\begin{array}{r}8020\\ \underline{990}\end{array}$

29. $\begin{array}{r}6421\\ \underline{5009}\end{array}$

30. $\begin{array}{r}8008\\ \underline{6009}\end{array}$

31. $\begin{array}{r}5017\\ \underline{3926}\end{array}$

32. $\begin{array}{r}9007\\ \underline{2647}\end{array}$

33. $\begin{array}{r}6249\\ \underline{3189}\end{array}$

34. $\begin{array}{r}5125\\ \underline{3642}\end{array}$

35. $\begin{array}{r}5237\\ \underline{2969}\end{array}$

36. $\begin{array}{r}5270\\ \underline{4139}\end{array}$

37. $\begin{array}{r}8012\\ \underline{7775}\end{array}$

38. $\begin{array}{r}7283\\ \underline{696}\end{array}$

39. $\begin{array}{r}3121\\ \underline{1900}\end{array}$

40. $\begin{array}{r}5300\\ \underline{769}\end{array}$

41. $\begin{array}{r}7230\\ \underline{47}\end{array}$

42. $\begin{array}{r}8272\\ \underline{5727}\end{array}$

43. $\begin{array}{r}5722\\ \underline{936}\end{array}$

44. $\begin{array}{r}8082\\ \underline{5081}\end{array}$

45. $\begin{array}{r}8333\\ \underline{4777}\end{array}$

46. $\begin{array}{r}6445\\ \underline{4207}\end{array}$

47. $\begin{array}{r}9217\\ \underline{3747}\end{array}$

48. $\begin{array}{r}8020\\ \underline{975}\end{array}$

49. $\begin{array}{r}4000\\ \underline{21}\end{array}$

50. $\begin{array}{r}3006\\ \underline{1598}\end{array}$

REVIEW DRILL CHART IN MULTIPLICATION

Do as many of these multiplications as you can in 7 min., counting each correct answer as 2%:

1. $\begin{array}{r} 236 \\ \underline{14} \end{array}$
2. $\begin{array}{r} 364 \\ \underline{34} \end{array}$
3. $\begin{array}{r} 519 \\ \underline{60} \end{array}$
4. $\begin{array}{r} 2736 \\ \underline{28} \end{array}$
5. $\begin{array}{r} 5686 \\ \underline{49} \end{array}$
6. $\begin{array}{r} 6080 \\ \underline{29} \end{array}$
7. $\begin{array}{r} 3929 \\ \underline{48} \end{array}$
8. $\begin{array}{r} 3077 \\ \underline{77} \end{array}$
9. $\begin{array}{r} 7083 \\ \underline{240} \end{array}$
10. $\begin{array}{r} 4033 \\ \underline{60} \end{array}$
11. $\begin{array}{r} 3041 \\ \underline{92} \end{array}$
12. $\begin{array}{r} 8756 \\ \underline{98} \end{array}$
13. $\begin{array}{r} 7182 \\ \underline{38} \end{array}$
14. $\begin{array}{r} 7685 \\ \underline{56} \end{array}$
15. $\begin{array}{r} 8129 \\ \underline{95} \end{array}$
16. $\begin{array}{r} 6237 \\ \underline{370} \end{array}$
17. $\begin{array}{r} 4842 \\ \underline{90} \end{array}$
18. $\begin{array}{r} 6908 \\ \underline{320} \end{array}$
19. $\begin{array}{r} 7400 \\ \underline{490} \end{array}$
20. $\begin{array}{r} 8000 \\ \underline{570} \end{array}$
21. $\begin{array}{r} 8837 \\ \underline{36} \end{array}$
22. $\begin{array}{r} 9867 \\ \underline{46} \end{array}$
23. $\begin{array}{r} 4907 \\ \underline{65} \end{array}$
24. $\begin{array}{r} 8071 \\ \underline{48} \end{array}$
25. $\begin{array}{r} 3080 \\ \underline{67} \end{array}$
26. $\begin{array}{r} 8843 \\ \underline{660} \end{array}$
27. $\begin{array}{r} 6969 \\ \underline{540} \end{array}$
28. $\begin{array}{r} 8383 \\ \underline{860} \end{array}$
29. $\begin{array}{r} 4707 \\ \underline{230} \end{array}$
30. $\begin{array}{r} 6782 \\ \underline{490} \end{array}$
31. $\begin{array}{r} 4979 \\ \underline{58} \end{array}$
32. $\begin{array}{r} 7854 \\ \underline{39} \end{array}$
33. $\begin{array}{r} 5279 \\ \underline{31} \end{array}$
34. $\begin{array}{r} 7096 \\ \underline{92} \end{array}$
35. $\begin{array}{r} 5129 \\ \underline{47} \end{array}$
36. $\begin{array}{r} 7484 \\ \underline{590} \end{array}$
37. $\begin{array}{r} 8300 \\ \underline{570} \end{array}$
38. $\begin{array}{r} 8366 \\ \underline{670} \end{array}$
39. $\begin{array}{r} 7389 \\ \underline{530} \end{array}$
40. $\begin{array}{r} 2929 \\ \underline{340} \end{array}$
41. $\begin{array}{r} 7281 \\ \underline{37} \end{array}$
42. $\begin{array}{r} 1342 \\ \underline{71} \end{array}$
43. $\begin{array}{r} 8876 \\ \underline{94} \end{array}$
44. $\begin{array}{r} 3280 \\ \underline{97} \end{array}$
45. $\begin{array}{r} 6277 \\ \underline{53} \end{array}$
46. $\begin{array}{r} 3080 \\ \underline{57} \end{array}$
47. $\begin{array}{r} 6600 \\ \underline{66} \end{array}$
48. $\begin{array}{r} 7474 \\ \underline{74} \end{array}$
49. $\begin{array}{r} 8299 \\ \underline{99} \end{array}$
50. $\begin{array}{r} 5802 \\ \underline{36} \end{array}$

REVIEW DRILL CHART IN SHORT DIVISION

Do as many of these divisions as you can in 6 min., writing fractions in the results instead of remainders, and counting each correct answer as $1\frac{2}{3}\%$*:*

1. $4\overline{)148}$
2. $6\overline{)396}$
3. $2\overline{)431}$
4. $6\overline{)789}$
5. $7\overline{)247}$
6. $3\overline{)776}$
7. $2\overline{)908}$
8. $7\overline{)644}$
9. $9\overline{)777}$
10. $5\overline{)357}$
11. $3\overline{)777}$
12. $8\overline{)736}$
13. $7\overline{)245}$
14. $8\overline{)999}$
15. $5\overline{)900}$
16. $8\overline{)328}$
17. $7\overline{)504}$
18. $3\overline{)347}$
19. $7\overline{)123}$
20. $6\overline{)325}$
21. $3\overline{)677}$
22. $2\overline{)809}$
23. $7\overline{)446}$
24. $3\overline{)709}$
25. $5\overline{)735}$
26. $3\overline{)707}$
27. $8\overline{)763}$
28. $7\overline{)542}$
29. $8\overline{)992}$
30. $5\overline{)905}$
31. $2\overline{)118}$
32. $9\overline{)333}$
33. $4\overline{)125}$
34. $8\overline{)345}$
35. $5\overline{)428}$
36. $3\overline{)708}$
37. $9\overline{)809}$
38. $8\overline{)426}$
39. $6\overline{)888}$
40. $4\overline{)848}$
41. $4\overline{)550}$
42. $8\overline{)376}$
43. $7\overline{)524}$
44. $8\overline{)299}$
45. $4\overline{)905}$
46. $5\overline{)225}$
47. $3\overline{)222}$
48. $5\overline{)126}$
49. $9\overline{)678}$
50. $4\overline{)979}$
51. $3\overline{)808}$
52. $9\overline{)918}$
53. $8\overline{)904}$
54. $5\overline{)375}$
55. $4\overline{)938}$
56. $4\overline{)560}$
57. $8\overline{)367}$
58. $7\overline{)452}$
59. $8\overline{)792}$
60. $3\overline{)905}$

REVIEW DRILL CHART IN LONG DIVISION

Do as many of the following divisions as you can in 30 min., writing fractions in the results instead of remainders, and counting each correct answer as $1\frac{2}{3}\%$*:*

1. $25\overline{)6250}$
2. $20\overline{)6520}$
3. $25\overline{)6025}$
4. $20\overline{)6025}$
5. $30\overline{)6250}$
6. $21\overline{)6250}$
7. $21\overline{)7028}$
8. $22\overline{)4742}$
9. $22\overline{)3742}$
10. $31\overline{)6910}$
11. $32\overline{)6728}$
12. $32\overline{)6736}$
13. $50\overline{)7850}$
14. $50\overline{)3925}$
15. $60\overline{)7320}$
16. $60\overline{)7350}$
17. $42\overline{)4284}$
18. $42\overline{)8484}$
19. $42\overline{)8421}$
20. $45\overline{)9945}$
21. $16\overline{)2727}$
22. $16\overline{)3760}$
23. $16\overline{)5368}$
24. $18\overline{)7380}$
25. $18\overline{)7308}$
26. $18\overline{)7317}$
27. $29\overline{)5829}$
28. $38\overline{)2936}$
29. $48\overline{)7286}$
30. $48\overline{)7296}$
31. $75\overline{)1575}$
32. $75\overline{)1600}$
33. $81\overline{)9999}$
34. $81\overline{)8910}$
35. $81\overline{)8919}$
36. $52\overline{)6262}$
37. $96\overline{)1920}$
38. $96\overline{)2016}$
39. $96\overline{)2064}$
40. $99\overline{)8732}$
41. $46\overline{)5060}$
42. $46\overline{)5083}$
43. $83\overline{)9130}$
44. $83\overline{)9213}$
45. $77\overline{)8877}$
46. $77\overline{)7788}$
47. $25\overline{)1505}$
48. $35\overline{)1505}$
49. $22\overline{)8866}$
50. $23\overline{)4830}$
51. $23\overline{)4853}$
52. $69\overline{)2369}$
53. $69\overline{)6923}$
54. $55\overline{)1111}$
55. $55\overline{)2750}$
56. $82\overline{)4141}$
57. $28\overline{)5656}$
58. $28\overline{)5887}$
59. $90\overline{)8190}$
60. $36\overline{)1836}$

REVIEW DRILL CHART IN FRACTIONS

Do as many of these operations as you can in 10 min., counting each correct answer as $1\frac{1}{2}\%$, and adding 1% to your score if you do them all:

1. $\frac{3}{4} + \frac{1}{2}$.
2. $\frac{3}{4} - \frac{1}{2}$.
3. $\frac{3}{4} \times \frac{1}{2}$.
4. $\frac{3}{4} \div \frac{1}{2}$.
5. $\frac{1}{2} \div \frac{3}{4}$.
6. $1\frac{5}{8} + \frac{1}{2}$.
7. $1\frac{5}{8} - \frac{1}{2}$.
8. $1\frac{5}{8} \times \frac{1}{2}$.
9. $1\frac{5}{8} \div \frac{1}{2}$.
10. $\frac{1}{2} \div 1\frac{5}{8}$.
11. $2\frac{7}{8} + 1\frac{3}{4}$.
12. $2\frac{7}{8} - 1\frac{3}{4}$.
13. $2\frac{7}{8} \times 1\frac{3}{4}$.
14. $2\frac{7}{8} \div 1\frac{3}{4}$.
15. $1\frac{3}{4} \div 2\frac{7}{8}$.
16. $5\frac{1}{8} + 2\frac{3}{4}$.
17. $5\frac{1}{8} - 2\frac{3}{4}$.
18. $5\frac{1}{8} \times 2\frac{3}{4}$.
19. $5\frac{1}{8} \div 2\frac{3}{4}$.
20. $2\frac{3}{4} \div 5\frac{1}{8}$.
21. $15 \times 2\frac{1}{2}$.
22. $15 \div 2\frac{1}{2}$.
23. $\frac{1}{2} + \frac{1}{8}$.
24. $\frac{1}{2} - \frac{1}{8}$.
25. $\frac{1}{2} \times \frac{1}{8}$.
26. $\frac{1}{2} \div \frac{1}{8}$.
27. $\frac{1}{8} \div \frac{1}{2}$.
28. $2\frac{3}{8} + \frac{1}{2}$.
29. $2\frac{3}{8} - \frac{1}{2}$.
30. $2\frac{3}{8} \times \frac{1}{2}$.
31. $2\frac{3}{8} \div \frac{1}{2}$.
32. $\frac{1}{2} \div 2\frac{3}{8}$.
33. $3\frac{5}{8} + 1\frac{3}{4}$.
34. $3\frac{5}{8} - 1\frac{3}{4}$.
35. $3\frac{5}{8} \times 1\frac{3}{4}$.
36. $3\frac{5}{8} \div 1\frac{3}{4}$.
37. $1\frac{3}{4} + 3\frac{5}{8}$.
38. $6\frac{3}{8} + 3\frac{3}{4}$.
39. $6\frac{3}{8} - 3\frac{3}{4}$.
40. $6\frac{3}{8} \times 3\frac{3}{4}$.
41. $6\frac{3}{8} \div 3\frac{3}{4}$.
42. $3\frac{3}{4} \div 6\frac{3}{8}$.
43. $7\frac{1}{2} \times 3\frac{3}{4}$.
44. $7\frac{1}{2} \div 3\frac{3}{4}$.
45. $\frac{1}{2} + \frac{1}{3}$.
46. $\frac{1}{2} - \frac{1}{3}$.
47. $\frac{1}{2} \times \frac{1}{3}$.
48. $\frac{1}{2} \div \frac{1}{3}$.
49. $\frac{1}{3} \div \frac{1}{2}$.
50. $3\frac{2}{3} + \frac{1}{2}$.
51. $3\frac{2}{3} - \frac{1}{2}$.
52. $3\frac{2}{3} \times \frac{1}{2}$.
53. $3\frac{2}{3} \div \frac{1}{2}$.
54. $\frac{1}{2} \div 3\frac{2}{3}$.
55. $5\frac{5}{6} + 1\frac{1}{2}$.
56. $5\frac{5}{6} - 1\frac{1}{2}$.
57. $5\frac{5}{6} \times 1\frac{1}{2}$.
58. $5\frac{5}{6} \div 1\frac{1}{2}$.
59. $1\frac{1}{2} \div 5\frac{5}{6}$.
60. $6\frac{2}{3} + 2\frac{5}{6}$.
61. $6\frac{2}{3} - 2\frac{5}{6}$.
62. $6\frac{2}{3} \times 2\frac{5}{6}$.
63. $6\frac{2}{3} \div 2\frac{5}{6}$.
64. $2\frac{5}{6} \div 6\frac{2}{3}$.
65. $17\frac{5}{8} \times 5\frac{7}{8}$.
66. $17\frac{5}{8} \div 5\frac{7}{8}$.

DIFFERENT EXPRESSIONS FOR FRACTIONS

Copy the following tables and fill the blank spaces:

	FRACTION	DECIMAL	PER CENT
1.	$\frac{1}{2}$	0.50	50 %
2.	$\frac{1}{4}$		
3.		0.75	
4.			10 %
5.	$\frac{1}{8}$		
6.		0.375	
7.			$62\frac{1}{2}$ %
8.	$\frac{7}{8}$		
9.		$0.33\frac{1}{3}$	
10.			$66\frac{2}{3}$ %
11.	$\frac{1}{6}$		
12.			$83\frac{1}{3}$ %
13.	$\frac{2}{5}$		
14.		0.60	
15.			80 %
16.	$\frac{1}{20}$		
17.			6 %
18.		0.08	
19.			12 %
20.		0.15	
21.	$\frac{1}{12}$		
22.			$6\frac{1}{4}$ %

	FRACTION	DECIMAL	PER CENT
23.	$1\frac{1}{4}$	1.25	125 %
24.		1.00	
25.			300 %
26.	$6\frac{1}{4}$		
27.	$6\frac{3}{4}$		
28.		2.125	
29.			225 %
30.		$2.33\frac{1}{3}$	
31.			$216\frac{2}{3}$ %
32.	$3\frac{2}{3}$		
33.		$4.66\frac{2}{3}$	
34.			200 %
35.		3.375	
36.	$2\frac{5}{8}$		
37.		6.875	
38.			$283\frac{1}{3}$ %
39.	$1\frac{1}{6}$		
40.		$2.08\frac{1}{3}$	
41.	5		
42.		7.00	
43.	$6\frac{5}{6}$		
44.			750 %

MISCELLANEOUS PROBLEMS

1. Mr. Wheeler works 6 da. each week and earns $31.50 per week. Last month, on account of illness, he lost 9 da. How much did he lose in wages?

2. A grocer charges 8¢ a pound for sugar and 50¢ a dozen for eggs. At a special sale one of his customers bought 6 lb. of sugar for 45¢ and $2\frac{1}{2}$ doz. eggs for $1. How much did the customer save in buying at the special sale?

3. At the rate of three for $1, find the cost of 1 doz. collars; of $\frac{1}{2}$ doz. collars; of 15 collars.

4. A family requires $3\frac{1}{2}$ qt. of milk and 1 pt. of cream daily. If they pay 12¢ a quart for the milk and 18¢ a half pint for the cream, how much do they pay for milk and cream during the month of July?

5. A family of four go to the "movies" on an average of two nights a week. If the tickets are 20¢ each, how much do they cost in a year? If the house rent of this family is $38 a month, for how many months would the money spent for "movies" pay the rent?

6. Mr. Howard, who lives 147 mi. from Albany, decides to make a trip there and back in his automobile. The price of gasoline is 17¢ a gallon, and his car will make 14 mi. on a gallon. On the trip he pays also 54¢ for oil. Find the cost of gasoline and oil for the trip to Albany and back.

7. A farmer who planted 40 A. (acres) of corn found that, by carefully selecting the seed corn, he could have raised 45 bu. instead of 30 bu. per acre. If he had done this and had sold the extra corn at 82¢ a bushel, he could have bought with the money some more stock for his farm. If the stock cost $60 a head, how many head could he have bought, and how much money would he have had left over?

II. PERCENTAGE

A SILENT READING LESSON

We have already studied *percentage*, which is the part of arithmetic that uses per cents. We found that per cents are simply another way of writing decimals, because per cent is only another name for hundredths. We learned that all the expressions 6%, $\frac{6}{100}$, 0.06, and $\frac{3}{50}$ have the same value, and that 6% is only another way of writing 0.06. Similarly,

$$\begin{aligned}100\% &\text{ means } \tfrac{100}{100}, \text{ or } 1;\\ 125\% &\text{ means } \tfrac{125}{100}, \text{ or } 1.25;\\ 400\% &\text{ means } \tfrac{400}{100}, \text{ or } 4.\end{aligned}$$

On page 9 we reviewed the relation of fractions to decimals and the relation of each to per cents. That is, relations like the following are already familiar to us:

$$\begin{aligned}10\% &= 0.10 &&= \tfrac{1}{10};\\ 25\% &= 0.25 &&= \tfrac{1}{4};\\ 50\% &= 0.50 &&= \tfrac{1}{2};\\ 62\tfrac{1}{2}\% &= 0.62\tfrac{1}{2} &&= \tfrac{5}{8}.\end{aligned}$$

We also know that other commonly used per cents may be expressed as fractions.

If we wish to find one of the above per cents of a number, it is much easier to use the fractional form. For example, if we wish to find 50% of $10, we simply take half of $10. Similarly, if we wish to find $37\frac{1}{2}\%$ of a given number, we take $\frac{3}{8}$ of it; if we wish to find $66\frac{2}{3}\%$ of a number, we take $\frac{2}{3}$ of it; if we wish to take 125% of a number, we may multiply it by $1\frac{1}{4}$, or we may add to the number $\frac{1}{4}$ of itself; and so on.

Sometimes, however, it is best to use decimals. For example, the fraction that is equal to 18% is $\frac{9}{50}$, and it is evidently easier to multiply by 0.18 than by $\frac{9}{50}$.

PER CENTS AS FRACTIONS

All work oral

1. By using these columns show that 20% = $\frac{1}{5}$, 40% = $\frac{2}{5}$, 60% = $\frac{3}{5}$, 80% = $\frac{4}{5}$, and 100% = 1.

				20%
			20%	20%
		20%	20%	20%
	20%	20%	20%	20%
20%	20%	20%	20%	20%
20%	40%	60%	80%	100%

2. Show that 20% is $\frac{1}{2}$ of 40%, $\frac{1}{4}$ of 80%, and $\frac{1}{5}$ of 100%.

3. Show that 40% is $\frac{2}{3}$ of 60%, $\frac{1}{2}$ of 80%, and $\frac{2}{5}$ of 100%.

4. Show that 60% is 20% + 40%, $1\frac{1}{2}$ times 40%, 3 times 20%, $\frac{3}{4}$ of 80%, and $\frac{3}{5}$ of 100%.

5. A foot is what fraction of a yard? A foot is what per cent of a yard?

6. A pint is what fraction of a gallon? A pint is what per cent of a gallon?

7. A nickel is what fraction of a dollar? A nickel is what per cent of a dollar?

8. If you wish to find $33\frac{1}{3}$% of a number, what fractional part of the number may you find?

9. If a man spends one fifth of his income for rent, what per cent does he spend for this purpose?

10. What per cent of the distance around a square is one side? two sides? three sides?

11. If there are 30 pupils in a class, and if 50% are boys, what fraction are boys? How many boys are there? How many girls are there?

12. How many dollars is $\frac{1}{5}$ of \$40? 20% of \$40? $\frac{1}{2}$ of \$40? 50% of \$40? $\frac{1}{4}$ of \$40? 25% of \$40?

USING PER CENTS

Numbers 1 to 16, oral

1. A man said that he gained 100% on some property which he sold. What does this mean?

2. A dealer said that he would sell some goods at 40% less than the marked price. Express this as a fraction.

3. A boy beginning to play tennis won only 25% of the games played the first week. Express this as a fraction.

Read as fractions, whole numbers, or mixed numbers:

4. 20%.
5. 30%.
6. 50%.
7. 60%.
8. 70%.
9. 90%.
10. 25%.
11. 125%.
12. 225%.
13. 200%.
14. 300%.
15. 325%.

16. One dealer sells an article for $33\frac{1}{3}\%$ more than it costs him, and another sells the same thing for $\frac{1}{3}$ more than it costs him. Is there any difference?

17. How much is $\frac{1}{25}$ of 50? 4% of 50? 4% of 60?

18. How much is $\frac{1}{4}$ of 440? 25% of 440? 25% of 3724?

19. How much is $\frac{1}{2}$ of 3628? 50% of 3628? 50% of 9376?

20. How much is $\frac{3}{4}$ of 6480? 75% of 6480? 75% of 3872?

21. How much is $1\frac{1}{4} \times 964$? 125% of 964? 125% of 896?

22. Instead of multiplying 72 by $12\frac{1}{2}\%$, what fractional part of 72 may we take? Do this, and thus find $12\frac{1}{2}\%$ of 72.

23. Instead of multiplying 640 by $37\frac{1}{2}\%$, what fractional part of 640 may we take? Do this, and thus find $37\frac{1}{2}\%$ of 640.

24. Find $87\frac{1}{2}\%$ of 160 and of 3200 in the shortest way.

25. Find $33\frac{1}{3}\%$ of 96 and of 9672 in the shortest way.

26. Find $3\frac{1}{4} \times 488$; 325% of 488; 325% of 872.

Per Cents as Decimals. If a baseball team has lost 32% of its games, how many hundredths of its games has it lost? How do you write 32% as hundredths?

Since 32%, 0.32, and $\frac{32}{100}$ all mean the same thing, we see that we can easily write per cents as decimals.

To express a per cent as a decimal, omit the per-cent sign and insert or move the decimal point two places to the left.

That is, when we omit the per-cent sign, we must indicate the hundredths in some other way, which we do by the position of the decimal point.

Thus, $1\frac{1}{2}\% = 0.01\frac{1}{2}$, or 0.015; $125\% = 1.25$; $0.6\% = 0.006$.

PER CENTS AS DECIMALS

Numbers 1 to 9, oral

1. If 2% of the people in your town are in the seventh grade of school, how many out of every 100 people are in the seventh grade? How many hundredths of the people in the town are in this grade?

2. If 2% of the pupils in your school are absent today, how many pupils are absent out of every 100? out of every 50?

3. How much is $\frac{1}{100}$ of \$200? 1% of \$200? 1% of \$1700?

4. How much is 0.06 of \$100? 6% of \$100? 6% of \$300?

Read the following per cents as decimals:

5. 35%. **6.** 19%. **7.** 75%. **8.** 120%. **9.** 325%.

10. Write $\frac{1}{10}$ as a decimal; as per cent.

11. Write $\frac{1}{10}\%$ as a decimal; as a fraction.

Write as decimals or as whole numbers:

12. 800%.
13. 0.5%.
14. 150%.
15. 0.005%.
16. $37\frac{1}{2}\%$.
17. $66\frac{2}{3}\%$.
18. 1.67%.
19. 3.75%.

Decimals as Per Cents. If a basketball team has won 0.6 of the games it has played, how many hundredths of the games has it won? This is what per cent?

That is, to express a decimal as per cent we ask how many hundredths it represents, as in the following examples:

1. Express 0.375 as per cent.

Here $0.375 = 37.5$ hundredths, or $37\frac{1}{2}$ hundredths. Therefore 0.375 is the same as 37.5%, or $37\frac{1}{2}\%$.

2. Express 0.00125 as per cent.

Here $0.00125 = \dfrac{125}{100{,}000} = \dfrac{0.125}{100} = 0.125\%$. Also, since $0.125 = \frac{125}{1000} = \frac{1}{8}$, we have $0.00125 = \frac{1}{8}\%$.

3. Express 6.55 as per cent.

Here $6.55 = \frac{655}{100}$, and therefore $6.55 = 655\%$.

To express a decimal as per cent, move the decimal point two places to the right and place the per-cent sign after the number.

Since 655. and 655 are the same, in cases like Ex. 3, above, we simply omit the decimal point.

DECIMALS AS PER CENTS

Numbers 1 to 5, oral

1. A pint, or 0.5 qt., is what per cent of a quart?

2. A peck, or 0.25 bu., is what per cent of a bushel?

3. A quart, or 0.125 pk., is what per cent of a peck?

4. What per cent of a dollar is 49¢? 49 dimes? $1?

5. Express 1 qt. as per cent of a pint; of 2 bu.; of 4 bu.

Write the following decimals as per cents:

6. 0.9.
7. 6.2.
8. 0.3.
9. 3.0.
10. 0.59.
11. 0.72.
12. 0.325.
13. 0.650.
14. $6.666\frac{2}{3}$.
15. 0.00375.

Fractions as Per Cents. If we wish to know the amount of a reduction of 20 % on a bill of goods, which is easier, to take $\frac{1}{5}$ of the amount of the bill or to take 20 % of it? Is it easier to divide by 5 or to multiply by 2?

Sometimes it is easier to use per cents than to use fractions. Since per cent means hundredths, to express fractions as per cents we have only to reduce them to hundredths.

For example, $\frac{4}{5} = \frac{80}{100}$, which is 0.80, or 80 %. Similarly, $\frac{2}{25} = \frac{8}{100} = 8\,\%$, and $2\frac{1}{2} = \frac{5}{2} = \frac{250}{100} = 250\,\%$.

To express a fraction as per cent, first reduce it to hundredths, omit the denominator, and then write the numerator followed by the per-cent sign.

FRACTIONS AS PER CENTS

Numbers 1 to 9, oral

1. 25¢ is what fraction of 50¢? what per cent of 50¢?

2. $33\frac{1}{3}$¢ is what fraction of $1? what per cent of $1?

Express the following fractions as per cents:

3. $\frac{1}{4}$. **4.** $\frac{1}{5}$. **5.** $\frac{1}{10}$. **6.** $\frac{3}{4}$. **7.** $\frac{2}{5}$. **8.** $\frac{7}{10}$.

9. $\frac{5}{8}$ of some games played is what per cent of the games?

10. An inch is what per cent of a foot? of a yard?

11. A pint is what per cent of a peck? of a bushel?

12. What per cent of a gallon is 2 qt.? $2\frac{1}{2}$ qt.? 4 qt.?

13. What per cent of a pound is 1 oz.? $3\frac{4}{5}$ oz.? 16 oz.?

14. What per cent of a rod is 1 ft.? 1 yd.? $5\frac{1}{2}$ yd.? 11 yd.?

15. If you have increased $\frac{1}{9}$ in weight during the past year, what per cent have you increased in weight?

16. If you have increased $12\frac{1}{2}$ % in weight during the past year, what fractional part have you increased?

Reading Per Cents. We already know that 0.75 may be read "seventy-five hundredths" or "seventy-five per cent." In business it is more common to write 75% than 0.75.

READING PER CENTS

Numbers 1 to 3, oral

1. If your school has twice as many pupils as it had 10 yr. ago, it has now what per cent as many as then?

2. Read the following numbers and tell which of them are equal: 2; $\frac{200}{100}$; 200%; $\frac{100}{50}$; 7; 700%.

3. Since $3.75 = \frac{375}{100}$, 3.75 is what per cent?

Write the following numbers in words:

4. 475%.
5. 4.75%.
6. 3.9%.
7. 39%.
8. 0.39%.
9. 390%.
10. 5.25.
11. 7.35.
12. 0.6%.
13. 0.06%.

Express the following per cents as decimals:

14. 225%. **15.** $\frac{1}{4}$%. **16.** 300%. **17.** $\frac{3}{8}$%. **18.** 0.4%.

Express as fractions or mixed numbers:

19. 78%. **20.** $6\frac{1}{4}$%. **21.** 325%. **22.** 475%. **23.** $33.3\frac{1}{3}$%.

Express the following numbers as per cents:

24. $\frac{3}{5}$. **25.** $\frac{1}{3}$. **26.** $\frac{3}{8}$. **27.** $\frac{4}{5}$. **28.** $2\frac{1}{2}$. **29.** $3\frac{3}{4}$.

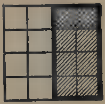
Express as decimals or whole numbers:

30. 28%.
31. 34%.
32. 0.6%.
33. $66\frac{2}{3}$%.
34. 600%.
35. 355%.
36. 250%.
37. 275%.
38. 325%.
39. 5000%.
40. 0.006%.
41. 4550%.

42. Arrange in order of size, beginning with the smallest: 64%, $\frac{18}{25}$, $\frac{3}{5}$, and $0.6\frac{3}{5}$.

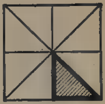
PER CENTS AND FRACTIONS

1. Read 0.50, using the words "per cent"; using the word "hundredths." How many halves are there in 1? Then how many times is 50% contained in 1? in 100%?



2. If this square is called 1, how many sixteenths of the square are shaded? What part of the square is shaded? How many hundredths of it are shaded? What per cent is shaded?

3. Read 0.25, using the words "per cent"; using the word "hundredths." How many fourths are there in 1? Then how many times is 25% contained in 1? in 100%?



4. If this square is called 1, how much is the shaded part? If the square is called 100, how much is the shaded part? What per cent of the square is shaded?

5. How many eighths are there in 1? Then how many times is $12\frac{1}{2}\%$ contained in 1?

6. Draw a square on the board and shade 10% of it. How many tenths of the square is 10% of it?

7. Draw another square on the board and shade 75% of it. How many fourths of the square is 75% of it?

8. How many times is $16\frac{2}{3}\%$ contained in 1? in 2?

9. How many times is $6\frac{1}{4}\%$ contained in 1? in 3?

10. How many times is 20% contained in 1? in 5?

11. How many times is 50% contained in 7? in 12?

12. How many times is 25% contained in 9? in 20?

13. Express $\frac{15}{100}$, 15%, and 0.15 as twentieths.

14. Express $\frac{3}{4}\%$ as a decimal; as a fraction.

15. Express $666\frac{2}{3}\%$ as a decimal; as an improper fraction.

A SILENT READING LESSON

Which is easier, to find $\frac{3}{8}$ of 72 games played or to find $37\frac{1}{2}\%$ of 72 games played?

Sometimes it is easier to use the fraction and sometimes it is easier to use the per cent. It is easier in this case, of course, to use $\frac{3}{8}$ than to use $37\frac{1}{2}\%$.

We have already learned the fractions that are equivalent to certain important per cents. The most important ones are

$50\% = \frac{1}{2}$

$25\% = \frac{1}{4}$

$75\% = \frac{3}{4}$

$12\frac{1}{2}\% = \frac{1}{8}$

$37\frac{1}{2}\% = \frac{3}{8}$

$62\frac{1}{2}\% = \frac{5}{8}$

$87\frac{1}{2}\% = \frac{7}{8}$

$6\frac{1}{4}\% = \frac{1}{16}$

$33\frac{1}{3}\% = \frac{1}{3}$

$66\frac{2}{3}\% = \frac{2}{3}$

$16\frac{2}{3}\% = \frac{1}{6}$

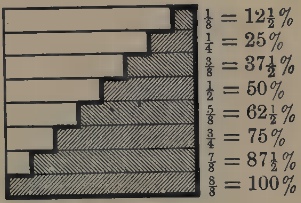
$83\frac{1}{3}\% = \frac{5}{6}$

$20\% = \frac{1}{5}$

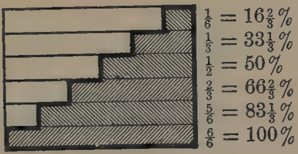
$40\% = \frac{2}{5}$

$60\% = \frac{3}{5}$

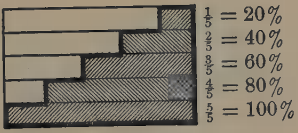
$80\% = \frac{4}{5}$



$\frac{1}{8} = 12\frac{1}{2}\%$
$\frac{1}{4} = 25\%$
$\frac{3}{8} = 37\frac{1}{2}\%$
$\frac{1}{2} = 50\%$
$\frac{5}{8} = 62\frac{1}{2}\%$
$\frac{3}{4} = 75\%$
$\frac{7}{8} = 87\frac{1}{2}\%$
$\frac{8}{8} = 100\%$



$\frac{1}{6} = 16\frac{2}{3}\%$
$\frac{1}{3} = 33\frac{1}{3}\%$
$\frac{1}{2} = 50\%$
$\frac{2}{3} = 66\frac{2}{3}\%$
$\frac{5}{6} = 83\frac{1}{3}\%$
$\frac{6}{6} = 100\%$



We should become so familiar with these equivalents that we can change at once from one to the other. We should never multiply by 25%, $12\frac{1}{2}\%$, $33\frac{1}{3}\%$, or any other per cent, when the fraction is easier for the purpose. That is, in finding $62\frac{1}{2}\%$ of \$1640 and $83\frac{1}{3}\%$ of \$720, we have

$$62\tfrac{1}{2}\% \text{ of } \$1640 = \tfrac{5}{8} \text{ of } \$1640 = 5 \times \$205 = \$1025,$$

and $$83\tfrac{1}{3}\% \text{ of } \$720 = \tfrac{5}{6} \text{ of } \$720 = 5 \times \$120 = \$600.$$

In finding 40% of \$725, however, there is nothing to be gained by using $\frac{2}{5}$ instead of 40%.

IMPORTANT PER CENTS

Numbers 1 to 9, oral

1. Recite the per-cent equivalents of the eighths.

2. Using the equivalent fractions, find the result of dividing $87\frac{1}{2}\%$ by $12\frac{1}{2}\%$.

3. Find the result of dividing 75% by $62\frac{1}{2}\%$.

4. Find the result of dividing $62\frac{1}{2}\%$ by 50%.

5. Recite the per-cent equivalents of the sixths.

6. Divide $83\frac{1}{3}\%$ by $33\frac{1}{3}\%$; $33\frac{1}{3}\%$ by $66\frac{2}{3}\%$.

7. Divide 50% by $16\frac{2}{3}\%$; 50% by 25%; 25% by 50%.

8. Recite the per-cent equivalents of the fifths.

9. Divide 1 by 80%; 80% by 1; 80% by 100%.

Draw figures as on page 19 and write the per-cent equivalents of

10. The sevenths.
11. The ninths.
12. The twelfths.
13. The sixteenths.
14. The tenths.
15. The twentieths.

Perform the following operations:

16. $50\% \div 40\%$.
17. $80\% \div 50\%$.
18. $25\% \div 60\%$.
19. $12\frac{1}{2}\%$ of 80%.
20. $83\frac{1}{3}\%$ of 60%.
21. 60% of $33\frac{1}{3}\%$.
22. $50\% \div 12\frac{1}{2}\%$.
23. $33\frac{1}{3}\%$ of 75%.
24. $66\frac{2}{3}\% \div 40\%$.

Find the value in each of the following cases:

25. 50% of \$1273.80.
26. 25% of \$3724.64.
27. $12\frac{1}{2}\%$ of \$5128.72.
28. $6\frac{1}{4}\%$ of \$23,376.16.
29. $37\frac{1}{2}\%$ of \$17,768.32.
30. $33\frac{1}{3}\%$ of \$77,163.69.
31. $66\frac{2}{3}\%$ of \$80,070.12.
32. $83\frac{1}{3}\%$ of \$71,108.10.
33. $112\frac{1}{2}\%$ of \$87,264.48.
34. $137\frac{1}{2}\%$ of \$96,328.32.

A SILENT READING LESSON

There are three important kinds of problems in percentage. The most important of these, and the one that we shall consider first, is the problem of finding some per cent of a number.

For example, suppose that a man who has a contract to remove 9600 cu. yd. of earth in building a canal reports that $23\frac{1}{4}\%$ of the earth has been removed.

In finding how many cubic yards this represents, we see that $23\frac{1}{4}\% = 0.23\frac{1}{4}$, and so we multiply 9600 by $0.23\frac{1}{4}$. The result, as shown at the right, is 2232; that is, 2232 cu. yd. of earth have been removed.

$$\begin{array}{r} 9600 \\ \underline{0.23\frac{1}{4}} \\ 2400 \\ 28800 \\ \underline{19200} \\ 2232.00 \end{array}$$

To find a per cent of a number, multiply the number by the given per cent expressed as a decimal.

In the above case a still easier way would be simply to multiply 96 by $23\frac{1}{4}$. We should always look for such short cuts, which save time and labor.

Just as we sometimes use special names like "multiplier" and "product" in multiplication, so in percentage we sometimes use special names for the different numbers. The case at the right, in which we find 40% of $300, shows how the names "base," "rate," and "percentage" are used.

Base,	$300
Rate,	0.40
Percentage,	$120.00

To find the percentage, multiply the base by the rate.

Sometimes the 40 in the above case is called the *rate per cent,* but generally we speak simply of the rate. All these terms, however, are seldom used outside the school, and even in school we ordinarily use the word "percentage" to mean that part of arithmetic which treats of per cents.

FINDING A PER CENT OF A NUMBER

Numbers 1 to 7, oral

1. In a school of 400 pupils 9% were in the seventh grade. How many pupils were in that grade?

2. In a school of 500 pupils 11% were in the third grade. How many pupils were in that grade?

3. In a school of 200 pupils 5% were tardy or absent last month. How many pupils were not tardy or absent?

4. If there should be an increase of $3\frac{1}{3}\%$ next year in a school of 300 pupils, how many more pupils would there be then? How many would there be in all?

5. In a school of 250 pupils there were 10% fewer pupils four years ago. How many pupils were there then?

6. If $16\frac{2}{3}\%$ of the pupils in a school of 240 pupils are to graduate this year, how many will graduate?

7. A man borrowed $200 and paid 6% of that sum for the use of it for a year. How much was this?

8. A taxicab company that uses an average of 1750 gal. of gasoline per week plans to equip its cabs with devices which are guaranteed to save 15% on gasoline. How many gallons must be saved to make this guaranty good?

9. In Ex. 8, how many gallons of gasoline will the taxicab company use per week after the cabs are equipped?

10. On a job requiring great haste, a contractor charged 12% of the cost for his services. If the cost of the job was $168,000, how much did the contractor receive?

11. An architect who furnished the plans and supervised the erection of a new school building costing $550,000 was paid 5% of the cost for his services. How much money did the architect receive on this job?

12. An overcoat that cost a dealer \$40 was sold at 40% above cost. Find the selling price.

13. Mollie had a test in spelling. There were 50 words in the test, and she missed 6% of them. How many words did she miss? How many did she spell correctly?

14. Mr. Russell has just received \$125 for some work which he did on a house. He puts 20% of this money in the bank and buys a Treasury Savings Certificate for \$84.60. How much money has he left?

15. Mr. Baker saved coal during the winter by using a little less each day and made up for it by taking better care of his furnace. If he used $14\frac{1}{2}$ T. last year and saved 9% this winter, how many tons did he save?

16. A manufacturer sells shoes so as to realize a gain of 11% on the manufacturing cost. If the manufacturing cost is \$3.70 a pair, how much is his gain on 1000 pairs?

17. Of 276 locomotives manufactured in a large plant, 75% were sold for \$74,125 each and the rest for \$72,825 each. How much was received for all?

18. A man can have a wall built for \$1.20 a foot, but he can have a better job done for 15% more. What will be the cost of building 54 ft. of the better kind of wall?

19. A farmer owned 1424 sheep and sold $12\frac{1}{2}$% of them. How many sheep did he sell? How many were left?

20. A man was paid $\frac{1}{2}$% of the cost for checking the plans of a new building costing \$475,000. How much did he receive?

21. In Ex. 20, how much more would the man have received if he had been paid $\frac{5}{8}$%?

22. A suit of clothes was marked \$40, but after the holidays the price was reduced 15%. How much was it reduced, and what was the price after the reduction?

MISCELLANEOUS PROBLEMS

Numbers 1 to 6, oral

1. If 20% of the pupils in a school of 425 pupils are in the seventh and eighth grades, how many are in those grades? How many are not in those grades?

2. If 25% of the pupils of a school of 420 pupils are in the first and second grades, how many are in those grades?

3. If 54% of the pupils in a school of 300 pupils are girls, how many girls are there in the school?

4. How much must be added to 250 ft. to increase it 50%? to increase it 100%?

5. How much must be added to 240 lb. to make it 125% of the present weight?

6. Water is standing in a tank to a depth of 6 in. If the depth is increased $16\frac{2}{3}$%, how deep is the water then?

7. How much butter fat is there in 425 lb. of milk in which the creamery test shows that 3.9% of the weight of the milk is butter fat?

8. A farmer takes 420 lb. of milk to a creamery and is paid 26¢ a pound for the butter fat. The tests show that the butter fat is 3.8% of the weight of the milk. Find the weight of the butter fat and the amount received by the farmer.

9. One girl who works in a store is paid $25 a week, and another is paid $24 a week. The first girl saves 6.2% of her salary, and the second girl saves 6.5%. Which girl saves more in a year, and how much more does she save?

10. A farmer who owns 90 hogs, which average 225 lb. each in weight, decides to sell 60% of them. If he sells the hogs at a price of $12 per hundredweight, how much money does he receive for them?

Another Problem in Percentage. The problem in percentage which is second in importance to finding a per cent is that of finding what per cent one number is of another.

We already know how to find what fractional part one number is of another. For example, we know that 1 is $\frac{1}{2}$ of 2; 2 is $\frac{2}{4}$ of 4, or $\frac{1}{2}$ of 4; 7 is $\frac{7}{10}$ of 10; and so on. That is,

To find what fractional part one number is of another, divide the first by the second.

We should see that a problem which asks what per cent one number is of another is only asking in another way what part one number is of another number. The result, however, is to be expressed with the per-cent sign.

FRACTIONAL PARTS

All work oral

1. A team lost 7 games out of 28 games played. The games lost were what part of the games played?

2. Mary's baby sister weighs 14 lb., and Mary herself weighs 70 lb. The baby's weight is what part of Mary's weight? Give the result as a fraction and also as a decimal.

In the following cases state what part the first number is of the second, giving the answer as a fraction and also as a decimal:

3. 6, 36.
4. 12, 36.
5. 18, 72.
6. 10, 60.
7. 8, 80.
8. 10, 80.
9. 10, 20.
10. 20, 200.
11. 3, 6.
12. $\frac{3}{10}$, 6.
13. 0.3, 0.6.
14. 0.30, 0.60.
15. 0.60, 0.30.
16. 300, 600.
17. $\frac{1}{4}$, 5.
18. $\frac{1}{2}$, 4.
19. 0.5, 4.
20. $\frac{1}{3}$, 9.
21. 2, 8.
22. 0.2, 8.
23. 2, 80.
24. 1, 20.
25. 5, 20.
26. 10, 20.
27. 20, 20.
28. 5, 25.
29. 25, 50.
30. 50, 25.

Finding What Per Cent One Number is of Another. If Fred made 6 base hits last June and was at bat 20 times, the base hits were what per cent of the number of times at bat?

If we think of this problem as asking what part the base hits were of the times at bat, we see that the result is $\frac{6}{20}$. If we follow the general rule of dividing one number by the other and expressing the result as a decimal, we have

$$6 \div 20 = 0.30.$$

Since 0.30 is the same as 30%, we thus see that the base hits were 30% of the times at bat; that is, we find the result as a decimal and then write it with the per-cent sign.

RELATIONS OF NUMBERS AS PER CENTS

All work oral

1. If Jack made 12 base hits and was at bat 24 times, what per cent of base hits did he make?

2. If a baseball team wins 16 games out of 20, what per cent of games does it win?

3. If 3 of the 30 pupils in a class are absent today, what per cent are absent?

4. If you answer correctly 15 questions out of 20 on an arithmetic test, what per cent do you answer correctly?

Find what per cent the first number is of the second:

5. 1, 2.
6. 1, 3.
7. 1, 4.
8. 2, 4.
9. 3, 4.
10. 4, 4.
11. 20, 40.
12. 30, 40.
13. 45, 90.
14. 15, 60.
15. 16, 32.
16. 32, 16.
17. 22, 44.
18. 44, 44.
19. 40, 60.
20. 72, 72.
21. 27, 36.
22. 27, 270.
23. 5, 25.
24. 2.5, 25.
25. $12\frac{1}{2}$, 25.
26. 25, 25.
27. 50, 75.
28. 50, 250.

Using Per Cents. A man sold at a profit of \$120 an automobile for which he had paid \$600. What per cent of the cost did he gain?

In finding what part \$120 is of \$600, we have

$$\$120 \div \$600 = 0.20 = 20\,\%.$$

To find what per cent one number is of another, divide the first by the second, finding the result as a decimal and then expressing it with the per-cent sign.

USING PER CENTS

1. \$130 is what per cent of \$2600? of \$3900?
2. \$11.25 is what per cent of \$375? of \$56.25?
3. \$10.12 is what per cent of \$253? of \$202.40?
4. \$29.10 is what per cent of \$465.60? of \$174.60?
5. A quart is what per cent of a gallon? of 25 gal.?
6. A rod is what per cent of a foot? of a yard?
7. $\frac{2}{3}$ is what per cent of $\frac{1}{4}$? $\frac{3}{8}$ is what per cent of $\frac{1}{4}$?
8. 25% is what per cent of 75%? of 125%? of 250%?
9. $33\frac{1}{3}\,\%$ is what per cent of $66\frac{2}{3}\,\%$? of 1? of $133\frac{1}{3}\,\%$?
10. $16\frac{2}{3}\,\%$ is what per cent of $33\frac{1}{3}\,\%$? of $66\frac{2}{3}\,\%$? of 1?

Find what per cent the first number is of the second:

11. 387, 2709. 12. 52.8, 31.68. 13. 56.7, 453.6.

14. If a man rents for \$360 a year a house that costs \$4000, the rent is what per cent of the cost?

15. If $12\frac{1}{2}$ T. of iron are obtained from 235 T. of ore, what per cent of the ore is iron?

16. Last year the population of a certain city increased from 26,275 to 28,530. What was the per cent of increase?

17. If the seventh grade devotes 45 min. a day to arithmetic and the sixth grade 42 min., what per cent more time does the seventh grade devote to arithmetic than does the sixth grade?

18. If a class had 16 examples to solve on Monday and 20 on Tuesday, what was the per cent of increase?

19. If you have read 69 pages in a book of 300 pages, what per cent of the pages have you read?

20. If the last chapter of a book is Chapter XXXII, and if you have read to the end of Chapter XXIV, what per cent of the chapters have you read?

21. A man's income is $2300 a year, and he spends $1886. What per cent of his income does he save?

22. In a town, 576 out of the 1200 pupils in school are boys. What per cent are boys?

23. From an investment of $13,500 a man receives an income of $1620. The income is what per cent of the amount which he invested?

24. The purity of gold is measured in carats, or twenty-fourths, 18 carats meaning $\frac{18}{24}$ pure gold. What is the per cent of pure gold in an 18-carat ring? in a 14-carat ring?

25. What is the per cent of pure gold in a watchcase that is 16 carats fine? in a chain that is 12 carats fine?

26. If 2 members of a class of 32 have been neither tardy nor absent during the year, and 4 others have not been tardy, what per cent have been neither tardy nor absent? What per cent have not been tardy?

27. A man's income is $6350 a year, and he saves $1087.50. What per cent does he save? What per cent does he spend? What is the sum of these two per cents? How does the answer to the last question check the first two results?

The Next Problem in Percentage. The third problem in percentage is that of finding the number of which a given number is a given per cent.

For example, if a ball team lost 18 games, which was 40% of the total games played, how many games were played?

We have $\frac{40}{100}$ of the total $= 18$.

Therefore $\frac{1}{100}$ of the total $= \frac{1}{40}$ of 18,

and the total $= \frac{100}{40}$ of 18;

or, expressed in another way, we have

$$\text{the total} = 18 \div \tfrac{40}{100}.$$

That is, the total number of games played by the team is

$$18 \div 0.40, \text{ or } 45.$$

Check. 40% of $45 = 18$.

To find the number of which a given number is a certain per cent, divide the given number by the per cent.

FINDING THE NUMBER

1. If 20% of the inhabitants of a certain city are school children, and there are 12,400 school children in the city, what is the total population of the city?

2. If 36% of the inhabitants of a certain city voted at a certain election, and there were 7200 ballots cast, what was the total population of the city?

3. A player on the seventh-grade baseball team reached first base 29 times, which was $33\frac{1}{3}\%$ of the number of times he was at bat. How many times was he at bat?

4. A player scored 24 times, which was $8\frac{1}{3}\%$ of the number of times he was at bat. How many times was he at bat?

5. If, in paying $16\frac{2}{3}\%$ of the expenses of a camping trip, you pay $12.75, what are the total expenses of the trip?

Find the numbers of which the following are the given per cents:

6. 44, 11%.

7. 75, 5%.

8. 24, 6%.

9. 98, 7%.

10. 44, 4%.

11. 37, $12\frac{1}{2}\%$.

12. 49, $3\frac{1}{3}\%$.

13. 53, $33\frac{1}{3}\%$.

14. 186, $37\frac{1}{2}\%$.

15. 325, $62\frac{1}{2}\%$.

16. 32.45, $16\frac{2}{3}\%$.

17. 57.8, 17%.

18. 22.22, 110%.

19. 65.72, 106%.

20. 622.6, 5.5%.

21. $4.65 is 15% of what amount? 20% of what amount?

22. $7.84 is 16% of what amount? 10% of what amount?

23. 12 ft. 5.6 in. is 11% of what length? 5% of what length?

24. 126 lb. 14 oz. is 35% of what weight?

25. If a farmer sold a horse for 80% of what it cost him and received $160, how much did the horse cost him? How much did he lose on the sale?

26. A poultry raiser set some eggs, and 15% of them failed to hatch. If the number that failed to hatch was 66, how many eggs did he set?

27. A man sells a building lot for $360, thereby gaining 20% on the cost. What per cent of the cost is the selling price? How much is the cost?

28. A man saves $1283.50 a year, which is 17% of his income. How much is his income?

29. A school has 20% of its pupils in the seventh grade, which has 24 pupils. How many pupils are there in the school? How many are not in the seventh grade?

30. In one New York State city a school had 112 pupils in the seventh grade. This number was $16\frac{2}{3}\%$ of the total number in the school. How many pupils were there in the school?

31. Solve Ex. 30, supposing that there were 119 pupils in the seventh grade, and that this was 17% of the total number.

Per Cents Greater or Less. Mr. Lane, who has been earning \$30 a week, has his wages raised to \$36 a week. His wages are now what per cent greater than before?

The increase is \$36 − \$30, or \$6. The question then reduces to this: \$6 is what per cent of \$30? This is like the case on page 27, and so we have

$$\$6 \div \$30 = 0.20 = 20\,\%.$$

That is, Mr. Lane's wages were raised 20 %.

Problems which ask us to find what per cent one number is less than another are solved in a similar way.

PER CENTS GREATER OR LESS

Numbers 1 to 12, oral

1. 101 is what per cent greater than 100?

2. 99 is what per cent less than 100?

3. \$1010 is what per cent greater than \$1000?

4. \$990 is what per cent less than \$1000?

5. \$106 is what per cent greater than \$100?

6. \$95 is what per cent less than \$100?

7. A man who is now earning \$6 a day formerly earned \$5 a day. What per cent have his daily wages increased?

8. A man who earned \$150 one month worked only part of the time the next month and earned \$100. His earnings were then what per cent less than before?

9. \$125 is what per cent greater than \$100?

10. \$250 is what per cent greater than \$200?

11. \$750 is what per cent less than \$1000?

12. 150 ft. is what per cent greater than 100 ft. and what per cent less than 200 ft.?

13. $150 is what per cent greater than $100?

14. $100 is what per cent less than $150?

15. A half mile is what per cent less than $\frac{3}{4}$ mi.?

16. $4080 is what per cent greater than $3400?

17. $21,000 is what per cent greater than $17,500?

18. $3500 is what per cent greater than $3000 and what per cent less than $4000?

19. $256.15 is what per cent greater than $235?

20. $216.20 is what per cent less than $235?

21. 40 in. is what per cent longer than 1 yd.?

22. A yard is what per cent longer than 1 ft.?

23. A foot is what per cent shorter than 1 yd.?

24. A mile is what per cent greater than 2640 ft.?

25. On a boiler which supplies steam for an engine there is a gauge which shows the pressure in pounds that the steam exerts against every square inch of the boiler surface. If the steam pressure increases from 120 lb. to 150 lb. per square inch, it is then what per cent greater than before?

26. A man bought a horse for $150, and sold it so as to gain $30. Find the per cent of gain on the cost.

Find the per cent of gain or loss on the cost in each case:

	COST	GAIN
27.	$425	$29.75
28.	$235	$16.45
29.	$320	$44.80
30.	$225	$11.25
31.	$575	$74.75

	COST	LOSS
32.	$725.50	$145.10
33.	$2750	$247.50
34.	$12,375	$1361.25
35.	$16,750	$3015
36.	$875.25	$140.04

Using What You Have Learned. We shall now take up some miscellaneous problems where it will be necessary for you to decide for yourself the method of solution. You will find it best first to ask yourself exactly what is given. Then consider carefully whether you are to find (1) a required per cent of a given number, (2) what per cent one of the given numbers is of the other, or (3) of what number the given number is the given per cent.

USING WHAT YOU HAVE LEARNED

1. A cubic foot of water weighs $62\frac{1}{2}$ lb. From a tank containing 16 cu. ft. there is drawn off an amount of water that weighs 125 lb. What per cent of the water is drawn off?

2. To a class of 32 pupils 8 more are admitted. The number admitted is what per cent of the original number? of the present number? If 10 pupils now leave the class, what per cent leave it?

3. In an arithmetic class it is planned to solve 550 problems in a certain number of days. All but 44 problems have been solved. What per cent of the problems remain to be solved? What per cent of them have been solved?

4. If 45% of the 240 pupils that attend a school are boys, and the rest are girls, how many girls are there?

5. A man purchased 530 head of cattle and sold 30% of them. How many head of cattle had he left?

6. If an automobile going at the rate of 14 mi. an hour increases its speed 37%, at what rate will it then be going?

7. If a man who earns $2400 a year has his salary increased 5%, how much does he then receive?

8. If a farmer sold 15 cows, which was $33\frac{1}{3}$% of the number he owned, how many cows did he own?

9. A cubic foot of platinum weighs 1220 lb., and a cubic foot of pressed gold weighs 10 lb. less. Platinum is what per cent heavier than pressed gold?

10. Of a salary of $24 a week a clerk spends 60% for board and room, 10% for clothes, and 15% for other expenses. If he saves the rest, how much does he save in 50 wk.?

11. If a baker uses 1278 lb. of flour in making bread and adds to the flour 426 lb. of liquid, the weight of the liquid is what per cent of the weight of the mixture?

12. There are 3 teaspoonfuls of sugar to a tablespoonful of sugar. A teaspoonful is what per cent of a tablespoonful?

13. There are 16 tablespoonfuls to a cup. A tablespoonful is what per cent of a cup?

14. A teaspoonful of butter is $\frac{1}{3}$ of a tablespoonful of butter. A tablespoonful is what per cent of a teaspoonful?

15. The record of the leading team in a baseball league in a recent year was 52 games won and 14 lost. What per cent of the games did the leading team win that year?

16. One of the best records in baseball of recent years was made when Brooklyn won the pennant with a total of 82 games won and 54 lost. What was the per cent of games won?

17. Two of the best averages for runs in baseball in a recent year were as follows: 597 times at bat, 139 runs; 543 times at bat, 145 runs. The runs in each case were what per cent of the number of times at bat?

18. If the expenses of a village were $66,500 last year and $66,005 this year, what is the per cent of decrease?

19. A man bought a horse for $225 and sold it at a loss of 8% on the cost. How much did he lose on the deal? How much did he receive for the horse?

III. HOW PER CENTS ARE USED

A SILENT READING LESSON

A merchant generally plans to sell his goods for enough more than their cost to pay for the goods, to cover the expenses of running his store, and to leave him a *profit* besides. If he fails to do this, he suffers a *loss*.

A merchant has many expenses, such as rent, heat, light, clerks' salaries, insurance, and so on. We shall call all these the *expenses*, or the *cost of doing business*, but the expression *overhead charges*, or simply *overhead*, is often used. In large stores the buying, selling, and operating expenses are sometimes considered separately. In city retail stores the total expenses often amount to about 30% of the total sales, and are sometimes as high as 45%. A merchant must allow for these expenses before he can say that he has made a profit.

If a merchant sells goods rapidly, so that he is continually buying new goods and selling them, we say that his *turnover* is rapid. In such cases, he can sell at a smaller profit on each article, because he sells so many of them. He can even afford to have *mark-down sales* to dispose of old stock. In obtaining a rapid turnover, good salesmanship and effective advertising are exceedingly valuable.

The merchant should have a larger profit than the man who simply puts his money in a savings bank and gets interest on it, for the merchant not only runs a much greater risk of losing the money invested, but is called upon to exercise keen business judgment in buying and selling goods.

Merchants usually speak of their profits or losses as per cents. The per cent is reckoned sometimes on the cost of goods and sometimes on the selling price. We shall first consider the plan of basing the per cent on the cost, beginning with simple transactions such as buying and selling a house.

Selling at a Profit. 1. Suppose that I bought a house for $8000 and sold it so as to gain 20% on what it cost me. How much did I gain?

The gain was 20% of $8000, or $\frac{1}{5}$ of $8000, and so, as shown at the right,

$\frac{1}{5}$ of $8000 = $1600, the amount of gain.

$$5\overline{)\$8000}\atop\$1600$$

2. What was the selling price in Ex. 1?

The selling price was the cost plus the gain; that is, I received $8000 (cost) plus $1600 (gain), and so, as here shown,

$8000 + $1600 = $9600, the selling price.

$$\begin{array}{r}\$8000\\ 1600\\ \hline \$9600\end{array}$$

3. In Ex. 2, find the selling price directly.

Since I gained 20%, it is evident that

$$\begin{array}{rl}100\,\% \text{ of the cost} &= \text{the cost}\\ 20\,\% \text{ of the cost} &= \text{the gain}\\ \hline 120\,\% \text{ of the cost} &= \text{the selling price}\end{array}$$

Hence, as shown at the right,

1.20 × $8000 = $9600, the selling price.

$$\begin{array}{r}\$8000\\ 1.20\\ \hline 160000\\ 8000\\ \hline \$9600.00\end{array}$$

SELLING AT A PROFIT

Given the cost and the per cent of gain, find the amount of gain and the selling price as in Exs. 1 and 2, above:

	Cost	Gain		Cost	Gain		Cost	Gain
1.	$700	10%	5.	$2000	15%	9.	$4200	8%
2.	$800	14%	6.	$2500	16%	10.	$3600	$8\frac{1}{4}$%
3.	$950	$12\frac{1}{2}$%	7.	$3000	$17\frac{1}{2}$%	11.	$4800	$9\frac{3}{4}$%
4.	$725	13%	8.	$4000	18%	12.	$7500	15%

13. Solve Exs. 4, 8, and 12 by the method given in the third illustrative example, above.

Selling at a Loss. 1. If I bought a house for $8000 and sold it at a loss of 20% of the cost, what was the selling price?

The selling price was the cost less 20% of the cost; that is,

$$\begin{array}{r} \$8000 \\ 0.80 \\ \hline \$6400.00 \end{array}$$

100% of the cost = the cost
20% of the cost = the loss
80% of the cost = the selling price

That is, I sold it for 80% of $8000, and so

0.80 × $8000 = $6400, the selling price.

2. If I sold a house and lost $1600, which was 20% of the cost, how much did the house cost me?

$$\begin{array}{r} \$1600 \\ 5 \\ \hline \$8000 \end{array}$$

Since 20% of the cost is $1600,
1% of the cost is $\frac{1}{20}$ of $1600,
and 100% of the cost is $\frac{100}{20}$ of $1600;
that is, 5 × $1600 = $8000, the cost.

3. If I sold a house that cost me $8000 and lost $1600, what per cent did I lose?

I lost $\frac{1600}{8000}$ of $8000, or $\frac{1}{5}$ of $8000. That is, since $\frac{1}{5}$ = 20%, I lost 20% on the cost of the house.

SELLING AT A LOSS

Copy the following tables and fill all the blank spaces:

	Cost	Loss	Amount of Loss
1.	$500	8%	
2.	$500		$30
3.		12%	$24
4.	$800	12½%	
5.	$600		$48

	Cost	Loss	Amount of Loss
6.	$2000	16⅔%	
7.		16⅔%	$200
8.	$3000		$600
9.	$5000	15%	
10.	$8000		$520

Profit or Loss on the Cost. **1.** A merchant's expenses average 30 % of the cost of goods. If he buys an article for $30 and wishes to make a profit of 10 % on the cost, at what price should he sell the article?

Since the cost is 100 % of itself, the selling price must be

100 % + 30 % + 10 %, or 140 %, of the cost,

and so 1.40 × $30 = $42, the selling price.

100 % of cost = cost	$30
30 % of cost = expenses	1.40
10 % of cost = profit	1200
140 % of cost = selling price	30
	$42.00 (selling price)

2. If the merchant in Ex. 1 sells the article for $33, what per cent does he lose on the cost?

The cost plus expenses is 130 % of the cost, and so

1.30 × $30 = $39, the cost + expenses.

Then $39 − $33 = $6, the amount of loss,

and $6 ÷ $30 = 0.20 = 20 %, the loss.

100 % of cost = cost	$30	$39	0.20
30 % of cost = expenses	1.30	33	$30)$6.00
130 % of cost = cost + expenses	900	$6 (loss)	
	30		
	$39.00		20 %. *Ans.*

Do not try to understand all of each solution at once. Begin with the first step and see if you understand why that was taken. Then take the next step, and so on. Do not ask your teacher for help until you have done some hard thinking.

PROFIT OR LOSS ON THE COST

1. Disregarding expenses, if you buy an article for $1 and sell it for $1.30, what is your per cent of gain? What would it be if you bought it for $1.25 and sold it for $1.75?

2. A dealer buys some goods for $750 and sells them so as to make a profit of $87.50. Find, to the nearest 0.1%, the per cent of profit.

3. If a grocer buys some canned goods for $2475 and sells them at a profit of $221.50, what is the per cent of profit?

4. A dry-goods merchant knows that his expenses average 25% of the cost of goods. If he buys some silk for $4800 and has to sell it for $5280, what is the per cent of loss?

5. A merchant's cost of doing business is 22% of the cost of goods. If he buys an article for $22.50 and wishes to make a profit of 12%, at what price should he sell the article?

6. Mr. Robinson bought a house for $5600 and sold it at a profit of 15%. Find the selling price.

7. If Mr. Robinson had sold the house mentioned in Ex. 6 at a loss of 15%, what would have been the selling price?

8. A merchant bought some goods for $5700, but had to sell them at a loss of $285. Find the per cent of loss.

9. A man bought a city lot for $24,000 and sold it at a profit of $16\frac{2}{3}$%. Find the amount of his profit and the price at which he sold the lot.

10. A man who bought an automobile for $2400 estimated that the use he got out of it was worth $360. He sold the car for $12\frac{1}{2}$% less than he paid for it. Find the profit or the loss.

11. A man bought some horses and sold them at a profit of $1960. If he made 20% on the deal, how much did he pay for the horses and for how much did he sell them?

Profit or Loss on the Selling Price. Formerly the per cent of profit or loss was generally computed on the cost of goods. Now, however, it is often the custom to figure the cost of the goods, the expenses, and the profit as per cents of the selling price. This method has been adopted almost universally by large retail stores and some other types of business concerns. In many establishments, however, the method of figuring profit on the cost is still used.

By keeping careful records of sales and expenses, a department store, for example, is able to know the selling price to be placed upon an article in order to cover expenses and to yield the desired profit. From a bookkeeping standpoint, there are many advantages in this method of figuring profit, but they are in general too technical to be considered in the schools.

PROFIT OR LOSS ON THE SELLING PRICE

All work oral

State the profit at 10% on each of the following selling prices:

1. \$1.
2. \$5.
3. \$8.
4. \$2.
5. \$4.
6. \$9.
7. \$15.
8. \$25.
9. \$30.
10. \$100.
11. \$300.
12. \$575.
13. \$2000.
14. \$2500.
15. \$7500.

State the loss at 6% on each of the following selling prices:

16. \$3.
17. \$5.
18. \$8.
19. \$1.
20. \$2.
21. \$6.
22. \$10.
23. \$30.
24. \$70.
25. \$100.
26. \$200.
27. \$500.
28. \$3000.
29. \$4000.
30. \$8000.

State the profit at 15% on each of the following selling prices:

31. \$1.
32. \$2.
33. \$3.
34. \$5.
35. \$7.
36. \$9.
37. \$10.
38. \$20.
39. \$30.
40. \$100.
41. \$200.
42. \$400.
43. \$1000.
44. \$2000.
45. \$3000.

Finding the Selling Price. In a store where the expenses average 24 % of sales, and the desired profit is 12 % of the selling price, what should be the selling price of an article costing $9.60?

The selling price (100 %) includes the cost, the expenses, and the profit, and so, as shown below, the cost is

100 % − 36 %, or 64 %, of the selling price.

Then $9.60 ÷ 0.64 = $15, the selling price.

24 % = expenses
12 % = profit
36 % = expenses + profit
100 % = selling price
36 % = expenses + profit
64 % = cost

$$\begin{array}{r} 0.64\overline{)\$9.60} \\ \$15. \\ 64\overline{)\$960.} \\ \underline{64} \\ 320 \\ \underline{320} \end{array}$$

Check. 0.36 × $15 = $5.40; $15 − $5.40 = $9.60.

FINDING THE SELLING PRICE

Copy the following table and fill the blank spaces:

	COST	EXPENSES	PROFIT	SELLING PRICE
1.	$240	30 %	10 %	
2.	$480	40 %	12 %	
3.	$750	35 %	15 %	
4.	$840	32 %	8 %	
5.	$1200	33 %	7 %	
6.	$1160	30 %	12 %	
7.	$5400	36 %	10 %	

PROFIT OR LOSS ON THE SELLING PRICE

1. If you sell an article for \$9, and your profit is 20% of the selling price, what is the profit?

2. A store's profit was \$3.96 on an article that sold for \$26.40. Find the per cent of profit on the selling price.

3. A merchant, whose expenses average 20% of the selling price, sells for \$13 an article that costs \$9. How much are the expenses in connection with the sale of this article? How much profit does the merchant make? What is the per cent of profit on the selling price?

4. If the expenses average 24% of sales, at what price, to the nearest 5¢, should a department store sell an article that cost \$4.50 so as to make a profit of 10% on the selling price?

5. A store with expenses averaging 28% of the selling price sells for \$17.50 an article that cost \$10.50. Find the per cent of profit on the selling price.

6. A dealer's expenses are 25% of sales, and he desires a profit of 12% of sales. At what price, to the nearest 10¢, should he sell an article costing \$12?

7. At what price should the dealer in Ex. 6 sell an article which costs \$4.50? \$6.75? \$12.25? \$17.60?

8. A dealer's expenses average 22% of sales, and he desires a profit of 14% of sales. If he buys silk at \$3.30 per yard, at what price, to the nearest 5¢, should he sell it?

9. A department store made a profit of 10% on an article which cost \$1.95 and sold for \$3. Find the per cent of the selling price allowed for expenses.

10. A merchant, whose expenses average 25% of the selling price, sells an article that cost \$10 at the cost price. Find the per cent of loss on the selling price.

A SILENT READING LESSON

We learned last year that, when one person acts as an *agent* for another, one of the ways by which the agent is paid for his services is by being given a *commission*; that is, by being paid a certain per cent of the money involved in the transaction, or a certain amount for each article bought or sold.

Thus, if an agent sells a house for a man for $6000 and is paid a commission of 5%, he receives 5% of $6000, or $300, for his services in connection with the sale.

We may speak of the 5% as the *rate* of commission and of the $300 as the *amount* of the commission.

In some kinds of business, agents are sent out to buy instead of sell goods. For example, suppose that a wool dealer sends an agent to the sheep-raising country to buy wool. If the agent buys $5000 worth of wool, and if his commission is 5%, he receives 5% of $5000, or $250, for his work.

We therefore see that an agent may be employed to sell on a commission, or to buy on a commission. In either case, we have this relation:

Amount of commission = *rate* × *amount of transaction.*

If an agent who is paid a commission of 5% sells a house for Mr. Brown for $6000, Mr. Brown receives $6000 less the amount of the agent's commission; that is,

In selling, the sum (or net amount) received is the amount of the sale less the amount of the commission.

Similarly, if an agent who is paid a commission of 5% buys $5000 worth of wool for Jones & Co., the total cost of the wool to the company is $5000 plus the amount of the agent's commission; that is,

In buying, the total cost is the amount of the purchase plus the amount of the commission.

Finding the Commission. If an agent sells a house for Mr. Henderson for \$8000 and receives a commission of $2\frac{1}{2}\%$, what is the amount of his commission?

$$\boxed{\begin{array}{rr} \$8000 & \$80 \\ \underline{0.02\frac{1}{2}} & \underline{2\frac{1}{2}} \\ & 40 \\ & \underline{160} \\ & \$200 \end{array}}$$

The amount is $2\frac{1}{2}\%$ of \$8000, but instead of multiplying \$8000 by $0.02\frac{1}{2}$, where we might make a mistake in the decimal point, it is easier to see that 1% of \$8000 is \$80, and that $2\frac{1}{2}\%$ of it is $2\frac{1}{2}$ times \$80, or \$200.

That is, the agent receives \$200.

We should always watch for such short cuts. Can you see the other short cut of simply dividing \$800 by 4? Can you see why it will give the same answer?

FINDING THE COMMISSION

All work oral

State the value of the following:

1. 2% of \$100.
2. 2% of \$200.
3. 4% of \$100.
4. 4% of \$300.
5. 6% of \$100.
6. 6% of \$500.

7. An agent sells a house for \$9000 and receives a commission of $2\frac{1}{2}\%$. Find the amount of the commission.

8. On a sale amounting to \$50,000 an agent receives a commission of 4%. Find the amount of the commission.

9. An agent who sells electric-light fixtures is paid a commission of 2%. Find the amount of his commission on a sale amounting to \$12,000.

State the amount of the commission on a sale amounting to

10. \$300 at 6%.
11. \$200 at 3%.
12. \$750 at 8%.
13. \$4000 at 5%.
14. \$3000 at 3%.
15. \$2500 at 9%.
16. \$3000 at 2%.
17. \$5000 at 4%.
18. \$15,750 at 3%.

Finding the Net Amount. A used-car dealer sells an automobile for Mr. Martin and charges a commission of 4%. If the dealer sells the car for $850, what is the amount of his commission, and what is the net amount received by Mr. Martin?

$850	$8.50	$850
0.04	4	34
	$34.00 (commission)	$816 (net amount)

Could we have multiplied $850 by 0.04 as easily as we multiplied $8.50 by 4? Either method may be used.

Why do we subtract $34 from $850?

COMMISSIONS AND NET AMOUNTS

1. Mr. Haskins has a car that he has used for some time. He pays a garage man $4\frac{1}{2}\%$ for selling it for him at a price of $1100. How much commission does the garage man receive? What is the net amount that Mr. Haskins receives?

In each of the following, which show the amounts of sales at different rates of commission, find the amount of the commission and the net amount received:

2. $50, 2%.
3. $75, 2%.
4. $75, 3%.
5. $85, 4%.
6. $60, $4\frac{1}{2}\%$.
7. $80, $3\frac{1}{4}\%$.
8. $80, $4\frac{3}{4}\%$.
9. $200, 2%.
10. $250, $2\frac{1}{2}\%$.
11. $320, $3\frac{3}{4}\%$.
12. $5000, 4%.
13. $4800, 5%.
14. $6200, $5\frac{1}{2}\%$.
15. $7200, $4\frac{3}{4}\%$.
16. $1250, 3%.
17. $1750, $2\frac{1}{4}\%$.
18. $1875, 4%.
19. $2250, $3\frac{1}{2}\%$.
20. $3375, 4%.
21. $3270, $4\frac{1}{2}\%$.
22. $15,000, 3%.
23. $20,000, 2%.
24. $25,000, 3%.
25. $30,000, $2\frac{1}{2}\%$.

A SILENT READING LESSON

Farmers and market gardeners often sell their vegetables and other farm produce through *commission merchants* or *brokers.* Both the commission merchant and the broker are agents, and each receives a commission for his work.

The commission merchant actually receives the produce and sells it; the broker does not receive it, but finds a purchaser to whom the farmer makes delivery. For our purposes we need consider only the work of the commission merchant, because the problems in both cases are very much alike.

The commission merchant *deducts* (a business term meaning "subtracts") the amount of his commission from the amount that he receives from selling the produce, and *remits* (sends) the net amount, called the *proceeds,* to his *principal* (the man for whom he acts). That is,

Proceeds = amount of sale – amount of commission.

Suppose, for example, that a market gardener ships (sends by truck or express) to a commission merchant a quantity of vegetables, and that the transportation charges are \$18. If the commission merchant sells the vegetables for \$500 and charges a commission of 8%, what are the proceeds?

To solve this problem, we first take 8% of \$500, and have

$$0.08 \times \$500 = \$40, \text{ amount of commission.}$$

Then, deducting the amount of the commission, we have

$$\$500 - \$40 = \$460, \text{ the proceeds.}$$

There still remains the question of the transportation charges of \$18. If they are paid by the commission merchant, he deducts \$18 more and remits the balance. If paid by the market gardener, he charges them up as expenses on this lot of vegetables; that is, in either case the net amount received for the vegetables is \$460 – \$18, or \$442.

PROBLEMS IN COMMISSION

1. At $2\frac{1}{2}\%$, find the amount of the commission on the sale of some furniture for \$2400.

2. At $5\frac{1}{2}\%$, how much is the commission on the sale of some wagons for \$4800? How much are the proceeds?

3. A commission merchant sells for a factory 4800 lb. of cheese at 32¢ a pound and deducts a commission of 5%. What sum does he remit to the factory?

4. An auctioneer sells property for Mr. Sinclair for \$750 and receives a commission of 8%. How much is the commission? How much is left for Mr. Sinclair?

5. A city dealer sends an agent to a fruit region to buy for him and pays him a commission of 2%. The agent's purchases amount to \$2000 at one grove, \$3200 at another, and \$1600 at a third. What is his total commission? What is the total cost to the dealer, including the commission?

6. A real-estate dealer sells 30 A. of land at \$400 an acre. How much is his commission at $3\frac{1}{2}\%$?

7. An agent sells \$90 worth of maps to our school, and his firm pays him a commission of $8\frac{1}{2}\%$. How much is his commission on the sale?

8. Some butter and eggs which a farmer shipped to the city are sold for \$116.50. The commission merchant charges the farmer 8% for selling. What are the proceeds?

9. At 8%, how much is the auctioneer's commission on a sale of furniture amounting to \$1425?

10. At 6%, how much is the architect's commission on a house which costs \$12,500 to build?

11. A commission merchant receives 8% for selling some produce for \$175. How much is his commission?

Further Work in Commission. **1.** A salesman who received a commission of 4% was paid $25.60 for selling a second-hand touring car. At what price did he sell the car?

$$0.04\overline{)\$25.60}\qquad \begin{array}{r}\$640.\\ 4\overline{)\$2560.}\end{array}$$

Since $25.60 is the result of multiplying the selling price by 4%, we find the selling price by dividing $25.60 by 0.04.

The answer is $640.

Amount of sale = amount of commission ÷ rate.

2. A salesman received $25.60 for selling a second-hand touring car for $640. What was the rate of commission paid him?

$$\begin{array}{r}0.04\\ \$640\overline{)\$25.60}\\ \underline{2560}\end{array}$$

Since $25.60 is the result of multiplying $640 by the rate, we find the rate by dividing $25.60 by $640.

The answer is 0.04, or 4%.

Rate = amount of commission ÷ amount of sale.

FURTHER WORK IN COMMISSION

All work oral

Given the amount of a commission and the rate paid, as follows, find the amount of the sale:

1. $2, 1%.
2. $2, 2%.
3. $8, 1%.
4. $3, 3%.
5. $4, 2%.
6. $10, 10%.
7. $5, 5%.
8. $50, 5%.

Given the amount of a sale and the amount of the commission, as follows, find the rate paid:

9. $1000, $10.
10. $1000, $30.
11. $1000, $80.
12. $2000, $20.
13. $2000, $40.
14. $2000, $100.
15. $3000, $60.
16. $3000, $120.
17. $3000, $180.

PROBLEMS IN COMMISSION

1. If you buy a car for $600, and the salesman's commission is 5%, how much commission does he receive?

2. If you buy a car for $500, and the salesman's commission is $30, what rate of commission is he paid?

3. If you buy a car from a salesman who is paid a commission of 6% by his employer, and if the salesman receives $42 on the sale, how much do you pay for the car?

4. If you buy some wool from an agent to whom you pay a commission of 6%, and if you pay him $42, how much in all do you pay for the wool?

5. A farmer ships some vegetables to a city dealer, who sells them for $380 and charges a commission of 10%, plus $6.80 for express. How much does the farmer receive?

6. Helen's father gave her a commission of 25% on all the raspberries that she picked and sold from his bushes. From this record of her sales, find how much Helen made:

July 10	25 qt. @ 25¢	July 15	12 qt. @ 16¢
11	15 qt. @ 20¢	17	15 qt. @ 12¢
12	20 qt. @ 20¢	18	17 qt. @ 15¢
13	18 qt. @ 18¢	19	20 qt. @ 20¢

7. If the amount of a sale of farm produce is $1280, and the rate of commission is 6%, how much is the commission?

8. If the amount of a sale is $1725, and the commission amounts to $138, what is the rate of commission paid?

9. Which receives the greater amount, a salesman who sells $10,250 worth of goods at a commission of $1\frac{1}{2}$%, or one who makes a sale amounting to $1375 at a commission of 15%?

REVIEW OF COMMISSION

Copy the following tables and fill the blank spaces:

	Amount of Sale	Rate of Commission	Amount of Commission
1.	$350	4%	
2.	$480		$24
3.		5%	$17
4.	$300		$12
5.	$750	6%	
6.		8%	$2.40
7.	$750		$15
8.	$500	4½%	
9.		12%	$240
10.	$800		$64

	Amount of Sale	Rate of Commission	Amount of Commission
11.	$825	4%	
12.		6%	$14.40
13.	$2500		$500
14.	$1200	3½%	
15.	$3600		$196
16.		8%	$33.60
17.	$1600	3¾%	
18.	$275		$4.50
19.		10%	$14.75
20.	$2500	4½%	

In each of the following cases, which show the amounts of sales and the net amounts received, find the amount and rate of commission paid:

21. $200, $188.
22. $300, $285.
23. $500, $480.
24. $600, $564.
25. $650, $624.
26. $750, $735.
27. $750, $727.50.
28. $825, $800.25.
29. $1275, $1224.

In the following cases, each of which shows the cost of a purchase and the cost plus the commission, find the amounts and the rates of commission paid:

30. $200, $212.
31. $200, $216.
32. $300, $315.
33. $500, $550.
34. $700, $756.
35. $900, $927.
36. $750, $772.50.
37. $950, $959.50.
38. $1650, $1674.25.

A SILENT READING LESSON

After the baseball season a boy may be glad to sell his catcher's mask for less than the original price, so that he can use the money toward buying a football. If he sells the mask for 25% less than the original price, he sells it at a *discount* of 25%; that is, the price at which he sells it is found by deducting 25%, or $\frac{1}{4}$, of the original price.

In the above case, the 25% is sometimes called the *rate* of discount. Such rates are often given as fractions instead of as per cents. For example, merchants frequently advertise special sales at which they are to sell goods at "$\frac{1}{4}$ off," at "$\frac{1}{3}$ off," or at other reductions in price.

By selling at a discount, dealers often dispose of goods that they do not wish to carry over until another season, or goods which are slightly soiled or damaged.

The price that we pay for goods at a store is usually marked on the goods. This is why it is called the *marked price*. If we buy goods at a discount, the amount which we pay after taking off the discount is called the *net price*.

Thus, if we buy a tricycle which was marked \$7, but has been reduced 10%, it costs us \$7 less 10% of \$7; that is, it costs us \$7 – 70¢, or \$6.30. In this case the discount is 10%, the amount of the discount is 70¢, and the net price is \$6.30.

Amount of discount = rate × marked price.
Net price = marked price – amount of discount.

If a discount is allowed on a bill, the amount paid in settling the debt is called the *net amount* of the bill.

In a printed list or catalogue such as manufacturers or wholesale dealers often issue, the prices are generally given without discounts. Such prices are called *list prices*, and in selling to retailers discounts from these prices are allowed.

Amount of discount = rate × list price.

FINDING DISCOUNTS

Numbers 1 to 18, oral

1. If a certain article is marked $60, and a discount of 10% is allowed, what is the amount of the discount?

2. If on a book marked $4 a dealer offers a discount of $\frac{1}{4}$, the net price is how many cents below the marked price?

3. If on an order for $300 worth of goods a merchant is allowed a discount of 30%, what is the net amount the merchant must pay?

Find the amount of a discount of:

4. 10% on $70.
5. 10% on $45.
6. 20% on $80.
7. 20% on $65.
8. $\frac{1}{4}$ on $36.
9. 25% on $48.
10. $\frac{1}{2}$ on $50.
11. 50% on $80.
12. 10% on $200.
13. 20% on $400.
14. 25% on $640.
15. 50% on $260.

16. If an article listed at $15 is sold to a dealer at a discount of 20%, how much does it cost him?

17. If a merchant marks a lot of suits at $45 each and sells them at $\frac{1}{3}$ off, what is the net price of each suit?

18. Watch advertisements in the newspapers to see if discounts are offered. Be prepared to state to the class tomorrow some possible reasons for such discounts.

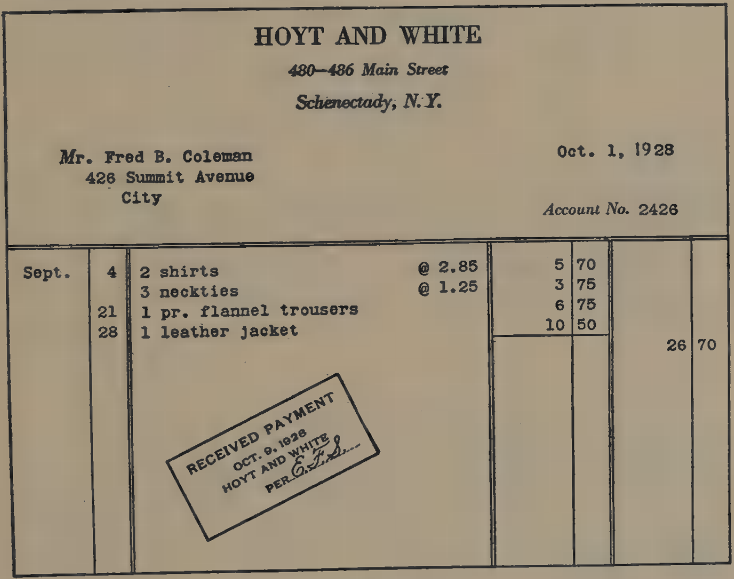
Find the net price of an article which is marked and sold at a discount, as follows:

19. $29.50, 20%.
20. $38.75, 20%.
21. $46.40, $\frac{1}{4}$.
22. $86.60, 25%.
23. $84.40, 25%.
24. $78.90, $\frac{1}{3}$.
25. $71.70, $33\frac{1}{3}$%.
26. $49.50, $16\frac{2}{3}$%.
27. $81.60, $\frac{1}{8}$.
28. $86.40, $12\frac{1}{2}$%.
29. $31.50, 5%.
30. $58.50, 10%.
31. $44.75, 20%.
32. $65.25, 20%.
33. $38.60, 10%.

A SILENT READING LESSON

When we purchase goods at a store and do not pay cash for them, the dealer sends us a *bill* for what we owe, usually on the first of the next month.

The following model shows such a bill, and on the bill is the dealer's receipt stamp to show that payment has been made:



HOYT AND WHITE
480–486 Main Street
Schenectady, N.Y.

Mr. Fred B. Coleman
426 Summit Avenue
City

Oct. 1, 1928

Account No. 2426

Sept.	4	2 shirts	@ 2.85	5	70		
		3 neckties	@ 1.25	3	75		
	21	1 pr. flannel trousers		6	75		
	28	1 leather jacket		10	50		
						26	70

RECEIVED PAYMENT
OCT. 9, 1928
HOYT AND WHITE
PER E.F.S.

Filling out a bill by inserting the amounts of the various purchases in the columns at the right is called *making the extensions* on the bill.

We sometimes hear the terms *debtor* and *creditor* used in speaking of bills. A debtor (Dr.) is one who owes money to another, and the creditor (Cr.) is the one to whom it is owed; that is, in the above case the debtor is Mr. Coleman and the creditor is Hoyt and White.

A SILENT READING LESSON (CONTINUED)

The bill that a manufacturer or a wholesaler sends to a retailer at the time that goods are shipped is often called an *invoice*. If the retail dealer is allowed a discount from the manufacturer's list prices, the amount of the discount generally appears on the invoice, as in the following model:

ALBANY, N.Y., Nov. 17, *1928*

DUFFY, WEST, AND BROWN
Manufacturing Jewelers
768 Webster Street

Sold to
R. E. Long Co.
124 First St.
Binghamton, N. Y.

Quantity	*Stock No.*	*Description*	*Price*					
3 doz.	406 A	Dessert forks	32	00	96	00		
1/6 doz.	327 P	Salad forks	33	00	5	50		
					101	50		
		Discount, 20%			20	30		
							81	20

Business men usually pay their bills by check. As we shall find, later on, a paid check is itself a receipt, and for this reason invoices and other bills paid in this way are not often receipted. A receipt may, however, be written separately, and in such cases would have a form something like this:

*$*81.20 *ALBANY, N.Y.,* Nov. 20, *1928*

Received of R. E. Long Co., Binghamton, N. Y.

Eighty-one and 20/100 *Dollars*

in payment of invoice of Nov. 17, 1928

Duffy, West, and Brown
Per C. T. Myers

Net Price by One Multiplication. We have learned that we can find the net price by first finding the amount of the discount and then subtracting this from the list price. However, when we take off 8% of a price, we have 92% of it left. We can therefore find the net price of an article by a single multiplication, if we wish.

For example, if the list price of an article is \$300, and the discount allowed is 8%, the net price is

$$(100\% - 8\%) \times \$300,$$

or

$$0.92 \times \$300;$$

that is, as shown at the right, it is \$276. Notice that we make the subtraction in the parentheses before we multiply.

100%	\$300 (list price)
8%	0.92
92%	\$276 (net price)

Net price = (100% − rate) × list price.

In problems we can now use either method, as required, but the one given on page 51 is sometimes simpler.

NET PRICE BY ONE MULTIPLICATION

Given the list price of an article and the rate of discount as follows, find the net price by one multiplication:

1. \$200, 2%.
2. \$350, 4%.
3. \$725, 4%.
4. \$560, 3%.
5. \$128, 5%.
6. \$2000, 2%.
7. \$2500, 3%.
8. \$3200, 8%.
9. \$4800, 6%.
10. \$7500, 8%.
11. \$25, 20%.
12. \$60, $33\frac{1}{3}\%$.
13. \$60, $16\frac{2}{3}\%$.
14. \$800, 25%.
15. \$600, $12\frac{1}{2}\%$.
16. \$400, 15%.
17. \$450, 14%.
18. \$620, 10%.
19. \$800, 16%.
20. \$900, $16\frac{2}{3}\%$.
21. \$1000, 12%.
22. \$1600, $12\frac{1}{2}\%$.
23. \$1200, 14%.
24. \$2000, 12%.

A SILENT READING LESSON

In some kinds of wholesale business two or more discounts are sometimes allowed on the same bill. For example, a dealer may buy $400 worth of hardware with discounts of 20% and 10%, often called "a discount of 20%, 10%." This means that 20% is first deducted from the amount of the bill, and 10% is then deducted from the remainder, as follows:

$$0.20 \times \$400 = \$80, \text{ and } \$400 - \$80 = \$320;$$
$$0.10 \times \$320 = \$32, \text{ and } \$320 - \$32 = \$288.$$

That is, the net amount of the bill for the hardware is $288.

This explains the general nature of the problem. Practically, however, we should take such short cuts as we can.

The solution is much shorter if we remember that 20% = $\frac{1}{5}$; that 10% = $\frac{1}{10}$; that if we take away $\frac{1}{5}$, we have $\frac{4}{5}$ left; and that if we take away $\frac{1}{10}$ from the result, we have $\frac{9}{10}$ left. That is, the solution becomes simply

$$\frac{9}{10} \text{ of } \frac{4}{5} \text{ of } \$400 = \frac{9 \times 4 \times \overset{80}{\cancel{\$400}}}{\cancel{10} \times \cancel{5}} = \$288.$$

Similarly, for a discount of $12\frac{1}{2}$%, $16\frac{2}{3}$%, we have

$$\frac{5}{6} \text{ of } \frac{7}{8} \text{ of } \$400 = \frac{5 \times 7 \times \overset{\overset{25}{\cancel{50}}}{\cancel{\$400}}}{\underset{3}{\cancel{6}} \times \cancel{8}} = \frac{\$875}{3} = \$291.66\tfrac{2}{3}.$$

That is, the net amount of the bill is $291.67.

In computing the amount of a discount, we disregard fractions less than $\frac{1}{2}$¢ and call $\frac{1}{2}$¢ or more a full cent.

The giving of several discounts, often called *chain discount*, is now becoming less common. The use of the single discount which is equivalent to two or more discounts obviously saves time and labor in bookkeeping.

SEVERAL DISCOUNTS

1. From $200 take 10%, and 10% from the remainder.

2. From $800 take 20%, and 10% from the remainder.

Find the net amounts of bills with discounts as follows:

3. $400, 20%, 20%.

4. $600, 25%, 5%.

5. $700, 25%, 10%.

6. $300, 15%, 2%.

7. What is the difference between a discount of 50% on $100 and one of 25%, 25%?

8. Is there any difference between a discount of 10%, 5% on $100 and one of 5%, 10%? How is it on $200? on $900?

9. What is the amount of the discount on a bill for $100 on which a discount of 10%, 8% is allowed? What single discount is equal to a discount of 10%, 8%?

Find the single discount that is equal to a discount of

10. 10%, 10%.

11. 10%, 12%.

12. 5%, 16%.

13. 8%, 12%.

14. 8%, 20%.

15. 10%, 20%.

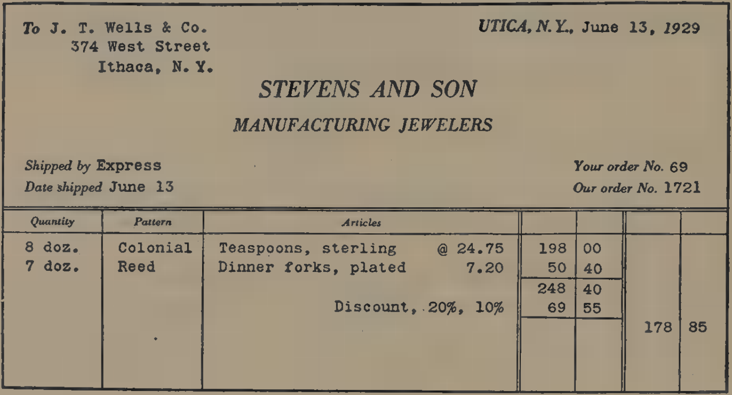
16. Find the net price of a tractor listed at $1220 less 10%, 5%, first by taking off the discounts separately, and then by taking a single discount as in Exs. 10–15.

17. An article listed in a catalogue at $278 is sold at a discount of 12%, with an extra discount of 2% for cash. Find the total amount of discount given a dealer who buys for cash.

18. A contractor received two bids for trim on a building that he was erecting. One was $3500 less 65%, 10%, and the other was $3400 less 50%, 16%. Which was the cheaper bid, and how much cheaper was it?

19. Find the net amount of a bill for $1000 less 10%, 5%, 2%; less 5%, 2%, 10%; less 2%, 10%, 5%.

Discounts on Invoices. The following model shows a form of a wholesaler's invoice for goods sold with two discounts:



To J. T. Wells & Co.
374 West Street
Ithaca, N. Y.

UTICA, N. Y., June 13, *1929*

STEVENS AND SON

MANUFACTURING JEWELERS

Shipped by Express
Date shipped June 13

Your order No. 69
Our order No. 1721

Quantity	*Pattern*	*Articles*				
8 doz.	Colonial	Teaspoons, sterling @ 24.75	198	00		
7 doz.	Reed	Dinner forks, plated 7.20	50	40		
			248	40		
		Discount, 20%, 10%	69	55		
					178	85

DISCOUNTS ON INVOICES

Write invoices for the following items, making up headings with dates and names like the above model:

1. 36 doz. files @ \$8.30. Discount, 30%, 20%.

2. 480 yd. silk @ \$2.50; 840 yd. velvet @ \$2.75; 425 yd. satin @ \$3.10; 635 yd. lawn @ 50¢. Discount, 25%, 10%.

3. 45 doz. locks with keys @ \$7.35; 9 doz. pairs hinges @ \$6.75; 36 doz. table forks @ \$8.40; 48 doz. table knives @ \$12.60. Discount, 20%, 10%.

4. 36 doz. locks @ \$6.30; 19 doz. mortise locks @ \$6.75; 28 doz. pairs hinges @ \$8.85. Discount, 25%, 8%.

5. 360 yd. taffeta @ \$1.37½; 9 gross pompons @ \$185; 6 doz. pieces braid @ \$31.80. Discount, 10%, 5%.

6. 780 yd. silk @ \$2.65; 440 yd. lawn @ 45¢; 560 yd. taffeta @ \$1.42. Discount, 10%, 5%, 5%.

Finding the Marked Price. Suppose that an article which I bought at a bargain sale for \$187.50 was sold to me at a discount of 25%. What was the original marked price?

The net price (\$187.50) is evidently the marked price less 25% of itself; that is, it is 75% of the marked price. Since \$187.50 is the result of multiplying the marked price by 75%, we divide \$187.50 by 0.75 to find the marked price, \$250.

$$\begin{array}{rl} 100\% &= \text{marked price}\\ \underline{25\%} &= \text{discount}\\ 75\% &= \text{net price} \end{array} \qquad 0.75\overline{)\$187.50} \qquad \begin{array}{r} \$250.\\ 75\overline{)\$18750.}\\ \underline{150}\\ 375\\ \underline{375} \end{array}$$

Marked price = net price ÷ (100% − rate).

In the above case it is easier to use fractions; that is,

$$\$187.50 \div \frac{3}{4} = \$\overset{62.50}{\cancel{187.50}} \times \frac{4}{\cancel{3}} = \$250.$$

Always watch for an opportunity to use such short cuts.

Finding the Rate of Discount. If in selling an article a discount of \$62.50 is allowed on the marked price of \$250, what is the rate of discount?

The amount of the discount is the rate times the marked price; that is, we have

$$\$62.50 = \text{rate} \times \$250.$$

To find the rate, therefore, we divide \$62.50 by \$250 and have

$$\$62.50 \div \$250 = 0.25;$$

that is, the rate of discount is 25%, or $\frac{1}{4}$.

Rate = amount of discount ÷ marked price.

Price List. The following shows the list prices of certain school supplies, with the discounts allowed to dealers:

Composition books,	$11.00 per gross, less 40%
Drawing compasses,	$5.40 per dozen, less 20%, 5%
Drawing paper, 9 × 12,	$3.80 per package, less 40%
Drawing pencils,	$11.42 per gross, less 30%
Penholders,	$8.40 per gross, less 30%, 5%
Pens,	$1.62 per gross, less 25%, 10%
Rulers,	$0.95 per dozen, less 25%
Thumb tacks,	$1.20 per 100, less 35%
Tubes of paste,	$10.10 per gross, less 20%, 10%

DISCOUNTS FROM LIST PRICES

1. A dealer orders 400 thumb tacks and 8 packages of drawing paper at the prices shown in the above list. After deducting the discount, how much does he pay for them?

2. How much does a dealer pay for 5 gross of penholders and 9 gross of pens? for 3 gross of pens and 2 doz. rulers?

3. For a class of 36 pupils a dealer buys drawing compasses and rulers, one for each pupil. How much do they cost him?

4. A dealer buys a gross each of rulers and drawing pencils. He sells the rulers at 15¢ each and the pencils at 12¢ each. His total receipts are how much more than the cost?

5. If a dealer buys a gross of tubes of paste and sells them at 15¢ each, his receipts are how much more than the cost?

6. On an order for 24 doz. rulers of a better kind, a dealer is allowed a discount of 25%. If he pays $22.50 for the lot, what is the list price per dozen?

7. In buying 72 doz. erasers at 45¢ a dozen, a dealer is allowed a discount amounting to $9.72 on the lot. What rate of discount is he allowed?

A SILENT READING LESSON

Another subject that we need to know about is *insurance.* We may never be insurance agents; but we all hope sometime to have property enough to insure, or it is probable that we may wish sometime to insure our lives.

If a man's home should be destroyed by fire, the loss would be very great to him and his family. It might take the man a long time to save money enough to buy or build a new home. It would be much better if people would join together, each contributing a small sum annually to a common fund. Then if the house of any one of them took fire, the owner could be paid, from the fund, enough to cover his loss.

This shows us the general nature of insurance. Usually, however, instead of a number of persons joining together, each pays a small annual sum, or *premium,* to an insurance company. The written agreement covering the details of the insurance is called the *policy.* The sum of money specified to be paid in case of loss is called the *face* of the policy.

The necessity for insuring against loss by fire is seen when we consider that the fire losses in this country in recent years have averaged over a million dollars a day. In times of disaster, such insurance is helpful not only to the one who insures but also to the entire community.

We should, if possible, make inquiry about the fire insurance on houses and other property in our vicinity and report to the class. We should also inquire about other kinds of insurance, such as life, accident, and health insurance, burglary insurance, and automobile insurance.

The growth of insurance on people's lives is seen from the fact that in 1850 the total insurance of this kind in our country was less than $70,000,000, whereas, in a recent year, the total amounted to more than $54,000,000,000.

Fire Insurance. The *rate* in fire insurance, unlike the rate in commission or discount, is not stated as a per cent. It is usually given as a certain sum for each $100 of insurance for a certain period, generally 1 yr., 3 yr., or 5 yr.

For example, in finding the premium, at $1.20 a year, for insuring a store for $4000 against loss by fire, we have

$$\$4000 = 40 \times \$100,$$

$$40 \times \$1.20 = \$48, \text{ the premium.}$$

Premium = number of hundreds in face × rate per $100.

This relation is usually expressed briefly, as follows:

Premium = face × rate.

If possible, bring an old fire-insurance policy to school and show it to the class.

FIRE-INSURANCE PREMIUMS

All work oral

1. At 90¢ a year, what is the annual premium on a policy for $1000? at 70¢ a year on a policy for $3000?

2. A man insured his house for $10,000 at a rate of $2.50 for 3 yr. What was the premium?

State the premium on a fire-insurance policy for

3. $1000 at 60¢.
4. $2000 at 70¢.
5. $2250 at $1.
6. $4000 at $2.
7. $2750 at $1.
8. $3000 at $2.
9. $7000 at $2.
10. $8000 at $2.
11. $3000 at $1.20.
12. $1000 at $2.25.
13. $10,000 at $0.80.
14. $10,000 at $1.50.

15. What is the usual length of time for which fire insurance is written in your city or town? What is the usual premium per $100 on the most common type of residential property? What conditions determine the rates of insurance?

Finding the Face of the Policy. The face of a policy is always stated in the policy itself. Even though the problem would hardly arise in business, it is interesting to see that we can easily find the face if we know the premium and rate.

For example, at 97¢ per \$100, what is the face of a fire-insurance policy when the premium is \$24.25?

Since 97¢ buys \$100 of insurance, the face is as many times \$100 as 97¢ is contained in \$24.25; that is,

$$\$0.97\overline{)\$24.25} \qquad \begin{array}{r} 25 \\ \$97\overline{)\$2425} \\ \underline{194} \\ 485 \\ \underline{485} \end{array}$$

$$\$24.25 \div \$0.97 = 25, \text{ the number of hundreds,}$$

and

$$25 \times \$100 = \$2500, \text{ the face.}$$

That is, briefly stated, we have the following relation:

Face = premium ÷ rate.

FACE OF THE POLICY

Find the face of a fire-insurance policy, given the premium paid and the rate per \$100 as follows:

1. \$17.60, 80¢.
2. \$28.80, 90¢.
3. \$137.50, \$1.
4. \$59.50, 85¢.
5. \$76.95, 95¢.
6. \$69, 75¢.
7. \$81.78, 87¢.
8. \$69, 92¢.
9. \$100.80, \$1.12.
10. \$85.40, 70¢.
11. \$81.78, 94¢.
12. \$80.36, 98¢.
13. \$80.36, 82¢.
14. \$80.36, \$1.64.
15. \$160.72, \$1.64.
16. \$160.72, 82¢.
17. \$101.50, 70¢.
18. \$101.50, \$1.45.
19. \$34.20, 76¢.
20. \$62.70, \$1.14.
21. \$72.80, \$1.12.
22. \$145, \$1.16.
23. \$146.25, \$1.17.
24. \$169.40, \$1.21.
25. \$356.70, \$1.23.
26. \$281.25, \$1.25.
27. \$233.45, \$1.15.

Finding the Rate. If it is necessary, we can easily find the rate when the face of a policy and the premium are known.

For example, if the face of a policy is \$8500 and the premium is \$93.50, what is the rate?

Since $\$8500 = 85 \times \$100,$

we see that each \$100 of insurance costs $\frac{1}{85}$ of \$93.50; that is, as shown in the work at the right, we have

$$\$93.50 \div 85 = \$1.10, \text{ the rate.}$$

$$\boxed{\begin{array}{r} \$1.10 \\ 85\overline{)\$93.50} \\ \underline{85} \\ 85 \\ \underline{85} \end{array}}$$

This shows us that to find the rate, we may divide the premium by the number of hundreds of insurance represented by the face of the policy; that is, stated briefly, we have the following relation:

Rate = premium ÷ face.

FINDING THE RATE

Find the rate per \$100, given the face of a fire-insurance policy and the premium as follows:

1. \$2000, \$18.
2. \$2000, \$14.
3. \$3000, \$24.
4. \$4000, \$34.
5. \$5000, \$35.
6. \$5000, \$45.
7. \$6000, \$60.
8. \$7000, \$77.
9. \$7500, \$75.
10. \$8000, \$100.
11. \$8000, \$120.
12. \$9000, \$90.
13. \$10,000, \$125.
14. \$12,000, \$150.
15. \$4000, \$48.
16. \$4500, \$49.50.
17. \$5700, \$62.70.
18. \$7800, \$93.60.
19. \$4200, \$54.60.
20. \$8400, \$109.20.
21. \$16,800, \$218.40.

22. Two neighbors were discussing fire insurance. One, who lived in a brick house, said that he paid a premium of \$21.60 for a policy for \$12,000. The other, who lived in a wooden house, paid \$19.80 for a policy for \$9000. Which paid the higher rate? Why are the rates different?

FIRE INSURANCE

1. At 98¢, what is the premium on a policy for $4000?

2. At $1.20, what is the premium on a policy for $6000?

3. At $1.25, what is the premium on a policy for $8000?

4. A building worth $15,000 is insured for $\frac{2}{3}$ of its value at $1.20. What is the premium?

5. If a man insures the contents of his store for $9000 at $1.25, what is the premium?

6. The premium at $1.50 for insuring a stock of goods is $105. What is the face of the policy?

7. A man insured his factory, valued at $225,000, for $\frac{4}{5}$ of its value. At $1.80, what was the premium?

8. A dealer insured his stock of goods, valued at $24,000, for $\frac{7}{8}$ of its value. At $1.45, what was the premium?

9. If a three-year policy for $3600 costs $59.40, what is the rate for the three years?

10. If the rate for 3 yr. is $2\frac{1}{2}$ times the annual rate, what is the annual rate in Ex. 9?

11. If a 3-year policy for $6000 costs $72, what is the rate for the 3 yr.? As in Ex. 10, what is the rate at which a 1-year policy would be written?

12. The premium paid by a manufacturer who insured his factory for $\frac{5}{8}$ of its value was $420. At a rate of $2, what was the face of the policy? What was the value of the factory?

13. A factory worth $42,000 is insured for 80% of its value, at $2. If fire caused a total loss, how much would the owner lose, including the premium paid?

14. A man insures his house for its full value, $5000, at $1.25. What is the premium? If fire causes a loss of 60% of the value, how much does he receive from the company?

Life Insurance. If a man has a family or others dependent upon him, he may provide protection for them, in case of his death, by insuring his life. Then, if he should die while the policy is in force, the insurance company will pay the face of the policy to the *beneficiary* that he has named.

The earlier a person takes out life insurance, the lower will be the premium, and the sooner will he cease to make payments if the policy calls for a definite number of payments. Every young person should consider taking out a life-insurance policy in some first-class company, because such insurance is valuable not only in providing protection for families but also in making provision for old age when a person's earning power is usually decreased.

Rates in life insurance are stated at so much per year for each $1000 of insurance. They vary with the age of the person insured, reckoned at the time he takes out the policy. The premiums are payable in advance.

LIFE INSURANCE

Find the premium on a life-insurance policy, given the face and the rate per $1000 as follows:

1. $2000, $27.10.
2. $3000, $25.64.
3. $2000, $21.80.
4. $4000, $32.45.
5. $3000, $18.72.
6. $5000, $28.64.
7. $6000, $31.65.
8. $8000, $27.30.
9. $15,000, $28.40.
10. $25,000, $22.20.
11. $35,000, $27.65.
12. $65,000, $24.12.

Find the face for a premium and rate per $1000 of

13. $262, $26.20.
14. $220, $55.
15. $225.40, $32.20.

Find the rate per $1000 for a face and premium of

16. $2000, $60.24.
17. $4000, $96.36.
18. $5000, $158.25.

A SILENT READING LESSON

The chief characteristics of the four leading kinds of life-insurance policies are as follows:

In an *ordinary life* policy the person insured agrees to pay during his lifetime a certain annual premium. Premiums may be paid also semiannually or quarterly, but usually there is a little saving by paying only once a year. The face of such a policy is not payable until the death of the one insured, and the payment is then made to the beneficiary.

In a *limited-payment life* policy the premiums are payable for a fixed number of years, often twenty. At the end of the specified time the policy is said to be *paid up*, but the face is not paid until the death of the person insured.

In an *endowment* policy the premiums are paid for a fixed number of years, often forty, thirty, or twenty. At the end of this period, the face is paid to the person insured, either all at once or in installments. The premiums on limited-payment life and endowment policies are naturally higher than those on ordinary life policies.

In a *term* policy the premiums are paid for a specified length of time, but the face of the policy is payable only if the person insured dies within the specified period. Such policies do not usually run for more than five or ten years.

There are various other forms of policies. Some are arranged so that a certain amount will be paid annually (an *annuity*) during the lifetime of the one for whose benefit the policy is written.

A man may also insure his health for a definite period or he may insure himself against death or injury by accident. The first of these forms of insurance is called *health insurance*; the second, *accident insurance*. The principle of accident and health insurance is substantially like that of life insurance.

LIFE INSURANCE

1. At $31.05 per $1000, how much is the annual premium that a man must pay on a 30-year endowment policy for $8000? How much would he pay in the 30 yr., not considering the question of interest on the several payments?

2. If a man takes out a 20-payment life policy for $10,000 at $27.24, how much, not considering interest, will he have paid when the policy matures; that is, after 20 payments?

3. How much is the annual premium on a 20-payment life policy for $14,000 if taken out by a man at such an age that the rate is $29.95? Find the total amount of the premiums for the 20 yr., not considering interest.

4. A man takes out a policy for $2500 in one company, at $24.12. If the next year he takes out a policy for $3500 in another company, at $24.89, how much does he then pay each year for life insurance?

5. At a rate of $29.53, a young man takes out an endowment policy for $10,000 on the 30-payment plan. How much is the annual premium? How much does he pay in premiums during the 30 yr., not considering interest on the payments? How much more than he has paid does he receive from the company when the policy matures?

6. A man took out a policy for $5000 at $26.40. He died just before the eighth annual payment was due. How much more did the beneficiary receive than the man had paid the company in premiums?

7. Twenty years ago a man took out a 20-payment life policy for $5000 at $25.41. How much has he paid in the 20 yr.? At his present age the rate is $38.51. If he takes out a new policy for $5000, how much more will he have to pay in 20 yr. on this policy than on the first one?

8. After a certain amount has been reserved for unexpected losses, the profits of mutual life-insurance companies are divided among the policy holders in proportion to the size of the policy which each one holds. Such payments, called *dividends,* may be taken in cash, they may be used to reduce the amount of the premium, or they will buy more paid-up insurance. At $21.15, what is the total amount of premiums paid on an ordinary life policy for $10,000 in 20 yr.? What is the net amount paid by the policy holder if the company during this period allows $74.75 in dividends on each $1000 of insurance?

9. For a 20-payment life policy taken out by a man 21 yr. old the rate is $25.41, and the dividends amount to $88.23 on each $1000 in 20 yr. If he uses the dividends to reduce the premiums, how much does he actually pay for a policy for $3000 by the time it is paid up?

10. Suppose the man in Ex. 9 waits until he is 50 yr. old, when the premium rate is $54.65, and the dividend rate is $198.60 per $1000. What would he then pay for a 20-payment life policy for $3000?

11. For a man 21 yr. old the rate on a 20-year endowment policy is $45.45, and the dividends amount to $189 per $1000 in 20 yr. What is the difference between the amount paid the company (less dividends) and the amount received at maturity on a policy for $5000?

12. A man pays $50 a year, in advance, for an accident policy for $10,000. If he keeps the insurance in force for 10 yr., how much does he pay the company?

13. A man pays $75 a year, in advance, for health insurance. The second year the company pays him $175 while he is ill. How much does he receive in excess of the premiums paid?

Workmen's Compensation Laws. In many states there are laws requiring that employers in certain industries shall pay compensation to a workman who is injured or killed while at work. Such industries generally include the construction or operation of railroads, electrical work, machine shops, subways, building operations, and the like. In the state of New York, farm laborers and domestic servants are not subject to compensation. Usually an employer insures his employees in some company which agrees to pay the required compensation.

The laws vary in different states, but in the state of New York the law specifies the following payments in case of death: $100 for funeral expenses; 30% of the workman's wages for the widow (to cease at her death or remarriage); 10% additional for each child until the age of 18 yr.; and 15% for each orphan until the age of 18 yr. The law also specifies certain per cents and periods of compensation in case of disablement, the award for total disability being two thirds of the wages.

The importance of the subject may be seen from the fact that in a recent year there were about 350,000 industrial accidents in the state of New York. The facts mentioned above need not be memorized, but should be referred to in solving the following typical problems.

WORKMEN'S COMPENSATION LAWS

1. A workman's wages are $28 a week. In case of his death, how much does his widow receive per week and for how long?

2. In Ex. 1, what is the total amount that will be paid to a child who is exactly 17 yr. old when the father is killed?

3. A workman's wages are $30.60 a week. If he has a disability allowance of $66\frac{2}{3}$% for 20 wk., how much will he receive per week? for the whole period?

A SILENT READING LESSON

We all owe a great deal to our parents, to our school, to the state in which we live, and to the United States. Each one of our several types of government, national, state, city, and town, spends large sums of money each year for educating us, erecting schoolhouses, protecting lives and property, building roads, and keeping public buildings in repair, — all of which help to make the conditions of our life such as to give us a chance to succeed in the world.

In return we owe it to the state to help support it and to make it strong and safe not merely for ourselves but for those who come after us.

One of the ways in which we can help in supporting the state is through the payment of *taxes*. To illustrate what a tax is, suppose that a class wishes to buy a picture for the schoolroom. There are 30 pupils in the class, and the picture that they wish to buy costs $4.20. Each pupil agrees to pay his share. The committee which is appointed to raise the money finds that the share of each member of the class is 14¢; that is, the total amount to be raised by the class is $4.20, and the tax on each pupil is 14¢. The committee appoints a *collector* to receive the money, and with the money which he collects the committee buys the picture.

Another way to support the state is to see, by taking an interest in politics, that the best men and women are elected to office, and so bring it about that the money raised by taxes is spent wisely. Experience has shown that it is a wise thing for every person, at the beginning of each year, to make out a budget in which he puts down his probable income and how he will spend it. It is now being generally recognized that it is desirable that the state and the nation should also budget their receipts and expenditures.

Taxes in a Village. Suppose that a village needs $75,000 for building a new schoolhouse, and its citizens vote to raise the amount by taxes. To raise so large a sum by taxing each citizen equally would be too heavy a burden upon some members of the community. Each citizen, therefore, is required to pay according to the value of his property; for example, a man who owns $10,000 worth of property pays twice as much as one who owns $5000 worth.

Taxes in a City. It takes a large amount of money to pay all the expenses of a city. The streets have to be cleaned, lighted, and kept in repair, the policemen have to be paid, the fire department has to be kept up, the city buildings have to be cared for, and the schools must be supported. Besides these there are many other expenses, and all of them must be met by money raised by taxes.

Taxes in the County, State, or Nation. In the same way, money is needed to pay the expenses of the county, the state, and the nation. All this money must be raised by taxes of some kind.

Tax Rate. Officials, called *assessors,* are generally elected or appointed to place a value, called the *assessed valuation,* on all property, such as land, buildings, automobiles, and other things, subject to taxation. After the total assessed valuation and the amount to be raised for all expenses is determined, it is a simple matter to determine the *tax rate.*

For example, if a city with a total assessed valuation of $2,500,000,000 must raise $75,000,000 to pay the year's expenses, each citizen should pay an amount equal to $75,000,000 ÷ 2,500,000,000, or $0.03, on each $1 worth of property that he owns. This rate of 3¢ on $1 may be expressed as 30 mills on $1, as $3 on $100, or as $30 on $1000, the last being the most common way.

Finding the Tax Rate. Suppose that a town with a total assessed valuation of \$6,400,000 must raise \$118,400 to meet the running expenses of the schools this year. What will be the school tax rate this year?

$$\begin{array}{r} \$0.0185 \\ 64\cancel{0}\cancel{0}\cancel{0}\cancel{0}\cancel{0})\overline{\$1.18400} \\ \underline{64} \\ 544 \\ \underline{512} \\ 320 \\ \underline{320} \end{array}$$

Since the tax on \$6,400,000 is \$118,400, the tax on \$1 is found by dividing \$118,400 by 6,400,000, as here shown. That is, to find the tax rate, we divide the amount to be raised by the number of dollars of assessed valuation, or briefly stated,

Rate = total to be raised ÷ assessed valuation.

The above result, \$0.0185, may be expressed as a tax rate of $18\frac{1}{2}$ mills on \$1, as \$1.85 on \$100, or as \$18.50 on \$1000.

Finding the Amount of the Tax Bill. It is now a simple matter to find the tax that each property owner must pay. For example, if the rate is \$18.50 per \$1000, in finding the amount paid on property assessed at \$12,000, we have

$$\$12{,}000 = 12 \times \$1000,$$

and $12 \times \$18.50 = \$222.00$, amount of the tax.

That is, the amount of the tax is the number of thousands of assessed valuation (or the number of hundreds at a given tax rate per \$100) times the rate per \$1000 (or per \$100), or briefly expressed,

Amount of tax = assessed valuation × rate.

Finding the Assessed Valuation. It is also an easy matter to find the assessed valuation of a piece of property when we know the rate and the amount of taxes paid. That is, from the last relation, given above, we see that

Assessed valuation = amount of tax ÷ rate.

PROPERTY TAXES

1. At $2.18 on $100, find the tax on a valuation of $8000.

Find the amount of the tax bill when the assessed valuation and the rate are as follows:

2. $8500, $1.46 on $100.
3. $9000, $2.10 on $100.
4. $16,500, $15.80 on $1000.
5. $25,000, $17.60 on $1000.

6. The assessed valuation of a city is $75,000,000, and the total amount to be raised is $1,845,000. Find the tax rate, expressing it as so many dollars and cents on each $1000.

7. At a tax rate of 19.6 mills on $1, what is the tax on property assessed at $10,000?

8. At a tax rate of $2.20 on $100, what is the tax on property assessed at $25,000?

9. At a tax rate of $16.30 on $1000, what is the tax on property assessed at $12,500?

Find the tax at $21.60 on $1000 on each of these valuations:

10. $4500.
11. $6500.
12. $8000.
13. $4600.
14. $7000.
15. $4500.
16. $8700.
17. $9700.
18. $6750.
19. $8500.
20. $12,000.
21. $25,000.
22. $32,500.
23. $38,500.
24. $45,000.
25. $75,000.

26. A man died, leaving an estate valued at $23,750, of which $22,000 was willed to his son and the rest to a nephew. The son did not have to pay any state inheritance tax on the first $5000, but paid 1% on the rest, and the nephew paid 5% on all he received. Find the total inheritance tax paid.

27. If a man's tax is $195, and the rate is $13 on $1000, what is the assessed valuation of his property? What is the assessed valuation of property which is taxed $234?

Collectors' Fees. A collector of taxes usually receives a fixed salary for his work. In some places he receives a certain per cent on the taxes collected, which is added to the taxpayer's bill in advance.

For example, if the collector's fee is 1% of the tax, and if a man's property is assessed at $8000 in a place where the tax rate is $18.50 per $1000, the total amount that the taxpayer must pay is found as here shown; that is, he must pay $149.48.

Tax per $1000,	$18.50
Number of thousands,	8
Amount of tax,	$148.00
	0.01
Collector's fee,	$1.4800
Amount of tax,	$148.00
Total amount,	$149.48

COLLECTING TAXES

When the collector is paid a salary, find the amount of the tax bill for a valuation and rate per $100 as follows:

1. $4000, $1.32.
2. $4500, $1.37.
3. $5200, $1.50.
4. $5500, $1.54.
5. $7000, $1.76.
6. $8000, $2.29.
7. $8500, $2.31.
8. $2700, $1.93.
9. $2500, $2.37.
10. $4500, $2.40.
11. $9000, $2.32.
12. $9500, $2.48.
13. $12,000, $1.87.
14. $12,500, $2.90.
15. $15,000, $2.45.

Find the amount of the tax bill, when the valuation, the rate per $1000, and the collector's fee are as follows:

16. $10,000, $12.60, 1%.
17. $15,000, $13.70, 1%.
18. $37,000, $13.20, 1%.
19. $47,000, $14.90, 1%.
20. $16,500, $18.70, $1\frac{1}{4}$%.
21. $17,500, $21.60, $1\frac{1}{2}$%.
22. $25,000, $23.40, $1\frac{1}{2}$%.
23. $27,500, $28.80, $1\frac{1}{2}$%.

Tax Table. To assist in the work of making out the tax bills, after the rate has been fixed, a *tax table* similar to the one below is often prepared. The following table, which is only a small specimen, is arranged for a tax rate of $1.78 on $100 (or of 17.8 mills on a dollar, or of $17.80 on $1000):

	0	1	2	3	4	5	6	7	8	9
0	0.00	0.178	0.356	0.534	0.712	0.890	1.068	1.246	1.424	1.602
1	1.78	1.958	2.136	2.314	2.492	2.670	2.848	3.026	3.204	3.382
[illegible]	3.56	3.738	3.916	4.094	4.272	4.450	4.628	4.806	4.984	5.162
[illegible]	5.34	5.518	5.696	5.874	6.052	6.230	6.408	6.586	6.764	6.942
4	7.12	7.298	7.476	7.654	7.832	8.010	8.188	8.366	8.544	8.722
5	8.90	9.078	9.256	9.434	9.612	9.790	9.968	10.146	10.324	10.502
[illegible]	10.68	10.858	11.036	11.214	11.392	11.570	11.748	11.926	12.104	12.282
7	12.46	12.638	12.816	12.994	13.172	13.350	13.528	13.706	13.884	14.062
8	14.24	14.418	14.596	14.774	14.952	15.130	15.308	15.486	15.664	15.842
9	16.02	16.198	16.376	16.554	16.732	16.910	17.088	17.266	17.444	17.622

In this table the hundreds' figure of the assessed valuation is shown in the column at the left, and the tens' figure at the top. Thus, at $1.78 on $100, the tax on $300 is $5.34; on $140 it is $2.49 (2.492); and on $90 it is $1.60 (1.602).

Using the Table. 1. At $1.78 per $100, how much tax must a man pay on property assessed at $8500?

We proceed as if we wished to find the tax on $850. In the table we look in the first column for 8, the first figure in $850, and then follow along this line to the column headed 5, the second figure in $850. We there find the number 15.130, which means that the tax on $850 is $15.13. Therefore, on $8500, which is 10 × $850, the tax is $151.30.

2. At $17.80 on $1000, how much tax must a man pay on property assessed at $17,500?

The tax on $17,000 is $302.60 (that is, it is 100 times the tax on $170, or 100 × $3.026), and the tax on $500 is $8.90. Hence the tax on $17,500 is $302.60 + $8.90, or $311.50.

USING THE TAX TABLE ON PAGE 76

1. At a tax rate of $1.78 on $100, what is the amount of the tax on property assessed at $6000? at $7000? at $7500? at $8600? at $9800?

2. At a tax rate of $17.80 on $1000, what is the amount of the tax on property assessed at $10,000? at $12,000? at $16,000? at $16,500? at $25,600?

3. At a tax rate of 17.8 mills on $1, what is the amount of the tax on property assessed at $7800?

4. At a tax rate of $0.0178 on $1, what is the amount of the tax on property assessed at $22,800?

At a tax rate of $17.80 on $1000, find the taxes on property assessed at the following amounts:

5. $9300.
6. $6700.
7. $8000.
8. $9000.
9. $8500.
10. $15,000.
11. $22,500.
12. $25,000.
13. $32,500.
14. $42,500.
15. $55,000.
16. $75,000.
17. $75,500.
18. $80,000.
19. $87,500.
20. $125,000.
21. $675,000.
22. $275,000.
23. $350,000.
24. $875,000.

25. Taxes in a certain city are payable in semiannual installments on May 1 and Nov. 1. If the whole tax is paid on May 1, a discount of half a year's interest at the rate of 4% a year is allowed on the second payment. If the rate is 17.8 mills on $1, and if the whole tax is paid on May 1, how much is due on a piece of property assessed at $3000?

26. A company owns four parcels of land in four different cities. One parcel is assessed at $35,000, the second at $8000, a third at $20,000, and the fourth at $40,000. If the tax rates in these cities are, respectively, $21.40, $17.80, $22.60, and $19.90 on $1000, what is the total of the company's taxes?

A SILENT READING LESSON

In order to make sure that all money collected for the village, city, county, state, or nation goes to the government for which it is intended, tax collectors have to *give bonds* that they will pay to the proper officers all money collected. This means that some company will insure the village, county, and so on against loss on the part of the collector. In the same way, the treasurer and any other officer through whose hands the money passes must give bonds for the honest and careful discharge of his duty.

A man may be perfectly honest, but he might lose the government's money through carelessness or through lack of ability to handle large sums. Against such dangers the government must protect itself.

In order to be able to discuss the question of local taxes in class, we should find out, if possible, what the property tax rate is in our community, what the chief expenses are, how our rate differs from some other rates in the state, and what the reasons are for this difference. At the same time we can find out what officials have to give bonds.

Part of the money raised by some states is collected through taxes on people's incomes. The general principle of such taxes is that those who receive large incomes during a year pay more taxes than those who receive less. In a recent year the rate of income tax in the state of New York was 1% on the first $10,000, 2% on the next $40,000, and 3% on larger incomes. A man whose taxable income was $30,000 a year had, at that time, to pay 1% of $10,000, or $100, and 2% of $20,000, or $400, making a total of $500.

Much of the income of the United States government comes from taxes of this kind, the rates for which are higher than those of the state of New York.

TAXING FOR GOOD ROADS

1. The highway tax, which was included in the county tax, on a certain farm for four successive years was as follows:

YEAR	ASSESSED VALUATION		HIGHWAY TAX RATE PER $1000
	Land	Buildings	
1927	$4600	$1300	$1.56
1928	4600	1300	1.78
1929	5000	1450	2.32
1930	5000	1450	1.96

The increase in the valuation of the land and buildings was due to the construction of good roads. If the roads had not been improved, and the valuation of the farm had remained unchanged, what would the total highway tax have been for the four years at the 1927 rate? How much would the owner of the farm have saved on his highway taxes?

2. At the end of 1930, after paying the year's taxes, the owner sold the farm for its assessed valuation in that year as given in Ex. 1. If the assessed valuation in 1927 represented fairly the value of the farm on that date, and if we consider the increase in value as due wholly to the building of good roads, did the owner gain or lose by their construction? Determine the gain or loss.

3. In the same county in which the farm in Ex. 1 was located, it cost $24,000 to maintain 60 mi. of macadam road and $1000 for the maintenance of 55 mi. of concrete road. Find the cost per mile of maintaining each type of road.

4. In Ex. 3, the cost of maintaining a mile of the macadam road was what per cent greater than the cost of maintaining a mile of the concrete road?

A SILENT READING LESSON

The expenses of the United States government naturally vary from year to year. The enormous cost of the World War made a corresponding increase in the annual expenses, because much of the national income must be devoted to paying for the war. But however large they may be, it is our duty to raise the money to pay the debts and expenses of the government. If the governments of the world would agree not to make war in the future, the cost of government could be greatly reduced throughout the world.

Recently the expenditures of our government averaged about $3,800,000,000 a year, or about $10,000,000 a day, but they are now being reduced from year to year. Some of the national receipts and expenditures were recently as follows:

Receipts	
Customs (duties on imported goods)	$546,000,000
Internal revenue (income tax, and so on)	2,800,000,000
Miscellaneous	666,000,000
Total receipts	$4,012,000,000

Expenditures	
War Department	$341,000,000
Navy Department	324,000,000
Pensions	230,000,000
Veterans' Bureau	101,000,000
Interest on the public debt	940,000,000
Other money appropriated by Congress	1,892,000,000
Total appropriations	$3,828,000,000

We can find out the receipts and expenditures for recent years by consulting one of the large almanacs, and can then discuss in class the reasons for such enormous expenditures.

Tariff. The United States collects a large part of its income by a tax on goods brought into the country. This income is called *customs revenue, tariff,* or *duty.* Duties are collected at *customhouses,* which are situated at important harbors on the coasts or important places on the Canadian and Mexican borders. Such places are called *ports of entry.*

Classes of Goods. Goods imported may be on the *free list*; that is, not subject to duty, as in the case of raw silk. They may be subject to *ad valorem* duty; that is, to a certain per cent on the value, as in the case of toys on which the duty specified in the present tariff law is 70% ad valorem. They may be subject to *specific* duty; that is, to a certain amount per unit of measure, as in the case of sheep on which the duty is $2 a head. They may be subject also to both ad valorem and specific duty, as in the case of perfumery on which the duty is 40¢ per pound and 75% ad valorem.

DUTIES ON IMPORTS

Numbers 1 to 10, oral

State the duty in each of the following cases:

1. At 1¢ per pound on 800 lb. of lard; on 8000 lb.
2. At 5¢ per pound on 100 lb. of cheese; on 1000 lb.
3. At 30% on $1000 worth of baseballs; on $1200 worth.
4. At 50% on straw hats worth $1000; worth $8000.
5. At 70% on dolls worth $1000; worth $3000.
6. At 50% on fur coats worth $1500; worth $20,000.
7. At $5 per dozen pairs on 450 doz. pairs of gloves.
8. At 40% on a shipment of violins worth $8000.
9. At 20% on a string of pearls worth $1250.
10. At 40% on a shipment of umbrellas worth $4000.

11. How much is the duty at 55% on a shipment of silk handkerchiefs worth $6500? Is this a specific duty or an ad valorem duty?

12. How much is the duty on 3500 bu. of barley at 20¢ per bushel? Is this a specific duty or an ad valorem duty?

13. At 24¢ a pound, find the duty on 2750 lb. of hops; on 12,500 lb.; on $2\frac{1}{2}$ T.

14. At 8¢ a pound, how much is the duty on 350 lb. of butter? on 1250 lb.? on $2\frac{1}{2}$ T.?

15. At 15¢ a bushel, how much is the duty on 8750 bu. of oats? on 9500 bu.? on 12,500 bu.?

16. A dealer imports 1500 lb. of maple sugar from Canada. How much is the duty at 4¢ per pound?

17. A wholesale grocery house imports from Canada 900 lb. of cheese. How much is the duty at 5¢ a pound?

18. A dealer imports 4750 lb. of butter. How much is the duty at 8¢ per pound?

19. The duty on honey is 3¢ a pound. If a dealer imports 1250 lb., how much is the duty?

20. A dealer imported 1800 lb. of raisins, paying duty at the rate of 2¢ per pound. How much duty was paid?

21. How much is the duty at 35% on a shipment of thread which is valued at $8750?

22. How much is the duty at 30% on a shipment of napkins which is valued at $4520?

23. How much is the duty at 40% on 2000 yd. of carpet valued at $2.50 a yard?

24. The duty on cigars is $4.50 per pound and 25% ad valorem. If a shipment of cigars weighs 550 lb. and is valued at $1650, how much is the total duty?

A SILENT READING LESSON

The largest part of the money needed to run the United States government is raised by a tax on incomes. The chief features of this form of taxation are as follows:

Each year every resident of the United States, whether a citizen or an alien, who has an income above a specified amount must pay the government a certain per cent of his income after deducting the exemptions that are allowed.

Persons whose incomes are large have to pay more, the rate of taxation increasing as the income increases. Certain types of business concerns also have to pay income taxes.

For example, suppose that a man's salary is $5000 in a year when the income-tax law specifies that $2500 of this is exempt from taxation on account of his being the head of a family, that the rate of taxation on the balance is 2%, and that he may deduct a credit of 25% because his income is earned instead of being derived from invested funds. His total income tax, $37.50, is then found as shown in the work at the right.

Salary,	$5000
Exempt,	2500
	$2500
Rate,	0.02
	$50.00
Less $\frac{1}{4}$,	12.50
Total tax,	$37.50

The details of the income tax are so complex, owing to the great number of different rates and to frequent changes in the laws, that we cannot take them up in school. We ought, however, to understand in a general way about such taxes, and if more information is desired we can find out the details for the current year at any bank.

A considerable part of our national revenue comes from taxes on tobacco. These, together with the income tax and certain other taxes, constitute the *internal revenue.*

INTERNAL REVENUE

1. If an unmarried man has an income of $4475.50, of which $1000 is exempt from taxation, find his income tax at 2%, deducting 25% for earned income.

2. Solve Ex. 1 for a married man with an income of $5650.80, of which $2900 is exempt from taxation.

3. In a year in which the rate was $12\frac{1}{2}$%, a company had to pay a tax on $374,285. How much was the tax?

4. Mr. Wilson has a salary of $20,000 and is allowed an exemption of $2500. If the first $4000 of the balance is taxable at 2%, the second $4000 at 4%, and the balance at 6%, what is the amount of the tax before deducting any credit? If he may now deduct a credit of $55 for earned income, but is obliged to pay an additional tax (called a *surtax*) of $220 because his income is more than $10,000, how much is the amount of his income tax?

5. A certain quality of cigar is taxed $3 per 1000, and an extra tax of $7 per 1000 is also collected. Find the total tax on an output of 275,000 cigars.

6. A certain quality of cigarette is taxed $3.60 per 1000, and an extra tax of $1.20 per 1000 is also collected. Find the total tax on an output of 3,250,000 cigarettes.

7. When tobacco is taxed at the rate of 8¢ per pound, with an extra tax of 5¢ per pound, what is the total tax on an output of 2,758,500 lb. of tobacco?

8. In one year the receipts from a special tax dropped from $89,000,000 to $62,000,000. Find the per cent of decrease.

9. When the rates were raised during the World War, the income-tax receipts increased in one year from $360,000,000 to $4,000,000,000. Find the per cent of increase.

A SILENT READING LESSON

We deal with the United States government more frequently through the post office than in any other way. Everyone needs to know the following general facts which relate to mailing (1) letters, (2) printed matter, and (3) parcels:

1. The rate for letters to be delivered in the United States, Canada, Great Britain, Ireland, and certain other countries is 2¢ per ounce, all fractions of an ounce being counted as a full ounce.

The rate on letters to countries not having the 2-cent rate is 5¢ for the first ounce and 3¢ for each additional ounce.

For letters sent in the United States by airplane the rate is 8¢, 16¢, or 24¢ an ounce, according to the distance traveled.

On government postal cards the rate to places in this country is 1¢, but on private mailing cards it is 2¢.

2. On newspapers and periodicals sent by individuals the rate to places in this country is 2¢ for each 2 oz. Publishers of newspapers and magazines pay special rates for mailing.

On other printed matter the rate to places in this country or abroad is 1½¢ for each 2 oz., with certain restrictions.

3. To points less than 300 mi. away, parcels weighing not more than 70 lb. may be sent by *parcel post*; for points more than 300 mi. away, the maximum weight allowed is 50 lb. There are various other restrictions, which may be learned on inquiry at any post office. The cost of sending by parcel post varies with the weight and distance, and parcels may be insured against loss by paying a small additional fee.

A piece of mail matter may be *registered* by paying an additional fee, thus insuring greater security, as well as reimbursement, up to a certain amount, in case of loss. *Special delivery* of a letter or parcel can also be obtained by payment of an additional fee.

Parcel Post. In the operation of the parcel post (fourth-class mail matter) the country is divided into zones or rings, the center of which is the place at which the parcel is mailed. Roughly speaking, the first zone includes distances up to 50 mi.; the second, from 50 mi. to 150 mi.; the third, from 150 mi. to 300 mi.; the fourth, from 300 mi. to 600 mi.; the fifth, from 600 mi. to 1000 mi.; the sixth, from 1000 mi. to 1400 mi.; the seventh, from 1400 mi. to 1800 mi.; the eighth, over 1800 mi. The basic rates for small parcels which are to be delivered in the first few zones are at present as follows:

Pounds	Local	Zones 1, 2	Zone 3	Zone 4	Zone 5	Zone 6
1	$0.05	$0.05	$0.06	$0.07	$0.08	$0.09
2	.06	.06	.08	.11	.14	.17
3	.06	.07	.10	.15	.20	.25
4	.07	.08	.12	.19	.26	.33
5	.07	.09	.14	.23	.32	.41
6	.08	.10	.16	.27	.38	.49
7	.08	.11	.18	.31	.44	.57
8	.09	.12	.20	.35	.50	.65
9	.09	.13	.22	.39	.56	.73
10	.10	.14	.24	.43	.62	.81

To the postage on each parcel as determined by the above table, a flat service charge of 2¢ is then added.

Postal Statistics. In a recent year there were about 52,000 post offices in the United States, and the length of the postal routes was over 460,000 mi. The income of the Post Office Department was about $533,000,000, and the expenditures were about $557,000,000, the loss being paid by the government. The number of stamps and postal cards issued was more than 16,000,000,000, which does not include much free and unstamped matter.

THE POST OFFICE

Numbers 1 to 12, oral

State the postage in each of the following cases:

1. A letter to Chicago, weighing $\frac{3}{4}$ oz.; weighing 2 oz.

2. A special-delivery (fee, 10¢) letter to San Francisco, weighing $1\frac{1}{4}$ oz.; weighing 6 oz.

3. A registered (fee, 15¢) letter to Boston, weighing $2\frac{1}{4}$ oz.

4. A registered (fee, 20¢), special-delivery (fee, 10¢) letter to New Orleans, weighing $3\frac{1}{2}$ oz.

5. A registered (fee, 15¢) letter to England, weighing 1 oz.

6. A newspaper to Galveston, weighing 3 oz.

7. A printed pamphlet to Sacramento, weighing 2 oz.

8. A letter weighing $1\frac{3}{4}$ oz., sent to Italy.

9. A letter weighing $2\frac{1}{2}$ oz., sent to Holland.

10. A letter weighing $\frac{3}{4}$ oz., sent to France.

11. A letter weighing $1\frac{1}{2}$ oz., sent by airplane mail to a place such that it takes the cheapest rate; the next higher rate; the highest rate.

12. A special-delivery (fee, 10¢) letter weighing $2\frac{1}{2}$ oz., sent by airplane mail to a place such that it takes the highest rate.

Using the table on page 86, find the postage in each of the following cases, remembering to add the 2-cent service charge:

13. A parcel weighing 6 lb., sent to a place in the fifth zone.

14. A parcel weighing 9 lb., sent to a place in the sixth zone.

15. A parcel weighing 8 lb., sent to a place in the local zone.

16. In a year in which our population was 105,000,000 we used about 15,400,000,000 postage stamps. Find the average number of stamps used per person.

MISCELLANEOUS PROBLEMS

1. A school football team won 7 out of 12 games played during a season. What per cent of the games played did the team win that season?

2. A merchant advertised a discount of 20% for a special sale. How much must you pay at the sale for a table that was originally marked $28?

3. A farmer raised 950 bu. of potatoes one year and 1120 bu. the second year. The second year's crop was what per cent greater than that of the first year?

4. A dealer tells me that my car has decreased 30% in value since it was purchased and is now worth $910. What was the original value of the car?

5. Walter says that 6% of 104 is more than $4\frac{1}{2}$% of 108; Tom says that it is not. Which one is right?

6. A farmer delivered to a creamery 420 lb. of milk, which tested 3.8% butter fat. If he was paid 34¢ a pound for the butter fat, how much money did he receive?

7. A class voted to give to a children's hospital 75% of the amount that was cleared at a school play. If the class cleared $86.40, how much was given to the hospital?

8. If you weighed 86 lb. when school was over last June, and if you gained 5% in weight during the summer vacation, how much did you weigh in the fall?

9. In dressing a calf for market 40% of the live weight of the animal is lost. A calf that weighs 120 lb., live weight, should weigh how much less when dressed for market?

10. A city which was asked to subscribe $35,000 for a certain charity raised $38,225. By what per cent did the amount raised exceed the amount requested?

IV. REVIEW AND DRILL

MINIMUM ESSENTIALS

1. Write in words the numbers 400,075, 40.0075, 4000.75, 0.475, and 0.00475.

Add, without writing in columns:

2. $8.20 + $5.50 + $7.75 + $6.73 + $4.30 + $25.50.

3. 7 ft. 3 in. + 8 ft. 6 in. + 9 ft. 8 in. + 2 ft. 3 in. + 3 ft. 5 in.

Subtract as follows:

4. $3\frac{3}{4} - 2\frac{15}{16}$. **5.** $890.60 − $389.96. **6.** $710.30 − $578.28.

Multiply or divide as indicated, giving the results of the divisions to the nearest 0.01:

7. 386 × $976.50. **8.** 1150.6 ÷ 29.4. **9.** 94.79 ÷ 92.6.

Find the discount on a bill for

10. $275 at 9%. **11.** $480 at $2\frac{1}{2}$%. **12.** $800 at $6\frac{3}{4}$%.

Find the value in each of the following cases:

13. 27% of $625. **14.** $3\frac{1}{4}$% of $550. **15.** $4\frac{1}{2}$% of $985.

Find the commission on the sale of property for

16. $275 at 2%. **18.** $220 at 3%. **20.** $8000 at 5%.
17. $350 at $4\frac{1}{2}$%. **19.** $380 at $3\frac{1}{2}$%. **21.** $9500 at $5\frac{1}{2}$%.

22. Find the annual premium on a life-insurance policy for $3000 at $24.50 per $1000.

Find the amount received at a sale of property when the cost and the gain or loss are as follows:

23. $8500; gain, 8%. **25.** $16,750; gain, $12\frac{1}{2}$%.
24. $7500; loss, 12%. **26.** $27,600; loss, $16\frac{2}{3}$%.

PROBLEMS WITHOUT NUMBERS

All work oral

1. If you buy a catcher's mitt and sell it at a certain per cent below cost, how do you find the selling price? How do you find the loss?

2. If you buy a dog and sell him at a certain per cent above what you paid for him, and know how much it has cost you to keep him while you owned him, how do you find whether you gained or lost, and the amount?

3. If you know the number of times a baseball player has been at bat during a season, and the number of hits he has made, how do you find the per cent of hits?

4. If you know the number of times a baseball player has been at bat during a season, and the per cent of runs he has made, how do you find the number of runs?

5. If you know the number of pupils in a school and the number in your room, how do you find the per cent that are in your room?

6. If you know a man's income and the amount which he pays for house rent, how do you find the per cent of his income that he pays for this purpose?

7. If you know the list price of an article which you wish to purchase, and if you are allowed certain discounts, what are you then able to find and how do you find it?

8. If you know the assessed valuation of a house and the tax rate, how do you find the amount of the tax bill?

9. If you know the face of a life-insurance policy and the rate per thousand, how do you find the annual premium?

10. Consider Ex. 9 for a fire-insurance policy at a known rate per hundred.

PROBLEMS FOR COMPLETION

1. A man buys a second-hand automobile for $520. After paying $140 for repairs, he is offered $750 for the car. Complete the problem in any reasonable way and solve it.

Complete and solve the following problems:

2. An aviator covered the distance from Cleveland to Bellefonte, Pa., which is 215 mi., in 83 min.

3. In an automobile race one car made 1.4 mi. in 51 sec., and another car made 1.7 mi. in 54 sec.

4. In New York State the legal weight of a bushel of sweet potatoes is 54 lb. A commission merchant sells 4565 lb.

5. Mr. Howie's wages were $32 a week, but have now been raised to $34.56 a week.

6. Carl is multiplying 15 by a certain number and gets 100 for the result.

7. An automobile driver notices that his speedometer has registered 4.8 mi. in 10 min. He is going to a place that is 36 mi. farther on and expects to keep up the same rate.

8. John finds that, on the average, he covers 26 in. at a step. He lives $\frac{3}{4}$ mi. from his school, and goes 5 da. a week.

9. A man has received a bill for goods amounting to $720 and is allowed a discount of 2% for prompt payment.

10. A sales manager tells a salesman that, with a commission of $4\frac{1}{2}$%, he ought to make $45 a week.

11. In a year when the tax rate is $1.20 per $100, a man pays a tax of $102 on his house.

12. A married man with a salary of $4200 a year is allowed an exemption of $2500. The income-tax rate is 2%, less a deduction of 25% for earned income.

Testing Your Alertness. In arithmetic you must be alert. To find how alert you really are, you will be asked to copy rows of figures like the ones below and to follow the directions that are given you.

The teacher will read three sets of directions that will be something like the following:

1. In the first row write 3 below each figure containing a curve line, write 7 above each one containing a right angle, and write 8 inside each one containing an acute angle.

2. In the second row draw lines above the squares in such a way that the lines will connect every pair of squares containing two numbers whose sum is 9, and then draw lines below the squares in such a way that these lines will connect every pair of squares containing two numbers whose sum is less than 7.

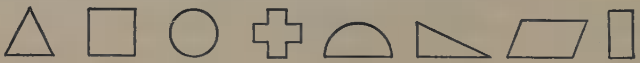
3. In the third row draw a line that will pass above every circle whose number is divisible by 2, and below every circle whose number is not divisible by 2. Put the letter X below every circle whose number is divisible by 3.

You will have to work quickly, and your scores will be kept to see if you improve in later trials.

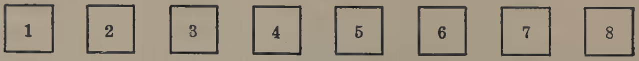
ALERTNESS TEST

Copy the following figures and, at a signal given by the teacher, do as you are directed, allowing 30 sec. for each row:

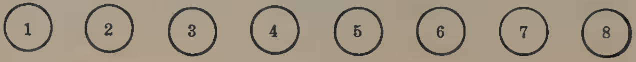
1.



2.



3.



MONEY IN FERTILIZERS

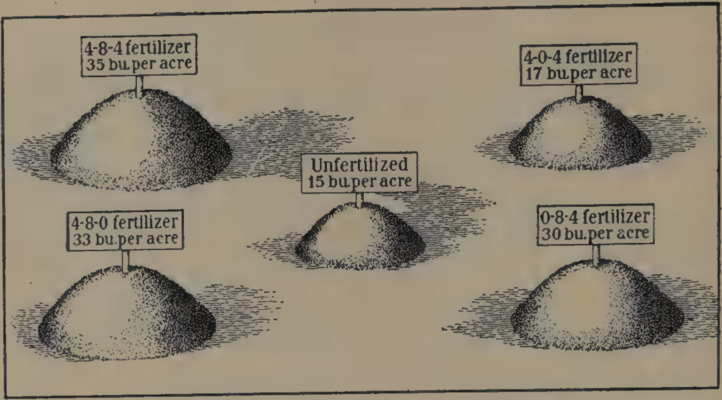
1. George's big brother came home from the agricultural college and decided that on their farm they ought to try more modern methods of feeding the plants. He told George that plants need nitrogen, phosphoric acid, and potash, and that if these are put into the soil by means of fertilizers, the plants grow better. George's brother tried applying fertilizer, costing $5.16, to an acre of corn, and the increase in the yield was 8.6 bu. At 90¢ a bushel at the farm, what was the value of the increase? The value of the increase was how much more than the cost of the fertilizer?

2. George's brother tried another kind of fertilizer, costing $7.64, on another acre of corn, and increased the yield by 7.2 bu. At 90¢ a bushel at the farm, was this experiment profitable or not? Find the gain or loss.

3. George's brother knew of another fertilizer, said to be excellent for wheat. They put some of this kind on an acre at a cost of $11.26. The increase in yield was 8.4 bu. At $1.05 a bushel, what was the gain or loss in this case?

4. The man who lived on the farm next to theirs used another kind of fertilizer for his wheat. He said that it was better than the kind in Ex. 3. George's brother tried it on an acre at a cost of $9.75, and the increase in yield was 9.6 bu. At $1.05 a bushel, how much was the gain or loss in this case? Was this fertilizer better than the kind in Ex. 3? How much more money per acre would this fertilizer return?

5. George's brother explained to him that a complete fertilizer is one containing all three of the plant foods, nitrogen, phosphoric acid, and potash. A 2–8–2 fertilizer is one that contains 2% of nitrogen, 8% of phosphoric acid, and 2% of potash. How many pounds are there of each of these foods in a ton of 2–8–2 fertilizer?



6. George's brother showed him the above picture, which shows the yields of wheat resulting from experiments with different fertilizers, and asked George to find how many pounds of nitrogen, phosphoric acid, and potash there are in 300 lb. (the amount used per acre) of the fertilizer that gave the 35-bushel yield. How many pounds are there of each?

7. In the same picture, what plant food is omitted in the fertilizer which gave the 17-bushel yield? How many pounds of each of the other plant foods are there in 300 lb. of this fertilizer? in a ton of this fertilizer?

8. In the same picture, what plant food is omitted in the fertilizer which gave the 33-bushel yield? in the fertilizer which gave the 30-bushel yield? How many pounds of each of the other plant foods are there in 300 lb. of each fertilizer? in a ton of each fertilizer?

9. Perhaps you know some practical farmer who can tell you something about the subject of fertilizers. If so, make a report to the class about it. If you live in a city, you may not know much about farming, but these problems will give you an idea of how plants have to be fed.

A SCHOOL-GARDEN PROJECT

1. For a school-garden project the teacher of a country school rented a piece of land 20 rd. long and 16 rd. wide, at a rental of \$12.50 an acre. The length of the garden ran east and west. She divided the entire piece into ten plots, beginning at the northwest corner, and assigned these plots as follows: to John, a strip 20 rd. long and 3 rd. wide; to William, Clarence, and Roger, each a strip 20 rd. long and 2 rd. wide; to Mary, Susan, Ella, and Ann, each a piece 5 rd. square; to Henry and Frank, each a strip 10 rd. long and 2 rd. wide. Draw a plan of the land with its divisions, using the scale of 1 in. to 4 rd.

2. The teacher paid \$4.10 for having the whole piece plowed, and \$2.50 a load for 5 loads of stable fertilizer. How much did the rent, plowing, and fertilizer cost?

3. From the result of Ex. 2 find the share that each of the pupils should pay, considering the fractional part of the land that each uses. That is, if John has $\frac{3}{16}$ of the land, he should pay $\frac{3}{16}$ of the expenses. If he does not make $\frac{3}{16}$ of the profits, that is his concern.

4. John planted his strip to early potatoes. He paid $1.80 for seed potatoes, $5.60 for marketing, and sold 18 sacks of potatoes at $4.60 a sack. Deducting his share of the expenses as found in Ex. 3, find his profit; that is, find how much he receives for his labor, since no account is taken in the expenses of the value of the time that he put in.

5. William planted his strip to tomatoes. He paid $16.55 for seed and for a hotbed and raised 108 boxes of tomatoes, which he sold at the garden at an average price of 90¢ a box. Find his profit, as in Ex. 4.

6. Clarence raised onion sets on his strip, paying $10.72 for seed and $29.68 for marketing. He sold 2986 lb. of onion sets at $3\frac{1}{2}$¢ per pound. Find his profit, as in Ex. 4.

7. Roger raised onions on his strip, paying $1.50 for seed. He sold the onions at the garden, selling 98 bags at 50¢ a bag. Find his profit, as in Ex. 4.

8. Henry raised celery on his strip, paying $12.80 for plants and $14.25 for marketing. He sold 262 bunches of celery at 25¢ a bunch. Find his profit, as in Ex. 4.

9. Frank raised lettuce and cabbage on his strip, paying $8.80 for seeds and plants and $13.20 for marketing. He sold 75 boxes of lettuce at 60¢ a box and 270 cabbages at 5¢ a head. Find his profit, as in Ex. 4.

10. From the results found in Exs. 4–9, find the total profit made by the six boys. Mark the plan which you drew in Ex. 1 so that each boy's strip will show what he raised and the profit that he made on it.

11. Considering the results in Exs. 4 and 5, would John probably have done better if on his strip he had planted tomatoes, as William did, instead of potatoes? Give the reasons on which you base your conclusion.

12. Mary raised onions, lettuce, and celery on her piece of land, paying $1.60 for seed and $11.58 for marketing. She sold 17 boxes of lettuce at 45¢ a box, 12 bags of onions at 50¢ a bag, and 246 bunches of celery at 20¢ a bunch. Find her profit, as in Ex. 4.

13. Susan raised radishes, beans, and spinach on her piece of land, paying $1 for seed and $7.50 for marketing. She sold 18 boxes of beans at 60¢ a box, 2200 bunches of radishes at 1¢ a bunch, and 41 boxes of spinach at 55¢ a box. Find her profit, as in Ex. 4.

14. Ella raised lettuce and cabbage on her piece of land, paying $6 for seed and plants and $14 for marketing. She sold 86 boxes of lettuce at 60¢ a box and 330 cabbages at 5¢ a head. Find her profit, as in Ex. 4.

15. Ann raised celery on her piece of land, paying $2.35 for seed and planting and $17.80 for marketing. She sold 286 bunches of celery at 25¢ a bunch. Find her profit, as in Ex. 4.

16. From the results found in Exs. 12–15, find the total profit made by the four girls. Complete the plan by marking the remaining strips as suggested in Ex. 10.

17. At the end of the year the question arose as to which boy or girl had made the most money per square rod. Find this profit in each case to the nearest cent and determine who stood highest, who stood next, and so on.

18. The question also arose as to the total profit of the group of the ten children, and what this profit amounted to per acre. Find these two amounts.

19. From Ex. 18 find the average profit made by the children. Find how much more than this average profit was made by the one who had the largest profit, and how much less by the one who had the smallest.

V. LITTLE EXAMINATIONS

I. 1. \$300 is what per cent more than \$200? than \$100?

2. \$200 is what per cent less than \$300? than \$400?

3. How much is $4\frac{1}{2}\%$ of \$350? of \$3500? of \$35,000?

4. How much is 150% of \$3500? $\frac{3}{4}\%$ of \$8000?

5. How much is $87\frac{1}{2}\%$ of \$7200? of \$6496?

II. 1. Find 0.2% of 3000, 2% of 300, and 20% of 30.

2. Express $3\frac{1}{2}$ qt. as per cent of a gallon.

3. Express an inch as per cent of a yard.

4. Express $33\frac{1}{3}\%$, $66\frac{2}{3}\%$, and $83\frac{1}{3}\%$ as fractions.

5. Express $\frac{3}{4}$, $\frac{7}{8}$, $\frac{1}{16}$, and $\frac{3}{5}$ as per cents.

III. 1. \$60 is what per cent of \$720? of \$7200? of \$72,000?

2. 9 in. is what per cent of 10 in.? of 10 ft.? of 10 yd.?

3. A pint is what per cent of 1 qt.? of 1 gal.? of 6 gal.?

4. 96 ft. is 8% of what distance? 4% of what distance?

5. 9 ft. 8 in. is 10% of what distance?

IV. 1. \$12.35 is 5% less than what amount of money?

2. \$204 is 15% less than what amount of money?

3. Find the discount on a bill for \$380 at 20%.

4. Find the discount on a bill for \$400 at 15%, 2%.

5. At 95¢ per \$100, find the face of a policy costing \$79.80.

V. 1. Find the commission at $5\frac{3}{4}\%$ on a sale totaling \$1650.

2. Find the tax on \$12,500 at \$22.60 per \$1000.

3. On a policy for \$9000 costing \$281.25 find the rate per \$1000.

4. If the tax on \$8200 is \$205, what is the rate?

5. Find the tax at 2% on a taxable income of \$3750.

CHAPTER II

I. BORROWING AND LENDING MONEY

A SILENT READING LESSON

Mr. Moore has worked in a grocery store for a number of years and decides to open a store of his own. He has saved some money, but he finds that it will take more than his savings to pay for the fixtures for the store and to buy the goods which he needs in order to start in business. He must therefore borrow some money. When a man borrows money, he pays for the use of it until such time as he pays it back.

The money paid for the use of money is called *interest*. For example, if Mr. Moore borrows $2000 for a year and pays $120 for the use of it, the $120 is interest.

Interest is very similar to rent. Rent is payment for the use of property, such as a house, and is usually paid at a fixed rate for a certain length of time. Interest is payment for the use of money and is usually paid at a fixed rate for a certain length of time.

The sum of money on which interest is paid is called the *principal*. That is, when Mr. Moore borrows $2000, the $2000 is the principal.

The principal plus the interest at any given time is called the *amount* of the debt at that time. For example, the amount of Mr. Moore's debt at the end of the year for which he borrowed the money is $2000 + $120, or $2120; that is, if he does not pay the interest until he pays back the principal, he will have to pay $2120 at that time.

Rate of Interest. Interest is usually specified as a certain per cent of the principal. The per cent is called the *rate of interest,* and means the rate for borrowing money for 1 yr. unless it is otherwise specified.

Interest for One Year. Finding interest for 1 yr. is therefore simply a case of finding a per cent of a number. For example, suppose that you borrow \$2000 at 6% for 1 yr.

The interest for 1 yr. is then 6% of \$2000; that is, as shown by the work at the right, it is \$120.

Principal,	\$2000
Rate,	0.06
Interest for 1 yr.,	\$120.00

Interest for 1 yr. = principal × rate.

In applying this relation, which is the usual form of expressing the rule for interest, we, of course, multiply the principal by the rate, and not the rate by the principal.

INTEREST FOR ONE YEAR

Without using pencil and paper, find the interest for 1 yr. on

1. \$100 at 5%.
2. \$100 at 6%.
3. \$1000 at 6%.
4. \$200 at 4%.
5. \$200 at 6%.
6. \$300 at 4%.
7. \$300 at 6%.
8. \$3000 at 5%.
9. \$4000 at 6%.

10. At 6% find the amount (principal plus interest) at the end of 1 yr. of a debt of \$2000; of \$4000; of \$7500.

Find to the nearest cent the interest for 1 yr. on

11. \$750 at 3%.
12. \$875 at 4%.
13. \$475 at 5%.
14. \$425 at 6%.
15. \$75 at $4\frac{1}{2}\%$.
16. \$85 at $3\frac{1}{2}\%$.
17. \$95 at $4\frac{1}{2}\%$.
18. \$65 at $2\frac{1}{2}\%$.
19. \$375.60 at 5%.
20. \$550.25 at 4%.
21. \$835.40 at 6%.
22. \$925.20 at 6%.

Interest for Several Years. If you pay $400 a year for house rent, you pay twice as much for 2 yr., three times as much for 3 yr., and so on for any number of years. In the same way, if you pay $60 for the use of $1000 for 1 yr., you pay twice as much for 2 yr., three times as much for 3 yr., and so on.

For example, suppose that you borrow $2000 at 6% for 3 yr.

The interest for 1 yr. is 6% of $2000, or $120, and for 3 yr. it is three times as much; that is, as shown at the right, it is $360.

Principal,	$2000
Rate,	0.06
Interest for 1 yr.,	$120.00
	3
Interest for 3 yr.,	$360.00

Interest = principal × rate × number of years.

INTEREST FOR SEVERAL YEARS

Find the interest for 2 yr. on

1. $250 at 6%.

2. $375 at 5%.

3. $480 at 6%.

4. $150 at 4%.

5. $280 at 5%.

6. $850 at 6%.

7. $1200 at $5\frac{1}{2}$%.

8. $1600 at 6%.

9. $2400 at $5\frac{1}{2}$%.

Find the interest for 3 yr. on

10. $250 at 4%.

11. $275 at 5%.

12. $360 at 6%.

13. $180 at $4\frac{1}{2}$%.

14. $260 at $5\frac{1}{2}$%.

15. $950 at $5\frac{1}{2}$%.

16. $75 at 6%.

17. $750 at 6%.

18. $7500 at $5\frac{1}{2}$%.

Find the interest on

19. $2500 for 2 yr. at 4%.

20. $3600 for 3 yr. at 5%.

21. $4800 for 4 yr. at 6%.

22. $7500 for 2 yr. at $4\frac{1}{2}$%.

23. $8500 for 3 yr. at $5\frac{1}{2}$%.

24. $12,500 for 4 yr. at 6%.

25. $18,000 for 2 yr. at 5%.

26. $25,000 for 3 yr. at $4\frac{1}{2}$%.

Interest for Part of a Year. If the interest for 1 yr. on a certain sum is \$50, for ½ yr., or 6 mo., it will be ½ of \$50, or \$25. Similarly, if the interest for 1 yr. is \$60, the interest for 3 mo., which is ¼ yr., will be ¼ of \$60, or \$15, and so on.

For example, to find the interest on \$6000 at 5% for 4 mo., we first find it for 1 yr., which is \$300.

Since 4 mo. = ⅓ yr., we then take ⅓ of the \$300; that is, as shown by the work at the right, the interest for 4 mo. is \$100.

Principal,	\$6000
Rate,	0.05
Interest for 1 yr.,	\$300.00
Interest for 4 mo.,	\$100.
	3)\$300.00

Interest = principal × rate × fraction of a year.

In practical work we simply write the 3 before the result of the multiplication, draw the line to represent division, and divide, writing the result below the number divided.

INTEREST FOR PART OF A YEAR

Find the interest for 6 mo. on

1. \$100 at 6%.
2. \$275 at 4%.
3. \$350 at 4%.
4. \$750 at 6%.
5. \$475 at 3%.
6. \$250 at 2½%.
7. \$350 at 6%.
8. \$550 at 4½%.
9. \$695.30 at 3%.
10. \$964.80 at 3%.
11. \$575.75 at 6%.
12. \$950.80 at 5%.

Find the interest on

13. \$250 for 4 mo. at 6%.
14. \$380 for 3 mo. at 5%.
15. \$850 for 2 mo. at 6%.
16. \$750 for 4 mo. at 7%.
17. \$1200 for 9 mo. at 6%.
18. \$1500 for 8 mo. at 5%.
19. \$2400 for 10 mo. at 6%.
20. \$2760 for 8 mo. at 7%.

General Rule for Interest. The plan of finding interest for any length of time is always the same; that is,

Find the interest for one year, and then multiply this by the number representing years or the fractional part of a year.

This rule may be expressed briefly, as follows:

Interest = principal × rate × time.

For example, in finding the interest on $250 for 10 mo. at 6%, we see that for 1 yr. it is 6% of $250, and for 10 mo. it is $\frac{10}{12}$ of this. That is, as here shown, the interest for 10 mo. is $12.50.

$$\boxed{\frac{\$25\cancel{0} \times \cancel{6} \times 1\cancel{0}}{1\cancel{0}\cancel{0} \times \underset{2}{\cancel{12}}} = \frac{\$25}{2} = \$12.50}$$

This example may be solved mentally by noticing that $\frac{10}{12}$, or $\frac{5}{6}$, of 6% is 5 × 1%, or 5%. We then have

$$5\% \text{ of } \$250 = 5 \times \$2.50 = \$12.50.$$

At present, in finding interest, we shall consider 30 da. as a month. The interest for 1 da. is then $\frac{1}{30}$ of the interest for 1 mo., for 5 da. it is $\frac{1}{6}$ of the interest for 1 mo., and so on.

We disregard a fraction less than ½¢ in any interest found, and call ½¢ or greater a full cent.

FINDING INTEREST

1. How much is the interest on $872, at 6%, for 1 yr.? for 6 mo.? for 4 mo.? for 8 mo.? for 10 mo.?

2. How much is the interest on $720, at 5%, for 1 yr.? for 6 mo.? for 10 mo.? for 1 mo.? for 9 mo.?

3. How much is the interest on $1080, at 4%, for 1 yr.? for 1 mo.? for ½ mo.? for 5 da.? for 1 da.?

4. How much is the interest on $225.50, at 4%, for 1 yr.? for 6 mo.? for 4 mo.? for 1 mo.? for 10 da.?

Interest for Years and Months. Suppose that Mr. Dean, needing $125 to pay for painting his house, borrows the money at 5%. If he does not pay the interest until 2 yr. 3 mo. later, how much interest is then due?

Since 2 yr. 3 mo. is the same as $2\frac{1}{4}$ yr., or $\frac{9}{4}$ yr., we arrange the work as here shown, cancel, and find the required interest to be $$14.06\frac{1}{4}$, which, disregarding the fraction, we give as $14.06.

$$\boxed{\frac{\overset{5}{\$\cancel{125}} \times 5 \times 9}{\underset{4}{\cancel{100}} \times 4} = \frac{\$225}{16} = \$14.06\tfrac{1}{4}}$$

Ordinarily we save time and avoid errors by using cancellation, but in the problem above we might have said

$$5\% \text{ of } \$125 = \$6.25, \text{ interest for 1 yr.,}$$

and $$2\tfrac{1}{4} \times \$6.25 = \$14.06\tfrac{1}{4}, \text{ or } \$14.06.$$

We do not frequently need to compute interest for years and months. Even if Mr. Dean, in the above case, did not pay back his loan for 3 yr. or more, he would probably have paid the interest every year or every 6 mo.

INTEREST FOR YEARS AND MONTHS

Find the interest on

1. $350 for $1\frac{1}{4}$ yr. at 6%.
2. $400 for $1\frac{1}{2}$ yr. at 5%.
3. $500 for $2\frac{1}{2}$ yr. at 5%.
4. $725 for $3\frac{1}{2}$ yr. at 4%.
5. $850 for $1\frac{3}{4}$ yr. at 6%.
6. $9600 for 2 yr. 6 mo. at 5%.
7. $825 for 3 yr. 3 mo. at 4%.
8. $950 for 2 yr. 8 mo. at 6%.
9. $305 for 1 yr. 9 mo. at 6%.
10. $800 for 1 yr. 2 mo. at $4\frac{1}{2}$%.

Find the interest on $875 for 2 yr. 3 mo. at

11. 5%.
12. 6%.
13. 3%.
14. 4%.
15. $5\frac{1}{2}$%.

FINDING INTEREST

1. Find the interest on $1500 for 1 yr. 2 mo. at 5%; at 6%; at 4%; at 5½%.

2. A dealer in automobiles buys 16 cars at $1150 each and agrees to pay for them in 9 mo., with interest at 6%. How much does he pay, including principal and interest?

3. Mr. Russell hires Mr. Baker to make some repairs on his house, agreeing to pay him $85 for the work. Mr. Baker agrees to wait 3 mo. for his money if he receives interest at 6%. How much is due Mr. Baker at the end of the 3 mo.?

4. Woodhull & Co. need $3650 to buy holiday goods. They borrow it on Oct. 15 and agree to pay it back in 3 mo., with interest at 5%. How much do they owe on Jan. 15?

5. Pierson & Co. buy $2250 worth of goods, agreeing to pay for the goods in 3 mo., with interest at 5%. How much do they owe at the end of the 3 mo.?

6. Mr. Holbrook buys from Mr. Jackson a second-hand automobile, agreeing to pay $580 in 4 mo., with interest at 6%. How much interest is due in 4 mo.?

7. A man borrows $2500, agreeing to pay interest at 6%. He pays the principal and interest at the end of 1 yr. 8½ mo. How much does he pay, including principal and interest?

8. A man buys 36 head of cattle at $90 a head and agrees to pay for them in 10 mo., with interest at 6%. How much does he pay, including principal and interest?

9. A man borrows $950, agreeing to pay interest at 5%. He pays the principal and interest 2 yr. 10 mo. later. How much does he pay, including principal and interest?

10. The day Edward is 16 yr. old his father invests $500 for him at 5%, the $500 and interest being due the day he is 21 yr. old. How much will be due at that time?

Interest for Years, Months, and Days. 1. If you borrow \$5000 at 6%, how much interest is due 8 mo. 6 da. later?

Since 8 mo. 6 da. = 246 da., and there are 360 da. in 12 mo. of 30 da., we find the interest, as shown below, for $\frac{246}{360}$ yr.:

$$\begin{array}{rr} 1 \text{ mo.} = & 30 \text{ da.} \\ & \underline{8\phantom{ \text{ da.}}} \\ 8 \text{ mo.} = & 240 \text{ da.} \\ & \underline{6 \text{ da.}} \\ 8 \text{ mo. } 6 \text{ da.} = & 246 \text{ da.} \end{array}$$

$$\frac{\$\overset{\overset{5}{\cancel{500}}}{\cancel{5000}} \times \cancel{6} \times \overset{41}{\cancel{246}}}{\cancel{100} \times \underset{\underset{\cancel{10}}{\cancel{60}}}{\cancel{360}}} = \$205$$

\$205. *Ans.*

2. Suppose that a man borrows \$400 from another man on Sept. 10, 1926, and agrees to pay the interest at 6% when he repays the principal. If he does not pay the loan until Aug. 7, 1927, how much interest is then due?

As shown below, we first find the time for which the \$400 draws interest. We then proceed in the usual way, as follows:

	yr.	mo.	da.
Aug. 7, 1927 =	1927	8	7
Sept. 10, 1926 =	1926	9	10
The time =		10	27

$$\begin{array}{rr} 1 \text{ mo.} = & 30 \text{ da.} \\ & \underline{10\phantom{ \text{ da.}}} \\ 10 \text{ mo.} = & 300 \text{ da.} \\ & \underline{27 \text{ da.}} \\ 10 \text{ mo. } 27 \text{ da.} = & 327 \text{ da.} \end{array}$$

$$\frac{\$\overset{20}{\cancel{400}} \times \cancel{6} \times \overset{109}{\cancel{327}}}{100 \times \underset{\underset{\cancel{20}}{\cancel{60}}}{\cancel{360}}} = \$21.80$$

\$21.80. *Ans.*

While we might have canceled further in this case, it is often better to leave the 100 in the denominator, because dividing by 100 is easily done by moving the decimal point.

3. Suppose that the man referred to in the second problem on page 106 had kept the $400 from Sept. 10, 1926, to Feb. 7, 1929. How much interest would be due then?

We first find that the $400 draws interest for 2 yr. 4 mo. 27 da. Disregarding the 2 yr., we then find the interest for 4 mo. 27 da., or 147 da., and to it add the interest for 2 yr., as shown below:

$$\begin{array}{rrrr} & \text{yr.} & \text{mo.} & \text{da.} \\ \text{Feb. 7, 1929} = & 1929 & 2 & 7 \\ \text{Sept. 10, 1926} = & \underline{1926} & \underline{9} & \underline{10} \\ \text{The time} = & 2 & 4 & 27 \end{array}$$

$$\begin{array}{rr} 1\text{ mo.} = & 30\text{ da.} \\ & \underline{4} \\ 4\text{ mo.} = & 120\text{ da.} \\ & \underline{27\text{ da.}} \\ 4\text{ mo. }27\text{ da.} = & 147\text{ da.} \end{array}$$

$$\frac{\$\overset{20}{\cancel{400}} \times \cancel{6} \times \overset{49}{\cancel{147}}}{100 \times \underset{\underset{\cancel{20}}{\cancel{60}}}{\cancel{360}}} = \$9.80$$

$$\$400 \times 0.06 \times 2 = \underline{\$48.00}$$

$$\$57.80 \quad \textit{Ans.}$$

This problem could have been solved by finding at one step the interest for 2 yr. 4 mo. 27 da., or for 867 da.; but in ordinary cases it is better to proceed as shown above, in order to keep the numbers in the computation as small as possible, and thus simplify the work.

We should understand that such cases in computing interest are becoming more and more unusual. People commonly borrow money at some bank, and banks use tables for computing interest. In ordinary business transactions, money is rarely borrowed for long periods, the usual time being only 30 da., 60 da., or 90 da. If a person borrows money for longer than 6 mo., the interest is generally paid at definite intervals,—usually every 3 mo., every 6 mo., or once a year.

FINDING INTEREST

Find the interest on $750 for 1 yr. 2 mo. 6 da. at

1. 4%. **2.** 5%. **3.** 6%. **4.** $4\frac{1}{2}$%. **5.** 7%.

Find the interest at $5\frac{1}{2}$% *for 1 yr. 3 mo. 10 da. on*

6. $375. **7.** $480. **8.** $560. **9.** $675. **10.** $950.

Find the interest at $4\frac{1}{2}$% *on $2750.75 for*

11. 1 yr. 6 mo. **12.** 1 yr. 8 mo. **13.** 2 yr. 9 mo. 16 da.

14. Find the interest on $1728.45 for 2 yr. 8 mo. at 6%.

15. Find the amount, at the end of 9 mo., of a loan of $3468.75 which bears interest at $5\frac{1}{2}$%.

Find the interest on

16. $725 for 1 yr. 6 mo. 8 da. at 6%.

17. $600 for 1 yr. 3 mo. 15 da. at 4%; at 5%.

18. $625.50 for 2 yr. 11 mo. 15 da. at 5%; at $5\frac{3}{4}$%.

19. $475.50 for 3 yr. 10 mo. 15 da. at 4%; at $4\frac{1}{4}$%.

20. $265.50 for 2 yr. 10 mo. 20 da. at $4\frac{3}{4}$%; at 5%.

21. $3562.50 for 3 yr. 2 mo. 5 da. at $5\frac{1}{2}$%; at $4\frac{1}{2}$%.

22. A man borrowed $1275 on Apr. 7, 1926, at 6%, and $1500 on Apr. 10, 1926, at 5%. He paid the principal and interest on both loans on Dec. 30, 1927. How much did he pay on that date?

23. A man borrowed $1500 on May 1, 1928, at 5%, and $2500 on July 5, 1928, at $4\frac{1}{2}$%. He paid the principal and interest on both loans on June 7, 1930. How much did he pay?

24. What is the total amount on Oct. 15, 1931, of $3200, which was borrowed Feb. 10, 1929, at 6%, and of $2800, which was borrowed Apr. 15, 1929, at 5%?

Six-Per-Cent Method. Because the most common rate of interest is 6%, and money is often loaned by banks for periods of 30 da., 60 da., or 90 da., it is frequently convenient to find the interest by what is called the "six-per-cent," or "sixty-day," method.

For example, in finding the interest on $420 for 60 da. at 6% by this method, since 60 da. = 2 mo. = $\frac{1}{6}$ yr., we have

$$\tfrac{1}{6} \text{ of } 6\% \text{ of } \$420 = 1\% \text{ of } \$420 = \$4.20.$$

Hence the interest at 6% for 60 da. is 0.01 of the principal, and, similarly, for 6 da. it is 0.001 of the principal. That is,

To find the interest at 6% for 60 da., move the decimal point two places to the left; for 6 da., move it three places to the left.

For other periods the interest can be found from that for 60 da. or 6 da.

For example, in finding the interest on $420 for 5 mo. 10 da. at 6%, the work should be done mentally except that the amounts may, if necessary, be written down, as here shown, and then added to find the required interest.

2 mo.,	$4.20
2 mo.,	4.20
1 mo.,	2.10
$\frac{1}{3}$ mo.,	.70
	$11.20

SIX-PER-CENT METHOD

Numbers 1 to 5, oral

Find the interest at 6% for 60 da. and also for 30 da. on

1. $250. **2.** $370. **3.** $1250. **4.** $1720. **5.** $2800.

Find the interest at 6% on

6. $750 for 3 mo.
7. $1250 for 5 mo.
8. $1450 for 7 mo.
9. $1860 for 9 mo.
10. $920 for 4 mo.
11. $870 for 1 mo.
12. $940 for 18 da.
13. $875 for 24 da.
14. $1500 for 165 da.
15. $2500 for 216 da.
16. $2750 for 132 da.
17. $3275 for 184 da.

Interest Table. Since interest tables, such as are used by banks, are generally made up in book form, it is impossible to give the complete tables in an arithmetic. Here is a small table which shows the interest on $100 at various rates:

Time	4%	5%	6%	7%	Time	4%	5%	6%	7%
1 da.	0.01	0.01	0.02	0.02	1 mo.	0.33	0.42	0.50	0.58
2 da.	.02	.03	.03	.04	2 mo.	.67	.83	1.00	1.17
3 da.	.03	.04	.05	.06	3 mo.	1.00	1.25	1.50	1.75
4 da.	.04	.06	.07	.08	4 mo.	1.33	1.67	2.00	2.33
5 da.	.06	.07	.08	.10	6 mo.	2.00	2.50	3.00	3.50
6 da.	.07	.08	.10	.12	12 mo.	4.00	5.00	6.00	7.00

Using the Table. To find the interest on $275 for 33 da. at 5%, we follow along the line in the table marked "1 mo." to the column marked "5%," and find 0.42. Then, on the line for 3 da. and under 5%, we find .04. We then proceed as follows:

Interest on $100 for 30 da. (1 mo.) at 5%,	$0.42
Interest on $100 for 3 da. at 5%,	.04
Interest on $100 for 33 da. at 5%,	$0.46
$275 = 2.75 × $100,	2.75
	230
	322
	92
Interest on $275 for 33 da. at 5%,	$1.2650

That is, the required interest is $1.26½, or $1.27.

With such a small table, the result may not be exact. If we find the interest in the usual way, we shall find that it is nearer $1.26 than $1.27. This table serves simply to show how a larger table is used.

USING THE INTEREST TABLE ON PAGE 110

Using the table, find the interest when the principal, the time, and the rate are as follows:

1. \$100, 4 mo., 7%.
2. \$100, 6 mo., 4%.
3. \$100, 5 da., 6%.
4. \$200, 1 yr., 5%.
5. \$300, 3 mo., 6%.
6. \$400, 6 da., 7%.
7. \$700, 4 mo., 4%.
8. \$850, 3 mo., 6%.
9. \$925, 2 mo., 6%.
10. \$775, 6 mo., 5%.
11. \$1200, 33 da., 6%.
12. \$1250, 3 mo. 5 da., 6%.
13. \$1225, 2 mo. 6 da., 5%.
14. \$1500, 35 da., 6%.
15. \$1850, 64 da., 4%.
16. \$1625, 6 mo. 2 da., 5%.
17. \$2500, 1 yr., 7%.
18. \$2500, 3 mo. 4 da., 6%.
19. \$3000, 2 mo. 6 da., 6%.
20. \$3250, 2 mo. 4 da., 4%.

21. Using the six-per-cent method, find the interest on \$250 for 90 da. at 6%. Check the result by using the table.

22. Using the method shown on page 106, find the interest on \$450 for 93 da. at 7%; that is, for $\frac{93}{360}$ yr. Check the result by using the table.

23. Using the table, find the interest on \$700 for 95 da. at 6%. Check the result by using the six-per-cent method.

24. Using the table, find the interest on \$1250 for 64 da. at 5%. Check the result by using the method shown on page 106.

25. In Exs. 21–24 three methods have been employed in finding interest. Find the interest on \$450 for 90 da. at 6% by each of the three methods, and state which one took the least time.

26. As in Ex. 25, time each method in finding the interest at 6% on \$1200 for 36 da.; on \$1250 for 63 da.; on \$1600 for 84 da.; on \$1450 for 126 da.

REVIEW OF INTEREST

Copy and complete the following statements:

1. The interest at 6% on $200 for 2 mo. is ——.

2. The interest at 5% on $200 for 2 mo. is ——.

3. The interest at 5% on $400 for 2 mo. is ——.

4. The interest at 5% on $400 for 6 mo. is ——.

5. At 4% the interest on $500 for 3 mo. is ——.

6. At 5% the interest on $750 for 3 mo. is ——.

7. At 6% the interest on $225 for 6 mo. is ——.

8. At 7% the interest on $1250 for 8 mo. is ——.

9. For 3 mo. the interest on $300 at 6% is ——.

10. For 2 mo. the interest on $450 at 5% is ——.

11. The interest at 6% on $1500 for 33 da. is ——.

12. The interest is —— on $250 for 64 da. at 6%.

13. The interest is —— on $425 for 92 da. at 5%.

14. On $300 the interest for 30 da. at 6% is ——; and if the time is 60 da., the interest is ——.

15. At 5% the interest on $500 for 60 da. is ——, and for half the time it is ——.

16. The interest on $500 for 60 da. at 3% is ——, and for half that time and double that rate it is ——.

17. The interest on $1000 for 6 mo. at 6% is ——; if the rate is half as much, if the time is twice as long, and if the principal is doubled, the interest is ——.

18. Doubling the principal —— the interest; doubling the time —— the interest; doubling the rate —— the interest; doubling any two of them multiplies the interest by ——; doubling all three multiplies the interest by ——.

PAYMENTS IN INSTALLMENTS

1. A man owes $100 and is given the choice of paying it in 60 da. with interest at 6%, or of paying $1.06 a day for 100 da. Which is the cheaper plan for him to take?

2. Mr. Roe buys a home costing $5000, paying $1000 down and the rest in eight installments of $500 each. The installments are due at the end of each 6 mo., with interest at 6% on the unpaid balance; that is, at the end of the first 6 mo. he owes interest on $4000, at the end of the second 6 mo. on $3500, and so on. After the last payment, how much has Mr. Roe paid in principal and interest?

3. A man buys a store costing $14,000 by paying $4000 down and the rest in five payments of $2000 each, due at the end of each year, with interest at 6% on the unpaid balance. How much does he pay in all for the store?

4. Suppose that you buy a radio set and have the choice of paying $110 at once or $10 a month for a year. If the use of the money is worth 6% to you, which is the cheaper plan for you? How much cheaper is it?

5. A dealer in radio supplies finds that if he sells for cash, he can put the money back into the business, where it will pay him 12% a year. Supposing that he is willing to wait 60 da. for payment, find the price that he should quote on a radio set which he ordinarily sells for $50, cash; that is, find the price which is equal to $50 plus the interest for 60 da. at 12% a year.

6. The dealer also sells the set mentioned in Ex. 5 on the installment plan, requiring $25 "down" (in cash) and the balance in six payments of $5 each, at the end of each month. If he puts each payment back into the business, how much better is this plan for him than the cash price of $50?

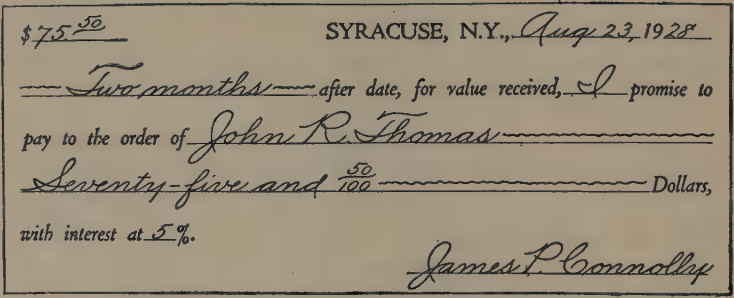
Promissory Note. When a person borrows money, the details of the transaction should be put in writing for the protection of all concerned. This is generally done by means of a written promise on the part of the borrower to repay the money to the lender at a certain time, either with interest at a specified rate or without interest. Such a written promise is called a *promissory note,* or simply a *note.*

The sum borrowed is stated in the note, and is called the *face* of the note.

The face plus the interest on a given date is called the *amount* of the note on that date.

A note should state the date, face, rate, person to whom payable, and date of *maturity* (date when it is due to be paid). It is customary to have the note state that it has been given for value received, but this is not necessary to make the note legal in most states.

The following form shows a simple type of promissory note:



$\$75^{50}$ SYRACUSE, N.Y., Aug 23, 1928

Two months after date, for value received, I promise to pay to the order of John R. Thomas

Seventy-five and $\frac{50}{100}$ Dollars,

with interest at 5%.

James P. Connolly

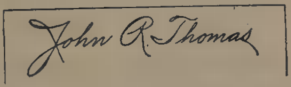
If, in the above note, the words "on demand" were substituted for "two months after date," the note would be called a *demand note.* Such notes are payable only after payment is demanded by the lender, and are generally used only in cases of borrowing for a few days or in cases where it is not possible to set a definite date of maturity.

Parties to a Note. The person named in a note as the one to whom it is payable is called the *payee*. The person who signs a note is called the *maker*.

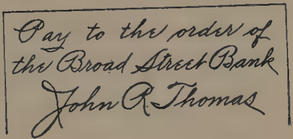
Indorsing a Note. If the payee wishes to sell a note which is payable to himself or order, so that he can get his money at once, he must *indorse* the note.

A note is indorsed by the payee's writing his name across the back. The indorser must pay the note if the maker does not. The two forms at the right show how Mr. Thomas might indorse the note on page 114.

The first indorsement is known as an *indorsement in blank*, and the second one as an *indorsement in full*.



John R. Thomas



Pay to the order of
the Broad Street Bank
John R. Thomas

Negotiable Note. A note that is payable to bearer, or that has been indorsed in blank, is a *negotiable note*; that is, it can be sold. If it is made or indorsed to the order of a particular person it is not negotiable without his indorsement.

Interest on Loans. The United States borrowed money before the World War at about 2%; during the war the rate began at $3\frac{1}{2}$% and increased to $4\frac{3}{4}$%. Savings banks pay depositors about 4% or 5%. In cities, borrowers usually pay from 4% to 6% for ordinary loans. Is this the rate where you live?

When a note bears interest, but the rate is not specified, the rate is the one fixed by the law of the state in which the transaction takes place. In many states this rate is 6%, and various penalties are prescribed by law for demanding a rate of interest higher than the legal rate. Since the law is liable to change, inquiry should be made as to whether or not a government stamp is still necessary on a note.

BORROWING MONEY ON NOTES

1. Suppose that Mr. R. J. Lee borrows \$275 from you today, giving his note due in 3 mo. with interest at 6%. Write the note and find how much he will have to pay when the note becomes due.

2. A purchaser of an automobile is offered the choice of paying half of \$2400 in cash and giving his note for the rest, due in 4 mo. at 5%, or of paying \$2300 in cash. How much less is the cost of the car if he pays cash?

3. A salesman starting on a trip borrowed \$750 on a 90-day note bearing interest at 5%. He returned on the day the note was due and immediately paid it. Write the note and find the amount of the note on the date of payment.

4. A merchant borrowed \$900 on May 4 and \$1150 on June 7, giving his note for each loan. He paid both notes the first of the following October, with interest at $4\frac{1}{2}$%. Counting 30 da. for each full month, from May 4 to Sept. 4 is 4 mo., or 120 da., and from Sept. 4 to Oct. 1 is 27 da. How many days did the first note run? How many days did the second one run? Find the total amount of the notes on Oct. 1.

5. A manufacturer held a note signed by one of his customers for \$3500. Interest at 6% was due on the note from May 16, 1928, to Feb. 26, 1929. The manufacturer sent the note to a collector, who was to receive 5% of the amount collected, but the customer failed and paid only 40% on his debts. How much did the manufacturer receive from the collector?

6. Write a note like the one on page 114, dating it today and taking \$150 as the principal and 6% as the rate. Use any names you choose for maker and payee and indorse the note to your order.

II. BANKING

A SILENT READING LESSON

If people had not earned and saved money, we should not have comfortable homes in which to live, mills and factories in which clothing and other things that we need are made, railroads to bring the city's food from the farms, or school buildings in which to obtain an education. All these things represent money which was saved by someone.

One of the best ways to save money is to make a *budget* in which we write down our allowance for a month, how much we shall spend, and how much we shall save. A family should do the same thing, making its budget for a year and doing its best to live up to the plan thus made.

We should all try to save at least a little of what we earn, and to see that what we have saved is kept safely. We may, for example, deposit our savings in a bank. There are several kinds of banks, and most people at some time have business dealings with one kind of bank or another. Only a part of the money deposited is kept by the bank; the rest is lent to business men or others who pay interest to the bank. On deposits of savings, banks generally pay a certain per cent of interest to the depositor.

One thing that is essential to everyone who hopes to succeed is a *bank account.* In most banks a person may open a savings account as soon as he begins to save even a small amount. When a person opens an account, he is given a *bank book* in which a record of his account is kept. Many schools organize school banks, electing officers and carrying on a banking business with imitation money.

We shall first learn more about a budget, which helps us to save money, and then shall learn about banks, which help to protect the money we have saved.

A SILENT READING LESSON (CONTINUED)

One advantage of a budget is that it places a check upon the tendency of a person, family, or business to spend money freely as it comes in, without thinking of future needs or of emergencies that are likely to arise.

In the case of families, experience has shown about what per cent of income may safely be assigned to various classes of expenditures. In a general way the expenses of the house may take about 30% of a small income, food 30%, clothing 10%, advancement 5%, health 5%, miscellaneous 10%, and investments 10%; but these figures will vary greatly.

We should prepare for our own use, but not for discussion in class, a budget for a month. At the end of the month we should see how closely the expenses come to our estimates and should then try to improve upon our budget.

There are many ways of making up a budget, but the following divisions of the expenses are typical:

1. *House*, including rent, taxes, insurance, heat, light, telephone. If a man owns his home, he does not pay rent but pays taxes and property insurance; if he rents a home, the owner pays for these items.

2. *Food*, including groceries, meat, milk, fruit, and so on.

3. *Clothing*, including suits, coats, underwear, shoes, hats, repairs, and similar items.

4. *Advancement*, including education, books, magazines, papers, travel, religion, charity.

5. *Health*, including recreation, bills of physicians and dentists, medicine, exercise, and so on.

6. *Miscellaneous*, including laundry, presents, small items.

7. *Savings*, including bank account, life insurance, and other ways of providing for future needs.

Form for a Budget. The model below is one of various convenient forms that may be used in preparing a budget.

I shall allow for my expenses during the next year the following amounts, and I will endeavor to keep within each allowance:

1. *House*

a. Rent (or taxes)			*d.* Light		
b. Insurance			*e.* Miscellaneous		
c. Heat			*Total*		

2. *Food*

a. Groceries			*d.* Fruit		
b. Meat			*e.* Other items		
c. Milk			*Total*		

3. *Clothing*

a. Suits and coats			*d.* Hats		
b. Underwear			*e.* Other items		
c. Shoes			*Total*		

4. *Advancement*

a. Education			*d.* Religion		
b. Reading matter			*e.* Charity		
c. Travel			*Total*		

5. *Health*

a. Recreation			*c.* Other expenses		
b. Doctor's bills			*Total*		

6. *Miscellaneous*

a. Laundry			*c.* Small items		
b. Presents			*Total*		

7. *Savings*

a. Bank account			*c.* Other kinds of saving		
b. Life insurance			*Total*		

BUDGETS

1. A family with an income of $1500 a year allowed 20% of the income for house, 25% for food, 20% for clothing, 20% for advancement, 10% for other expenses, and the rest for savings. How much money was allowed for each purpose?

2. A family with an income of $2500 a year allowed 20% for house, 25% for food, 15% for clothing, 25% for advancement, 10% for other expenses, and the rest for savings. How much money was allowed for each purpose?

3. The per cents stated in Exs. 1 and 2 are the results of careful studies of family expenses where the incomes range from $1500 to $4000. The changes in the cost of living do not materially affect the relative per cents of the income expended for various purposes. Solve Ex. 2, assuming that the income is $2800; is $3200; is $4000.

4. A study of family expenditures in a manufacturing community showed that 19% of the income was paid for house, 43% for food, 11% for clothing, 22% for all other expenses, and the rest for savings. If a family has an income of $1200 a year, how much money is spent for each purpose?

5. Using the form given on page 119, make up a budget for an income of $3000 a year, allotting the specified per cent to each item. 1. House: rent, 18%; heat, 5%; light, 2%; miscellaneous, 2%. 2. Food: groceries, 12%; meat, $6\frac{1}{2}$%; milk, 2%; fruit, $1\frac{1}{2}$%; other items, 3%. 3. Clothing: suits and coats, 6%; underwear, 2%; shoes, 2%; hats, 1%; other items, 4%. 4. Advancement: education, 1%; reading matter, $2\frac{1}{2}$%; travel, $2\frac{1}{2}$%; religion, 2%; charity, 2%. 5. Health: recreation, 3%; doctor's bills, 2%; other expenses, 2%. 6. Miscellaneous: laundry, 2%; presents, 3%; small items, 2%. 7. Savings: bank account, 4%; life insurance, 3%; other kinds of saving, 2%.

A SILENT READING LESSON

A boy who puts 1¢ each day into a toy bank will save enough in 6 mo. to buy a fielder's glove or two baseball bats. A girl who saves 10¢ a day will save enough in 2 mo. to buy a dress or a pair of good shoes. A man who puts aside $1 a day will perhaps save enough in a year to make the first payment on a lot on which to build a home.

A boy who deposits $1 a week for 10 yr. in a bank, which pays interest at 4% a year and adds it to the amount on deposit every 6 mo., will have over $600 in 10 yr. A man who deposits $10 a week in the same bank will have over $6300 at the end of the 10 yr., — an amount well worth having.

People generally get the things that they need in order to live comfortably, or the things that help them to enjoy life, by saving a little at a time. There are many people who never learn how to save money wisely. Many spend their money as they earn it, because of the constant temptation to live extravagantly, which includes not only living beyond one's means but also spending money foolishly.

We should all begin early to save, no matter if it is but a few dollars or a few cents a year. Saving money is just as important as earning it.

After a depositor has opened a savings account at a bank, as described on page 117, he may make deposits whenever he wishes. Each time he takes his book and money to the bank. The bank clerk (or *teller*, as he is often called) takes the money and writes in the book the amount deposited. Similarly, when money is drawn out, the amount is entered in the book. Deposits may also be sent by mail.

We should inquire at home or of some banker about trust companies, which, in addition to doing a regular banking business, take charge of funds and manage and settle estates.

SAVING MONEY

1. Beginning on the day that he is 16 yr. old, a boy saves 30¢ a day for 300 da. a year and deposits it in a savings bank. Not counting interest, how much will he have in the bank when he becomes 21 yr. old?

2. A man gives his daughter $5 on the day that she is 1 yr. old, $10 on her next birthday, $15 on her next birthday, and so on until she is 21 yr. old, depositing the money for her in a savings bank. Not counting interest, how much will the girl have in the bank when she is 21 yr. old?

3. A man saved $5 a week for 50 wk. of the year in which he was 21 yr. old. In each following year he increased his savings 10% over the preceding year. How much did he save during the year in which he was 29 yr. old?

4. A merchant saves $4125 the first year he is in business. The second year he increases his savings one third. The third year they are only 85% as much as the second year. The fourth year they increase 30% over the third year. How much does he save in the 4 yr.?

5. A man is paid a salary of $45 a week for 50 wk. in a year. His monthly expenses are $35 for house rent, 60% as much for clothing, $60 for food, and 30% as much for other necessary expenses as for food. How much can the man deposit a year in the savings bank?

6. Two years ago a clerk in a store had a salary of $28 a week and was also paid a commission of 2% on the goods he sold. That year he worked 48 wk. and sold $6350 worth of goods. Last year his salary was increased 25%, but the rate of commission was not changed. If he worked 49 wk. and sold $7750 worth of goods, how much was his income last year? How much was his income increased?

Interest at Savings Banks. Savings banks are closely watched by the state and are therefore very reliable. They usually pay interest every 6 mo. or every 3 mo. If not drawn out by the depositor, this interest is added to the principal, and the total amount then draws interest.

When interest due is added to the principal, and the total amount draws interest, the interest is called *compound interest.*

The method of finding compound interest is much like that of simple interest. For example, suppose that we wish to find the interest on \$2000 for 2 yr. at 4% a year, compounded semi-annually.

Arranging the work as here shown, we find the amount to be \$2164.87 and the interest to be \$164.87.

The interest here is found exactly, but since savings banks pay interest only on the dollars and not on the cents, a bank would pay a depositor only \$164.84 in this case.

Disregard a fraction less than $\frac{1}{2}$¢ in any interest; call $\frac{1}{2}$¢ or more a full cent.

$$\begin{array}{rl}
\$2000, & \text{first principal}\\
0.02, & \text{rate for 6 mo.}\\
\hline
\$40, & \text{int. first 6 mo.}\\
2000 & \\
\hline
\$2040, & \text{amt. after 6 mo.}\\
0.02 & \\
\hline
\$40.80, & \text{int. second 6 mo.}\\
2040.00 & \\
\hline
\$2080.80, & \text{amt. after 1 yr.}\\
0.02 & \\
\hline
\$41.62 & (\$41.616)\\
2080.80 & \\
\hline
\$2122.42, & \text{amt. after } 1\tfrac{1}{2} \text{ yr.}\\
0.02 & \\
\hline
\$42.45 & (\$42.4484)\\
2122.42 & \\
\hline
\$2164.87, & \text{amt. after 2 yr.}\\
2000.00, & \text{original principal}\\
\hline
\$164.87, & \text{int. for 2 yr.}
\end{array}$$

Simple interest in the above example would be \$160, or \$4.87 less than the compound interest. If simple interest is collected when due and is at once reinvested, the investor receives the same as compound interest.

Savings-Bank Account. Some savings banks compute the interest due each Jan. 1 and July 1 on the smallest balance on deposit during the preceding 6 mo., and add it to the amount on deposit. The following is a specimen account at a bank where the interest rate is 4%, payable semiannually:

Date		Deposits		Interest		Payments		Balance	
1928									
July	1	600	50					600	50
July	20	75						675	50
Sept.	6					120		555	50
Dec.	7	60						615	50
Dec.	20					65		550	50
1929									
Jan.	1			11				561	50
May	9	200						761	50
July	1			11	22			772	72

The smallest balance during the first interest period (July 1, 1928, to Jan. 1, 1929) is $550.50, and interest is computed on exact dollars only. At 2%, the interest on $550 is $11, and hence $11 is credited in the interest column for Jan. 1 and added to the balance. In the second period the smallest balance is $561.50, and the interest is 2% of $561, or $11.22.

Some banks allow deposits to begin drawing interest on the first of each month; others on the first of each quarter; others, as above, on the first of each half year. In a bank which allows interest from the first of each quarter and credits it every 6 mo., we should have, in the above case,

Oct. 1, 1928, 1% of $555 = $5.55
Jan. 1, 1929, 1% of $550 = 5.50
Jan. 1, 1929, credit of interest = $11.05

SIMPLE AND COMPOUND INTEREST

At simple interest, and also at interest compounded annually, find the amount (principal plus interest) of

1. $2500 for 4 yr. at 5%.
2. $4000 for 3 yr. at 6%.
3. $3500 for 3 yr. at 4%.
4. $2250 for 4 yr. at 3%.
5. $1750 for 3 yr. at $3\frac{1}{2}\%$.
6. $625.50 for 4 yr. at $3\frac{1}{2}\%$.
7. $10,000 for 6 yr. at 4%.
8. $425.50 for 4 yr. at $4\frac{1}{2}\%$.

At interest compounded semiannually, find the amount of

9. $200 for 3 yr. at 4%.
10. $650 for 2 yr. at 6%.
11. $700 for 3 yr. at 3%.
12. $500 for 2 yr. at 4%.
13. $1000 for 2 yr. at $4\frac{1}{2}\%$.
14. $2000 for 2 yr. at $3\frac{1}{2}\%$.

15. John has $500 on deposit in a savings bank on Jan. 1. Assuming that no money is withdrawn, how much interest is due on the following July 1 if the bank pays 4%? if it pays $3\frac{1}{2}\%$? if it pays $3\frac{1}{4}\%$? if it pays $3\frac{3}{4}\%$?

16. A school has an account at a savings bank in which are deposited the savings of all the pupils, and a committee of the pupils keeps the account of each pupil. If $475 is deposited on June 30, and the bank pays 4%, how much interest will be due on the following Jan. 1?

17. Edwin has $50 in a savings bank on July 1, and deposits $10 on the last day of each month for 6 mo., beginning July 31. The bank allows 4% on sums on deposit the first of each month, and credits interest on Jan. 1 and July 1; that is, on Jan. 1 the balance will be increased by the interest due Jan. 1, which is computed for 6 mo. on $50, for 5 mo. on the first $10, for 4 mo. on the second $10, and so on to the December deposit, which draws no interest. Find Edwin's balance on Jan. 1 following the last deposit.

18. Which brings the more interest in 2 yr., \$1250 drawing simple interest at $4\frac{1}{2}\%$, or \$1250 deposited at the beginning of a year in a savings bank which pays 4%, compounded semiannually, and disregards the cents in the principal in figuring the interest? How much is the difference?

19. On Jan. 1 a man deposits \$2000 in a savings bank which pays interest at 4%, compounded semiannually. If he leaves the money in the bank for 2 yr., how much interest does he receive on his deposit?

20. Mrs. Hall deposits \$40 each month in a savings bank that pays 4%, compounded on Jan. 1 and July 1, on all sums on deposit on the first day of each month. If she deposits the money just before the first of the month, beginning the last of January, what will be her balance on the following Jan. 1?

21. In Ex. 20, if the bank credits interest on Jan. 1 and July 1 on all sums on deposit on Jan. 1, Apr. 1, July 1, and Oct. 1, what will be Mrs. Hall's balance on Jan. 1?

22. A man's account at a savings bank shows the following deposits: Jan. 2, 1928, \$950; Mar. 22, 1928, \$40; Apr. 12, 1928, \$65; Sept. 10, 1928, \$25; Mar. 8, 1929, \$125. The account shows also the following withdrawals: Feb. 1, 1928, \$35; Aug. 2, 1928, \$75; Sept. 7, 1928, \$125; Dec. 13, 1928, \$95; Jan. 2, 1929, \$385. The bank pays 4% on the smallest balance on deposit during an interest period, and compounds the interest on Jan. 1 and July 1. Find how much the man has to his credit on July 2, 1929.

23. A man deposits \$200 in a savings bank on Jan. 1, \$300 on Feb. 1, \$100 on May 1, \$400 on Aug. 1, and \$350 on Nov. 1. The bank allows interest at 4% from the first of each month, compounding it on July 1 and on Jan. 1. How much will the man have to his credit on Jan. 1 following these deposits? How much will he have on the following July 1?

Postal Savings System. The United States government conducts a savings bank, known as the *Postal Savings System*, in connection with the post office. Although every savings bank is supervised by the state in which it is located, many persons prefer a smaller rate of income from the Postal Savings System, because our national government guarantees the safety of their money.

Any person of the age of 10 yr. or over may open an account, and may make deposits (which must be in exact dollars), until the balance reaches $2500, exclusive of accumulated interest.

Interest is paid by the government at the rate of 2% for each full year that the money remains on deposit, beginning on the first day of the month following the one in which the deposit is made. Interest is not paid for any fraction of a year. The government does not compound the interest on these deposits, but if the interest is withdrawn when due and is at once deposited, the depositor will receive the equivalent of compound interest on his money.

A person may exchange his deposits for *postal savings bonds*, which are like promissory notes that bear interest. These bonds bear interest at $2\frac{1}{2}$%, payable every 6 mo., and are issued in denominations of $20, $100, and $500. The exchange may be made on Jan. 1 or July 1 of any year.

The depositor receives a *postal savings certificate of deposit* for the amount of each deposit which he makes. These certificates are issued in the following denominations: $1, $2, $5, $10, $20, $50, $100, $200, and $500.

Since no deposit of less than $1 is allowed, the government sells *postal savings stamps* at 10¢ each. These stamps may be attached to a *postal savings card*, which has spaces for ten stamps. The card with one stamp already on it costs 10¢. When the card is filled up, it may be used as a deposit of $1, or the $1 may be collected in cash.

POSTAL SAVINGS SYSTEM

Numbers 1 to 16, oral

Find the interest for 1 yr. at 2% on a deposit of

1. \$20. **2.** \$50. **3.** \$100. **4.** \$400. **5.** \$800.

Find the interest for 2 yr. at 2% on a deposit of

6. \$10. **7.** \$40. **8.** \$200. **9.** \$700. **10.** \$1000.

Find the interest for 1 yr. at $2\frac{1}{2}\%$ received by a man who holds postal savings bonds for

11. \$20. **12.** \$100. **13.** \$300. **14.** \$500. **15.** \$1000.

16. Using as large denominations as possible, state the denominations of the certificates of deposit which you will receive for a deposit of \$87.

17. Saving 10¢ on each of the 312 week days of a certain year, a boy filled his postal savings card as fast as he could and then bought certificates of deposit. How much money did he have on deposit with the post office at the end of the year?

18. If you deposit \$4 on June 7 and \$16 on June 15, 1929, how much interest will be due on July 1, 1930?

19. If you deposit \$10 the last day of each month for 1 yr., beginning Jan. 31, 1928, how much do you have on deposit at the post office on Jan. 1, 1929? What is the first date on which any interest is due you, and how much is then due? How much interest is due on Feb. 1, 1930?

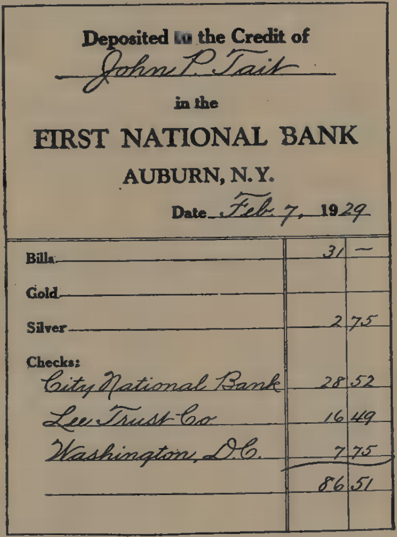
20. If in Ex. 19 you make no withdrawals of interest, how much interest is due you on Aug. 1, 1930?

21. On July 1 Mr. Watson exchanges \$500 in postal savings certificates for a postal savings bond paying $2\frac{1}{2}\%$. Find how much interest is due on this bond on the following Jan. 1.

A SILENT READING LESSON

Instead of requiring the bank book to be presented, some banks pay money from a depositor's account upon presentation of a written order called a *check*. Such a bank is sometimes called a *commercial bank*, to distinguish it from a savings bank. An account in such a bank is often called a *checking account*, and, if small, does not draw interest. The convenience of such an account is more important than the interest that the money might earn.

It is much safer to send a check from one place to another than to send cash.



Deposited to the Credit of
John P. Tait
in the
FIRST NATIONAL BANK
AUBURN, N.Y.
Date Feb. 7, 1929

Bills	31	—
Gold		
Silver	2	75
Checks:		
City National Bank	28	52
Lee Trust Co	16	49
Washington, D.C.	7	75
	86	51

When a man makes a deposit of money or of checks at his bank, he fills out a *deposit slip*, as here shown, and hands it in with his deposit. Most banks request that the name of the bank on which the check is drawn be given on the deposit slip when the two banks are located in the same city or town. In other cases the name of the place in which the other bank is located is generally sufficient.

When the money to be deposited is taken to the bank, the teller who receives it checks the deposit with the items on the deposit slip to see that it is correct. When a deposit is made by mail, the bank fills out a printed receipt form and sends it to the depositor.

DEPOSIT SLIPS

Numbers 1 to 10, oral

State the amount of each of the following deposits:

1. $15.50 3.75	**3.** $12.75 2.25	**5.** $23.80 12.70	**7.** $12.00 13.75	**9.** $62.50 2.50
2. $10.00 5.75 1.25	**4.** $22.50 2.50 5.00	**6.** $45.00 5.75 1.25	**8.** $16.75 10.00 3.25	**10.** $75.50 4.50 20.00

Fill out a deposit slip for each of the following cases, using your name as depositor in some bank of which you know the name:

11. Bills, $254; silver, $40; check on First National Bank, $87.50; check on Traders Bank, $127.50.

12. Bills, $423; gold, $275; silver, $135.75; check on Cotton Exchange Bank, $342.60; check on City Trust Co., $67.50.

13. Bills, $135; check on Chemical National Bank, $325; check on Third National Bank, $63.73.

14. Bills, $1726; gold, $125; silver, $100; check on Traders Bank, $335.50; check on Second National Bank, $175.40.

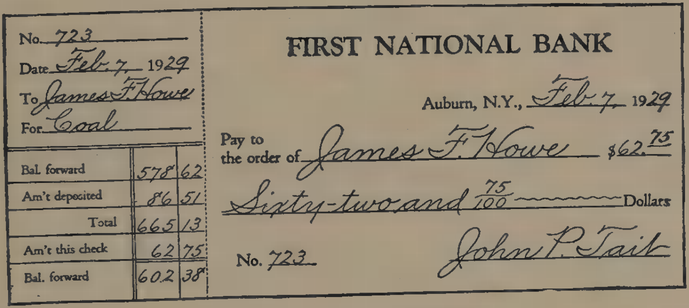
15. Bills, $145; gold, $480; silver, $350.75; check on Merchants National Bank, $255; check on Farmers Trust Co., $162.50; check on Second National Bank, $275.50.

16. Gold, $50; checks on First National Bank, $27.62, $41.75, $32.80; check on Lincoln Trust Co., $37.42; check on a Chicago bank, $67.85; check on a Detroit bank, $135.20.

17. Bills, $146; silver, $275.50; check on First National Bank, $43.50; check on State Trust Co., $250; check on a San Francisco bank, $125.60; check on a St. Louis bank, $37.65; check on a Rochester bank, $42.50.

Checks. As we have already learned, a check is an order written by the depositor directing his bank to pay out money from his account. A check is as good as money if the one who signs it has enough funds in the bank to pay the check when it is presented.

A *check book* containing blank checks and stubs is furnished to each depositor by the bank. When the check is filled out, it is detached, but the stub is left in the book. On the stubs the depositor puts down the amounts of the checks that he draws and of the deposits that he makes. He thus keeps a record of his bank balance, as shown in the following model:



No. 723
Date Feb 7, 1929
To James F. Howe
For Coal

Bal. forward	578	62
Am't deposited	86	51
Total	665	13
Am't this check	62	75
Bal. forward	602	38

FIRST NATIONAL BANK

Auburn, N.Y., Feb 7, 1929

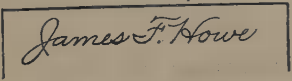
Pay to the order of James F. Howe $62.75

Sixty-two and 75/100 Dollars

No. 723 John P. Tait

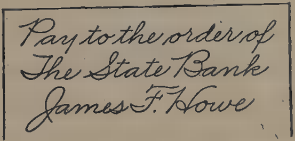
Stub Check

The person to whom a check is payable is called the *payee.* To collect the check, the payee must *indorse* it by writing his name across the back, as here shown.



James F. Howe

If the payee wishes to make the check payable to some other person, for example, to the bank at which he wishes to deposit the check to his account, he may indorse it as here shown. In this case only the State Bank can collect the check, and to do so must indorse it below the preceding indorsement.



Pay to the order of
The State Bank
James F. Howe

DEPOSITS AND CHECKS

1. On Feb. 7 Mr. Tait, a page of whose check book is shown on page 131, makes out check No. 724 to the Corner Grocery Co. in payment of a bill of $14.63. Write the check and fill in the stub, showing his bank balance after paying this bill.

2. A man has $350 in the bank on the first of the month. During the month he pays by check the following bills: rent, $40; telephone, $3.50; milk, $4.27; groceries, $22.17; meat, $17.67; light, $3.86; coal, $72.20; insurance, $18.76. What is his balance on the last day of the month?

3. If your deposits in a bank have been $68.45, $92.30, $47.60, and $96.70, and you have drawn checks for $8.55, $23.65, $8.58, $48.75, and $34.60, what is your balance?

4. A merchant having $980.75 in the bank makes deposits during the next week of $185.50, $97.85, $135.50, $86.85, and $236.80. He draws checks for $89.65, $37.20, $93.60, $15.20, $248.70, and $39.80. What is his balance?

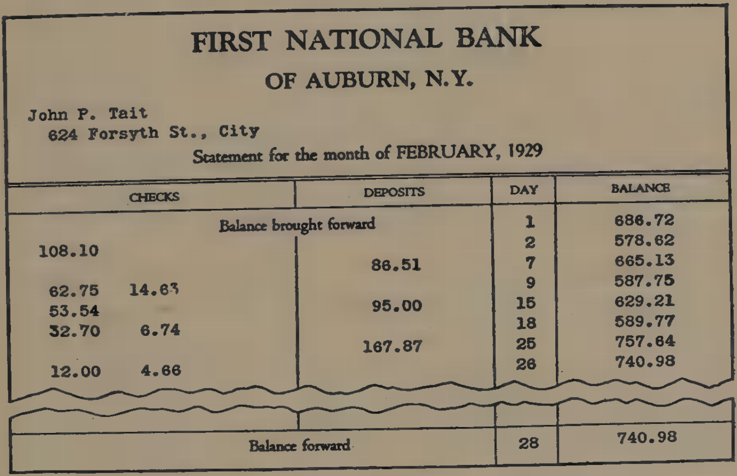
5. A man whose bank balance is $825.60 deposits checks for $75.80, $126.75, $234.80, and $42.80. He draws checks for $36.48, $7.85, $47.50, $128.75, $125.80, and $24.75. What is his balance?

6. A grocer having $828.50 in the bank deposits $567.80, $245.50, $89.65, $482.86, $429.50, and $376.50, and draws checks for $427.50, $38.95, $67.82, $568.70, and $122.58. He also pays by check one invoice for $173.96 and another for $86.75. What is his balance?

7. A restaurant keeper having $1026.92 in the bank makes deposits of $488.75, $928.75, $386.48, $442.80, $196.85, and $327.75, and draws checks for $96.75, $286.75, $342.80, and $438.50. He also pays back by check a loan of $230 on which interest is due for 4 mo. at 5%. What is his balance?

Pass Book. When an account is opened, the bank usually gives the depositor a *pass book,* in which is entered the amount deposited. Whenever an additional deposit is made at the bank the book is handed to the receiving teller, who enters the amount and returns the book to the depositor.

Statement. While some banks balance the pass book, most large banks use it simply as the customer's receipt for money deposited and, at the end of each month, send the depositor a *statement* of his account in some such form as the following:



FIRST NATIONAL BANK

OF AUBURN, N.Y.

John P. Tait
624 Forsyth St., City

Statement for the month of FEBRUARY, 1929

CHECKS		DEPOSITS	DAY	BALANCE
		Balance brought forward	1	686.72
108.10			2	578.62
		86.51	7	665.13
62.75	14.63		9	587.75
53.54		95.00	15	629.21
32.70	6.74		18	589.77
		167.87	25	757.64
12.00	4.66		26	740.98
		Balance forward	28	740.98

The deposits and other credits are added to the balance from the previous month's statement, and the checks are deducted as they are paid by the bank. The column at the right shows the daily balance, and the last entry shows the balance to be carried to the next month's statement.

With the statement the bank returns the checks that it has paid during the month. The depositor checks this statement with his own record, as kept in his check book.

BANK STATEMENTS

1. The bank statement for October shows that a firm has a balance of $7632.96 on Oct. 31. The pass book shows the following deposits in November: *6*, $3758.60; *9*, $2843.85; *12*, $2976.42; *16*, $4826.75; *20*, $2928.85; *24*, $3174.30; *29*, $2672.29. The check book shows the following checks issued in November: *2*, $689.85; *3*, $392.90, $167.75; *5*, $3228.45, $7.90, $654.68; *9*, $850, $525.35; *14*, $796.80, $2285.40; *20*, $948.65; *22*, $3689.85; *29*, $34.80, $340, $10.26, $3.92, $52.30, $128.75, $68.38. What is the firm's balance on Nov. 30?

2. For Ex. 1 make out the statement which the bank sends the firm on Nov. 30, using a form like the one on page 133 and inserting the name of some bank and firm in your vicinity. Consider that the checks issued were paid by the bank on the day they were drawn, although they may actually have been paid some time thereafter.

3. A merchant's bank balance was $14,296.35 on Oct. 1. Deposits were made during the month as follows: *8*, $2978.30; *14*, $2285.85; *18*, $2862.75; *22*, $2438.70; *25*, $3162.62; *28*, $1657.37. Checks were drawn as follows: *6*, $697.28, $1128.67, $231.27; *13*, $268.92, $765.43, $1.47; *16*, $1428.83, $647.26, $2786; *19*, $11.28, $350, $591.23; *25*, $1100; *29*, $6.27, $285, $2.89, $15.75, $28.50, $528.92. Make out the bank statement for October as in Ex. 2.

4. Make out a bank statement for December, as in Ex. 2, using the following items: Balance, *Dec. 1*, $2385.60. Deposits: *2*, $495.65; *3*, $229.86; *6*, $308.75; *8*, $445.65; *10*, $638.80; *28*, $1275.30. Checks: *3*, $50, $74.28, $61.97; *7*, $221.62, $45; *8*, $4.27, $940, $78.26; *14*, $16.30, $28.16, $97.80; *18*, $1225, $87.30; *25*, $1.47, $6.89, $3.26; *29*, $41.60, $12.30, $2.48, $75.

A SILENT READING LESSON

Business houses usually have need of large sums of money at certain times of the year, and it is better for them to borrow it for two or three months than to keep such an amount on hand for a whole year. Borrowing money, therefore, is often good business policy for the merchant, contractor, or manufacturer, although it is poor policy for any person to borrow money simply to gratify some extravagant desire.

It is now the custom for business houses and for many private individuals to borrow money at banks. The banks obtain the money for these loans from their depositors, who find it safer and more convenient to keep their money in a bank than to lock it up in their homes or offices. The banks are paid for their work by charging borrowers a higher rate of interest than they pay to depositors.

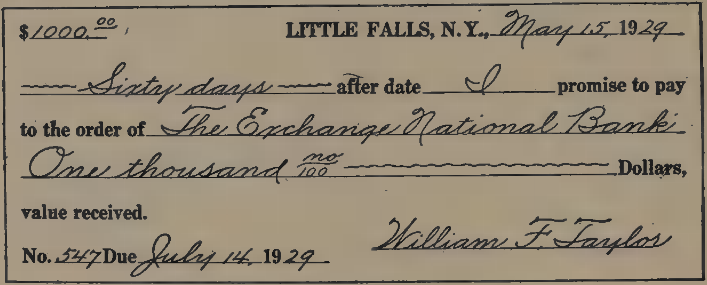
If a business man wishes to borrow money from a bank, the bank will sometimes lend it to him on the strength of his financial standing alone.

A reputation for honesty and promptness in business dealings is a very valuable asset for an individual or for a company. Start life with a good reputation and never lose it.

If a man's financial standing is not such as to permit an unsecured loan, he may deposit with the bank sufficient *collateral,* which means property of value such as stocks and bonds about which we shall learn more later. The bank has the right to sell the collateral in case the note is not paid.

Instead of depositing collateral, the borrower may get some financially sound person to indorse his note, the indorser thereby agreeing to pay it if the maker does not. Some large city banks have about a third of their loans secured by collateral, and the rest of their loans are made on notes either with or without indorsers.

Borrowing on a Note. Suppose that Mr. Taylor wishes to borrow $1000 for about 2 mo. If the prospects for his business are satisfactory, and if he has a good reputation in financial matters, the bank with which he does business may lend him the money without requiring a deposit of collateral. Mr. Taylor will then fill out a note like the following model:



$1000.00 LITTLE FALLS, N.Y., May 15 1929

Sixty days after date I promise to pay

to the order of The Exchange National Bank

One thousand no/100 Dollars,

value received.

No. 547 Due July 14 1929 William F. Taylor

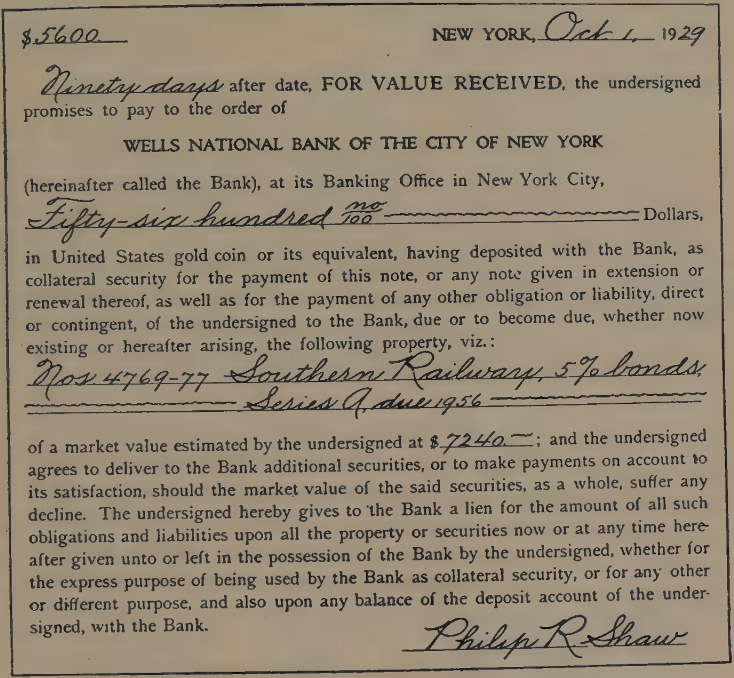
The bank keeps this note until the date of maturity, which is 60 da. after May 15, 1929, or July 14, 1929. The bank then charges the note to Mr. Taylor's account, unless he has made some other arrangement for paying it, notifies him that it has done so, and returns the note to him.

If Mr. Taylor wishes to keep the money for longer than 60 da., the bank will probably have him make out a new note for 30 da., 60 da., or 90 da. more, and will cancel the old one.

If Mr. Taylor's financial standing is not such as to warrant lending the money to him without some security, he may be asked to get an indorser. In such a case he would make the note payable to some person known to the bank, and this person would indorse it to the bank, thus promising to pay the money if Mr. Taylor failed to do so.

Sometimes, instead of indorsing a note in this way, the indorser signs his name below that of the maker; that is, there are then two makers. Such a note is called a *joint note.*

Note Secured by Collateral. The following model shows a typical form of a note secured by the deposit of collateral:



$5600   NEW YORK, Oct 1, 1929

Ninety days after date, FOR VALUE RECEIVED, the undersigned promises to pay to the order of

WELLS NATIONAL BANK OF THE CITY OF NEW YORK

(hereinafter called the Bank), at its Banking Office in New York City,

Fifty-six hundred no/100 Dollars,

in United States gold coin or its equivalent, having deposited with the Bank, as collateral security for the payment of this note, or any note given in extension or renewal thereof, as well as for the payment of any other obligation or liability, direct or contingent, of the undersigned to the Bank, due or to become due, whether now existing or hereafter arising, the following property, viz.:

Nos 4769-77 Southern Railway, 5% bonds, Series A, due 1956

of a market value estimated by the undersigned at $7240.— ; and the undersigned agrees to deliver to the Bank additional securities, or to make payments on account to its satisfaction, should the market value of the said securities, as a whole, suffer any decline. The undersigned hereby gives to the Bank a lien for the amount of all such obligations and liabilities upon all the property or securities now or at any time hereafter given unto or left in the possession of the Bank by the undersigned, whether for the express purpose of being used by the Bank as collateral security, or for any other or different purpose, and also upon any balance of the deposit account of the undersigned, with the Bank.

Philip R. Shaw

In the above case the collateral deposited with the bank consists of nine Southern Railway bonds for $1000 each. When we study bonds, we shall understand why they are property of value that can be pledged in this way. Banks usually require that the face of a note shall be not greater than about 80% of the market value of the collateral.

It is not necessary to try to remember much about such a note. We simply need to know what it means when we hear some one speak of "putting up collateral" to secure a loan.

Interest by the Six-Per-Cent Method. The six-per-cent method, mentioned on page 109, is so commonly used in banking, where the notes often bear 6% and run for 30 da., 60 da., or 90 da., that it will help us to review it briefly and to learn a little more about it.

Briefly stated, the principle of the method is as follows:

To find the interest at 6% for 60 da., move the decimal point two places to the left; for 6 da., move it three places to the left.

In applying such a rule, we, of course, remember that a sum like \$250 is the same as \$250. or \$250.00; that is, any whole number may be thought of as having a decimal point.

For example, the interest at 6% for 60 da. on \$250 is \$2.50, and on \$765.50 it is \$7.655, or \$7.66. Similarly, at 6% the interest for 6 da. on \$1250 is \$1.25, and on \$1875 it is \$1.875, or \$1.88.

For 30 da. the interest is half that for 60 da.; for 90 da. it is one and a half times that for 60 da. Thus, the interest on \$1200 at 6% for 30 da. is $\frac{1}{2}$ of \$12, or \$6, and for 90 da. it is \$12 + \$6, or \$18.

NOTES BEARING INTEREST AT 6%

Find the interest at 6% on a note for

1. \$120 for 60 da.
2. \$270 for 90 da.
3. \$525 for 30 da.
4. \$2575 for 90 da.
5. \$3625 for 90 da.
6. \$4250 for 30 da.
7. \$3850 for 60 da.
8. \$4000 for 60 da.
9. \$7500 for 90 da.
10. \$160 for 2 mo. 6 da.
11. \$450 for 2 mo. 12 da.
12. \$725 for 2 mo. 15 da.
13. \$850 for 2 mo. 20 da.
14. \$550 for 1 mo. 15 da.
15. \$1275 for 1 mo. 20 da.
16. \$6250 for 1 mo. 10 da.
17. \$7500 for 3 mo. 15 da.
18. \$8000 for 3 mo. 20 da.

Six-Per-Cent Method for Other Rates. We may use the six-per-cent method to find interest at other rates in common use.

For example, since 5 is $\frac{1}{6}$ less than 6, the interest at 5% on a given principal is $\frac{1}{6}$ less than the interest at 6%. The work at the right shows how we apply this principle in computing the interest due on $1260 for 90 da. at 5%.

Interest for 60 da. at 6%,	$12.60
Interest for 30 da.,	6.30
Interest for 90 da. at 6%,	$18.90
Deduct $\frac{1}{6}$,	3.15
Interest for 90 da. at 5%,	$15.75

Similarly, the interest at $5\frac{1}{2}$% is $\frac{1}{12}$ less than the interest at 6%, the interest at $4\frac{1}{2}$% is $\frac{1}{4}$ less than the interest at 6%, the interest at 4% is $\frac{1}{3}$ less than the interest at 6%, and so on for other rates of interest.

SIX-PER-CENT METHOD FOR OTHER RATES

Find the interest at 5% for these principals and periods:

1. $780, 30 da.
2. $840, 60 da.
3. $86.40, 60 da.
4. $79.20, 30 da.
5. $1200, 90 da.
6. $1560, 90 da.

Find the interest at 4% for these principals and periods:

7. $240, 30 da.
8. $630, 60 da.
9. $68.80, 60 da.
10. $72.24, 30 da.
11. $1800, 90 da.
12. $2436, 90 da.

Find the interest at $5\frac{1}{2}$% for these principals and periods:

13. $720, 30 da.
14. $960, 60 da.
15. $60.72, 60 da.
16. $97.68, 30 da.
17. $5600, 90 da.
18. $4048, 90 da.

Find the interest at $4\frac{1}{2}$% for these principals and periods:

19. $160, 30 da.
20. $920, 60 da.
21. $54.80, 60 da.
22. $62.48, 30 da.
23. $6000, 90 da.
24. $8648, 90 da.

Bank Discount. When a man borrows money from a bank the bank may require him to pay the interest in advance, although not all banks do so. Interest paid in advance is called *discount*, and the period for which discount is charged is called the *term of discount.*

The word "discount" is thus used both in banking and in buying goods. The arithmetic involved is the same in both cases; that is, it is finding a per cent of a number.

Proceeds of a Note. The face of a note less the discount is called the *proceeds* of the note.

For example, suppose that a 90-day note for $500 is discounted by a bank at 6%.

By the six-per-cent method, as shown in the first step at the right, the discount (interest) is found to be $7.50.

As shown in the second step, the proceeds are then found to be $492.50.

$5.00	$500.00
2.50	7.50
$7.50	$492.50

BANK DISCOUNT

When discounted at 6%, find the proceeds of a note for

1. $75 for 60 da.
2. $90 for 30 da.
3. $50 for 90 da.
4. $250 for 90 da.
5. $275 for 90 da.
6. $425 for 60 da.
7. $1500 for 60 da.
8. $1800 for 63 da.
9. $4500 for 93 da.

Find the discount at 5% when the face of the note and the term of discount are as follows:

10. $40, 30 da.
11. $70, 60 da.
12. $80, 90 da.
13. $175, 66 da.
14. $250, 78 da.
15. $900, 36 da.
16. $1250, 30 da.
17. $2275, 60 da.
18. $4750, 90 da.
19. $6200, 30 da.
20. $3560, 30 da.
21. $4550, 60 da.
22. $7725, 90 da.
23. $8250, 60 da.
24. $6575, 90 da.

BANK DISCOUNT

1. Make out a note for $1200, dated today and payable to your order in 60 da. with interest at 6%, and sign it with the initials X.Y.Z.

2. Make out a 30-day note without interest for $675, dated today and payable to the order of the County National Bank, and sign it A.B.C. If the bank discounts the note today at 6%, what are the proceeds?

3. Make out a 90-day note without interest for $1250, dated today and payable to the order of the Blank Trust Co. If the note is discounted today at 6%, what are the proceeds?

4. A man's bank account shows deposits of $37.50, $82.50, $75, $100, $50, $300, $40, $125, $500, and checks drawn for $13.75, $62, $5, $125.50, $82.75. He needs $2000 to start him in business and wishes to keep about $500 in the bank. How much, to the nearest $100, should he borrow?

5. If, in Ex. 4, the man's bank discounts his note for this sum for 90 da., what are the proceeds?

6. A merchant has to pay a debt of $1275 less 10%. His balance in the bank is $672.80, and he has $107.50 in cash in his safe. He wishes to leave about $300 in the bank and about $65 in his safe. How much, to the nearest $50 must he borrow? If he gives a 90-day note for this sum, and the note is discounted the same day at 6%, the proceeds are how much more or how much less than the exact sum which he needs?

Find the proceeds when the face of a note, the period of discount, and the rate of discount are as follows:

7. $875, 30 da., 6%.

8. $975, 90 da., 5%.

9. $425, 30 da., 6%.

10. $2565, 90 da., 5%.

11. $3250, 90 da., $5\frac{1}{2}$%.

12. $4250, 30 da., $4\frac{1}{2}$%.

Discounting an Interest-bearing Note. A note is like any other commodity, something that may be sold. Banks, being dealers in money, are the institutions to which to sell notes. Thus, if you hold a note given you by someone else and need the money before the note is due, you may sell it to a bank. The bank will, of course, charge you discount for advancing the money on the note.

For example, suppose that you hold T. F. Hoyt's note for $500, dated Sept. 1 and due Dec. 1, with interest at 5%. If you discount the note at your bank on Sept. 26 at 6%, what are the proceeds?

Since the interest on the note will be paid to the bank, the first step is to find the amount of the note on Dec. 1. From a table the bank finds that there are 66 da. from Sept. 26 to Dec. 1 (Sept., 4 da.; Oct., 31 da.; Nov., 30 da.; Dec., 1 da.), and computes the discount for this term on the amount of the note. Deducting this discount, as shown in the last step of the following solution, the proceeds paid you are $500.68:

Face of note,	$500	60 da.,	$5.0625
Rate of interest,	0.05	6 da.,	.50625
	4)$25.00	Discount,	$5.56875
Interest due,	$6.25	Amount,	$506.25
Face,	$500.00	Discount,	5.57
Amount, Dec. 1,	$506.25	Proceeds,	$500.68

Federal Reserve Banks. If banks wish to borrow money, they may do so, subject to certain restrictions, from the *Federal Reserve Banks* which have been established by the government in different parts of the country. That is, briefly stated, these government banks may be said to act as banks for other banks.

DISCOUNTING NOTES

Find the discount and the proceeds for each of the following notes, which draw no interest, when the face, the term of discount, and the discount rate are as follows:

1. $750, 60 da., 5%.
2. $225, 60 da., 5%.
3. $500, 60 da., 6%.
4. $450, 60 da., 5%.
5. $225, 90 da., 5%.
6. $5575, 90 da., 6%.
7. $7500, 30 da., 5%.
8. $4750, 30 da., 6%.
9. $42.50, 30 da., 6%.
10. $125.50, 60 da., 6%.
11. $275.25, 60 da., 6%.
12. $375.50, 90 da., 6%.
13. $457.50, 30 da., 5%.
14. $296.50, 30 da., 5%.
15. $287.60, 90 da., 5%.
16. $375.40, 90 da., 5%.
17. $2750 90 da., 4½%.
18. $3275, 90 da., 4½%.

19. The discount at 6% on a note for 90 da. is $18.75, and on another note for 30 da. the discount is $32.50. If neither note draws interest, what is the face of each note?

20. On Oct. 4 a dealer bought $1750 worth of goods, giving a 90-day note bearing interest at 5%. On Oct. 10 the holder of the note discounted it at 6%. Find the proceeds.

Find the proceeds of each of the following notes:

	FACE OF NOTE	DATE OF NOTE	DATE DUE	RATE OF INTEREST	DATE OF DISCOUNT	RATE OF DISCOUNT
21.	$300	July 1	July 31	4%	July 16	6%
22.	$800	Sept. 1	Oct. 31	5%	Sept. 11	6%
23.	$550	Oct. 1	Dec. 30	4%	Dec. 2	6%
24.	$325	Mar. 1	Apr. 30	5%	Mar. 17	6%
25.	$1000	Nov. 1	Dec. 1	6%	Nov. 1	6%
26.	$1250	June 1	Aug. 30	5%	July 23	5%

REVIEW OF BANKING

1. Find the interest on $1750 for 60 da. at 6%, first by the six-per-cent method, then by cancellation, and finally by the ordinary method of finding the interest for 1 yr. and then for the fractional part of a year. Write a statement telling the advantage of the six-per-cent method over the other methods.

2. Using the three methods mentioned in Ex. 1, find the interest on $675 for 90 da. at 6%; on $1425 for 30 da. at 6%.

3. A firm wishes to borrow about $5000, and a bank offers to lend the money at 6%. If the firm discounts a 60-day note for $5100, how much more than $5000 will the firm actually receive after the discount is paid?

4. A real-estate dealer buys some property for $15,000. He pays $4800 down and borrows the rest from his bank on a 90-day note at 6%. How much discount does he pay?

5. How much greater, if any, is the discount on a note for $1250 discounted for 60 da. at 6% than the discount on a note for $2500 discounted for 30 da. at 6%?

6. A man who wishes to borrow $4875 gives his bank a 30-day note for $4900. If the bank discounts the note at 6%, will the proceeds be more or will they be less than the amount he needs? How much more or how much less will they be?

7. On Apr. 1 a dealer buys 16 automobiles at $1285, paying $8060 cash and giving a 90-day note, bearing interest at 6%, for the balance. If the manufacturer who holds the note discounts it at 6% on Apr. 16, how much discount does he pay? What are the proceeds of the note?

8. If a discount of 2% is allowed for cash on a bill of $1850, and a bank will discount a 30-day note for $1850 at 6%, how much does a merchant gain by borrowing the money at the bank and taking advantage of the cash discount?

A SILENT READING LESSON

If Mr. Brasch, who has an account at the Central National Bank, sends a check to Mr. Locke, who has an account at the West End Trust Co. in the same city, Mr. Locke would probably deposit the check at the West End Trust Co.

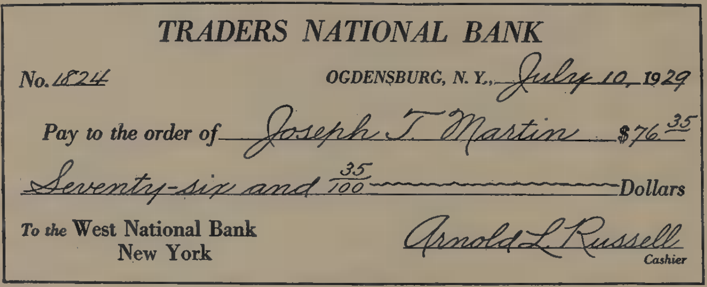
In a business day the West End Trust Co. may thus take in a good many checks which are drawn on the Central National Bank. Each day representatives from the different city banks meet at a place known as the *clearing house*, where they exchange checks and settle the balances due one another. In this way Mr. Brasch's check is returned to the Central National Bank, where it is charged to his account.

A similar procedure takes place in case the two persons live in different places. For example, a man in San Francisco might deposit at his bank a check drawn on a New York bank. If the San Francisco bank wished to increase its deposit with a Chicago bank with which it does business, the check might be sent to the Chicago bank and there credited to the account of the San Francisco bank. The Chicago bank, in turn, would send it to a New York bank with which it does business and receive credit for the amount. The New York bank on which the check is drawn would receive the check through the clearing house, as described above.

Some idea of the importance to modern business of checks and similar forms of transmitting funds may be gained from the fact that the New York City "clearings" alone often total more than a billion dollars a day.

Sometimes banks make a small collection charge to pay for the expense and work of collecting checks drawn on banks at distant points. Checks drawn on banks in large commercial centers, such as New York, Chicago, Philadelphia, and so on, are often collected for a depositor without any charge.

Paying Bills by Bank Draft. Suppose that Joseph T. Martin of Ogdensburg wishes to order farm tools from the Farmers Specialty Co. of Buffalo. He might send his personal check, but if the company does not know him it may not be willing to accept it. Accordingly he asks his bank in Ogdensburg, the Traders National Bank, for a "draft on New York" for the desired amount. The Traders Bank has money on deposit in the West National Bank of New York, and so issues the following order, called a *bank draft*, against this account:



TRADERS NATIONAL BANK

No. 1824 *OGDENSBURG, N. Y.,* July 10, 1929

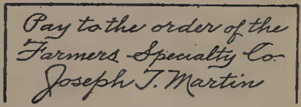
Pay to the order of Joseph T. Martin $\$76^{35}$

Seventy-six and $\frac{35}{100}$ *Dollars*

To the West National Bank
New York

Arnold L. Russell
Cashier

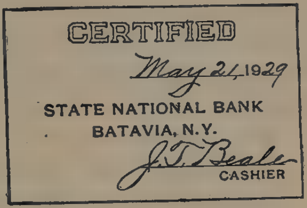
Mr. Martin then indorses the draft as here shown and sends it to the Farmers Specialty Co. with his order. Since the draft is really a check of the Traders National Bank instead of a personal check, there will be no question of its payment. Drafts on New York are accepted freely in all parts of the country and are used extensively in paying bills.



Pay to the order of the
Farmers Specialty Co.
Joseph T. Martin

A draft made by a cashier upon his own bank is usually called a *cashier's check*, and may be used for transmitting funds in the same way as a bank draft. While a personal check is a kind of draft, because it directs a bank to pay money, the word "draft" is not used in this connection. In fact, the word "check" is now often used in place of "draft."

Certified Check. To avoid any doubt as to a man's having sufficient money in the bank with which to pay a personal check when it is presented, he may have the check *certified*, which means that the bank guarantees the payment of the check. To do this the man takes the check to his bank and has the cashier, or some other officer, stamp it across the face in some such way as here shown. The bank at once takes the money from the man's account and holds it in a separate account until the check is returned for payment.


CERTIFIED
May 21, 1929
STATE NATIONAL BANK
BATAVIA, N.Y.
J. T. Beale
CASHIER

Letter of Credit. If a person is traveling, either in this country or abroad, it is necessary for him to obtain money in various places. It is difficult for him to cash personal checks in places where he is not known, and if he has to carry with him sufficient money to cover all his expenses, he runs the risk of theft or loss. One of the methods of avoiding such difficulties is that of carrying a *letter of credit.*

For example, a person may deposit \$5000 with a bank that sells letters of credit, and this bank will, for a reasonable compensation (usually 1% or less), give him a letter, signed by the bank officials and addressed to a large number of other banks, telling them that it guarantees the payment of any drafts drawn by the holder of the letter up to a total of \$5000.

When a draft is drawn on a letter of credit, the bank that pays the money writes on the letter a statement of the amount drawn. The holder of the letter is identified by his signature, which is either on the letter or, more commonly, in a book of instructions accompanying the letter.

Express money orders and *travelers' checks* are often used in place of the letter of credit, but the principle of identification by means of a comparison of signatures is the same.

Paying Bills by Money Order. In case you do not have a checking account at some bank and wish to send money to a distant place, you can easily do so by means of a *money order*. Such orders may be purchased at a post office, or from an express company, by payment of a small fee in addition to the face of the order. The order is mailed to the person to whom you are sending the money, and he can obtain the money at his post office or express office.

The fees for issuing postal money orders for use in this country are as follows:

For sums not exceeding \$2.50, 5¢; for amounts from \$2.51 to \$5, 7¢; \$5.01 to \$10, 10¢; \$10.01 to \$20, 12¢; \$20.01 to \$40, 15¢; \$40.01 to \$60, 18¢; \$60.01 to \$80, 20¢; \$80.01 to \$100, 22¢.

MONEY ORDERS

Numbers 1 to 16, oral

State the cost (face plus fee) of a postal money order for

1. \$25.
2. \$60.
3. \$80.
4. \$100.
5. \$31.50.
6. \$19.90.
7. \$52.75.
8. \$86.50.
9. \$92.30.
10. \$69.95.
11. \$16.30.
12. \$90.50.
13. \$92.75.
14. \$60.32.
15. \$80.41.
16. \$33.70.

17. When the United States government charges 22¢ for issuing a postal money order for \$100, what per cent of the face does it charge?

18. If you wish to send \$345 by means of postal money orders, you will have to buy three orders for \$100 each and one for \$45. How much, including both the face and the fee in each case, do you have to pay for the four orders?

19. When an express company charges 18¢ for issuing an express money order for \$75, what per cent of the face does the company charge?

III. MEASUREMENTS

A SILENT READING LESSON

Last year we learned how to measure the lengths of lines and how to find the area of each one of the following figures:

Square

Rectangle

Parallelogram

Triangle

Before we study more about measurements, we should review this work. In the cases of the square, the rectangle, and the parallelogram, as we already know,

The area is the product of the base and height,

or briefly, $$\mathit{Area} = \mathit{base} \times \mathit{height}.$$

For example, for a square that is 6 in. on a side, we have

$$6 \times 6 \text{ sq. in.} = 36 \text{ sq. in., the area.}$$

Similarly, if we wish to find the area of a rectangle that is 8 in. long and 6 in. wide, we have

$$8 \times 6 \text{ sq. in.} = 48 \text{ sq. in., the area.}$$

In the same way, if we wish to find the area of a parallelogram with a base of 9 in. and a height of $2\frac{1}{2}$ in., we have

$$9 \times 2\tfrac{1}{2} \text{ sq. in.} = 22\tfrac{1}{2} \text{ sq. in., the area.}$$

In finding the area of a triangle, we have this relation:

$$\mathit{Area} = \tfrac{1}{2} \times \mathit{base} \times \mathit{height}.$$

Thus, in finding the area of a triangle with a base of 7 in. and a height (or altitude) of 3 in., we have

$$\tfrac{1}{2} \times 7 \times 3 \text{ sq. in.} = 10\tfrac{1}{2} \text{ sq. in., the area.}$$

In the following review of the common measures we should refer to the tables on pages 419–420, if necessary.

REVIEW DRILL CHART ON THE TABLES OF MEASURES

State rapidly the number of inches in

1. 1 ft. **2.** 6 ft. **3.** 1 yd. **4.** $\frac{1}{2}$ yd. **5.** $\frac{3}{4}$ yd.

State rapidly the number of feet in

6. 24 in. **7.** 1 yd. **8.** 1 rd. **9.** 1 mi. **10.** 7 yd.

State rapidly the number of yards in

11. 3 ft. **12.** 9 ft. **13.** 1 rd. **14.** 36 in. **15.** 18 in.

State rapidly the number of rods in

16. $16\frac{1}{2}$ ft. **17.** $5\frac{1}{2}$ yd. **18.** 11 yd. **19.** 33 ft. **20.** 1 mi.

State rapidly the number of ounces in

21. 1 lb. **22.** 2 lb. **23.** $\frac{1}{2}$ lb. **24.** $\frac{1}{4}$ lb. **25.** $\frac{3}{4}$ lb.

State rapidly the number of pounds in

26. 16 oz. **27.** 32 oz. **28.** 24 oz. **29.** 8 oz. **30.** 1 T.

State rapidly the number of pints in

31. 1 qt. **32.** $\frac{1}{2}$ qt. **33.** $2\frac{1}{2}$ qt. **34.** 1 gal. **35.** $\frac{1}{2}$ gal.

State rapidly the number of quarts in

36. 1 gal. **37.** 6 pt. **38.** 1 pk. **39.** 3 gal. **40.** 1 bu.

State rapidly the number of pecks in

41. 1 bu. **42.** $\frac{1}{2}$ bu. **43.** $1\frac{1}{2}$ bu. **44.** 8 qt. **45.** 32 qt.

State rapidly the number of square rods in

46. 1 A. **47.** $\frac{1}{2}$ A. **48.** $\frac{1}{4}$ A. **49.** $1\frac{1}{2}$ A. **50.** 2 A.

State rapidly the number of square feet in

51. 144 sq. in. **52.** 1 sq. yd. **53.** 5 sq. yd. **54.** $1\frac{1}{2}$ sq. yd.

REVIEW DRILL CHART IN ADDING MEASURES

Perform the following addition rapidly:

1. 2 ft. 7 in.
3 ft. 9 in.

2. 5 ft. 4 in.
1 ft. 8 in.
3 ft. 7 in.

3. 2 lb. 7 oz.
3 lb. 9 oz.

4. 5 lb. 4 oz.
1 lb. 8 oz.
3 lb. 7 oz.

5. 2 yd. 7 in.
3 yd. 9 in.

6. 5 yd. 4 in.
2 yd. 3 in.
1 yd. 8 in.
3 yd. 7 in.

7. 5 ft. 2 in.
3 ft. 8 in.
7 ft. 6 in.
7 in.
3 ft. 5 in.
2 ft. 8 in.

8. 2 qt. 1 pt.
5 qt. 1 pt.

9. 2 qt. 1 pt.
3 qt. $1\frac{1}{2}$ pt.
4 qt. $\frac{3}{4}$ pt.

10. 2 bu. 1 pk.
5 bu. 1 pk.

11. 2 bu. 1 pk.
3 bu. $1\frac{1}{2}$ pk.
4 bu. $\frac{3}{4}$ pk.

12. 2 pk. 1 qt.
5 pk. 1 qt.

13. 2 pk. 1 qt.
3 pk. $2\frac{1}{2}$ qt.
3 pk. $1\frac{1}{2}$ qt.
4 pk. $\frac{3}{4}$ qt.

14. 2 qt. 1 pt.
2 qt. $\frac{1}{2}$ pt.
6 qt. $1\frac{1}{2}$ pt.
5 qt. 1 pt.
$\frac{3}{4}$ pt.
2 qt. $1\frac{1}{4}$ pt.

15. 4 T. 1700 lb.
6 T. 1500 lb.

16. 5 T. 1400 lb.
6 T. 1200 lb.
9 T. 1400 lb.

17. 4 mi. 1700 yd.
6 mi. 1500 yd.

18. 5 mi. 1400 yd.
6 mi. 1200 yd.
9 mi. 1400 yd.

19. 4 mi. 1700 ft.
6 mi. 1500 ft.

20. 5 mi. 1400 ft.
2 mi. 1800 ft.
6 mi. 1200 ft.
9 mi. 1400 ft.

21. 5 T. 800 lb.
1 T. 1200 lb.
1800 lb.
3 T. 1600 lb.
4 T. 1700 lb.
1800 lb.

REVIEW DRILL CHART IN SUBTRACTING MEASURES

Perform the following subtractions rapidly:

1. 7 ft. 2 in.
3 ft. 7 in.

2. 7 yd. 2 in.
3 yd. 7 in.

3. 7 rd. 2 in.
3 rd. 7 in.

4. 7 mi. 2 ft.
3 mi. 7 ft.

5. 7 lb. 2 oz.
3 lb. 7 oz.

6. 7 bu. 2 qt.
3 bu. 7 qt.

7. 7 gal. 2 pt.
3 gal. 7 pt.

8. 8 rd. 2 yd.
5 rd. 3 yd.

9. 6 ft. 1 in.
3 ft. 9 in.

10. 8 ft. 2 in.
7 ft. 8 in.

11. 9 bu. 20 qt.
6 bu. 25 qt.

12. 9 yd. 20 in.
6 yd. 25 in.

13. 9 rd. 20 in.
6 rd. 25 in.

14. 9 mi. 20 rd.
6 mi. 25 rd.

15. 9 mi. 20 ft.
6 mi. 25 ft.

16. 9 T. 20 lb.
6 T. 25 lb.

17. 9 hr. 20 min.
6 hr. 25 min.

18. 9 mo. 20 da.
6 mo. 25 da.

19. 8 yd. 19 in.
28 in.

20. 9 mi. 30 ft.
6 mi. 75 ft.

21. 4 yr. 170 da.
1 yr. 200 da.

22. 4 mi. 170 yd.
1 mi. 200 yd.

23. 4 mi. 170 rd.
1 mi. 200 rd.

24. 4 T. 170 lb.
1 T. 200 lb.

25. 4 T. 200 lb.
1 T. 170 lb.

26. 4 mi. 170 ft.
1 mi. 200 ft.

27. 4 mi. 200 ft.
1 mi. 170 ft.

28. 4 sq. yd. 170 sq. in.
1 sq. yd. 200 sq. in.

29. 3 sq. ft.
1 sq. ft. 100 sq. in.

30. 3 sq. yd.
1 sq. yd. 100 sq. in.

REVIEW DRILL CHART IN MULTIPLYING MEASURES

Perform the following multiplications rapidly:

1. 2 ft. 9 in.
2

2. 2 yd. 9 in.
2

3. 2 rd. 9 in.
2

4. 2 lb. 9 oz.
2

5. 4 ft. 1 in.
20

6. 4 lb. 1 oz.
20

7. 4 gal. 1 qt.
20

8. 4 bu. 1 pk.
20

9. 7 ft. 6 in.
15

10. 8 lb. 2 oz.
24

11. 3 ft. 2 in.
24

12. 3 lb. 2 oz.
24

13. 3 yd. 2 in.
24

14. 3 bu. 2 pk.
24

15. 7 ft. 3 in.
48

16. 7 gal. 3 qt.
48

17. 7 yd. 3 in.
48

18. 7 gal. 3 pt.
48

19. 9 ft. 3 in.
50

20. 7 ft. 9 in.
12

21. 2 mi. 1800 ft.
3

22. 2 T. 1800 lb.
3

23. 5 hr. 48 min.
6

24. 5 mi. 48 rd.
6

25. 5 T. 48 lb.
6

26. 5 min. 48 sec.
6

27. 5 yr. 48 wk.
6

28. 5 wk. 48 hr.
6

29. 5 hr. 48 sec.
6

30. 5 da. 48 min.
6

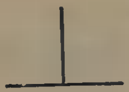
REVIEW DRILL CHART IN DIVIDING MEASURES

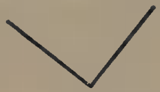
Perform the following divisions rapidly:

1. 18 ft. 6 in. by 2.
2. 18 lb. 6 oz. by 2.
3. 19 ft. 8 in. by 4.
4. 19 lb. 8 oz. by 4.
5. 23 yd. 9 in. by 5.
6. 23 ft. 9 in. by 5.
7. 14 bu. 3 pk. by 6.
8. 14 ft. 3 in. by 6.
9. 14 lb. 3 oz. by 6.
10. 14 yd. 3 in. by 6.
11. 14 gal. 3 qt. by 6.
12. 14 gal. 3 pt. by 6.
13. 23 lb. 2 oz. by 8.
14. 23 yd. 2 ft. by 8.
15. 23 yd. 2 in. by 8.
16. 23 bu. 2 pk. by 8.
17. 23 ft. 2 in. by 8.
18. 23 rd. 2 ft. by 8.
19. 23 rd. 2 in. by 8.
20. 23 gal. 2 qt. by 8.
21. 23 gal. 2 pt. by 8.
22. 23 mi. 2 yd. by 8.
23. 23 mi. 2 rd. by 8.
24. 23 mi. 2 ft. by 8.
25. 8 ft. by 2.
26. 8 ft. by 2 ft.
27. 8 ft. 2 in. by 2.
28. 8 ft. 2 in. by 2 ft.
29. 8 ft. 2 in. by 2 in.
30. 8 lb. 2 oz. by 2 lb.
31. 8 lb. 2 oz. by 2 oz.
32. 8 yd. 2 in. by 2 yd.
33. 8 yd. 2 in. by 2 in.
34. 8 gal. 2 pt. by 2 gal.
35. 8 gal. 2 pt. by 2 pt.
36. 27 lb. 2 oz. by 12 lb.
37. 27 lb. 2 oz. by 12 oz.
38. 27 ft. 2 in. by 12 ft.
39. 27 ft. 2 in. by 12 in.
40. 27 yd. 2 ft. by 12 yd.
41. 27 yd. 2 ft. by 12 ft.
42. 47 yd. 2 ft. by $\frac{1}{2}$ yd.
43. 47 yd. 2 ft. by $\frac{1}{4}$ ft.
44. 47 yd. 2 ft. by $\frac{1}{8}$ in.
45. 50 lb. 8 oz. by $\frac{1}{2}$ lb.
46. 50 lb. 8 oz. by $\frac{1}{2}$ oz.
47. 50 ft. 8 in. by $2\frac{1}{2}$ ft.
48. 50 ft. 8 in. by $2\frac{1}{2}$ in.

Ruler and Compasses. We can draw a straight line by means of a ruler, and a circle by means of compasses. If we do not have a ruler, we can fold a piece of paper and use the folded edge. If we do not have compasses, we can draw the circle around the rim of a coin.

The following figures illustrate *perpendicular lines,* which are straight lines that form *right angles* with each other like any two adjoining sides of a rectangle or of a square:





The easiest way of drawing two perpendicular lines is to draw a right angle, and there are several ways of doing this. A carpenter's square may be used, but those who have much drawing to do use a triangle of celluloid or other stiff substance, one angle of which is a right angle. If you fold a piece of paper, and then fold it again so that one part of the crease lies along the other part, the two creases will form a right angle, and you can use this for drawing perpendicular lines. Try it and see how it works.

DRAWING FIGURES

1. Draw a straight line 1 in. long and at either end draw a line perpendicular to the 1-inch line.

2. From any point draw two straight lines, each 1 in. long, and each perpendicular to the other.

3. Draw a triangle with its base 1 in. long.

4. Draw a square with each side 1 in. long.

5. Draw a rectangle with a base of 2 in. and a height of 1 in.

6. Draw a four-sided figure with its opposite sides as nearly parallel as you can make them.

RECTANGLES, TRIANGLES, AND PARALLELOGRAMS

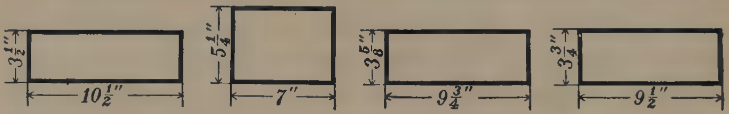
1. Draw a rectangle 3″ long and 2″ high; that is, one which is 3 in. by 2 in. Find the area. Divide the rectangle into square inches and show that the answer you give is correct.

2. A rectangle is $2\frac{1}{2}$″ long and $1\frac{1}{4}$″ wide. Find the perimeter (the distance around the rectangle) and the area.

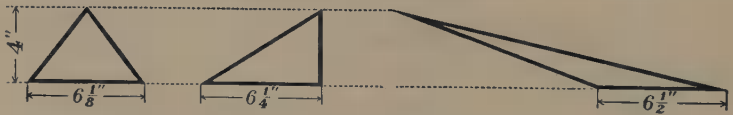
3. A parallelogram has a base of 2″ and a height of $1\frac{1}{2}$″. If possible, find both the perimeter and the area.

4. What do we mean when we say that we "draw a figure to scale"? If we represent 10′ (10 ft.) by 1″, by what should we represent 5′? 1′? 20′?

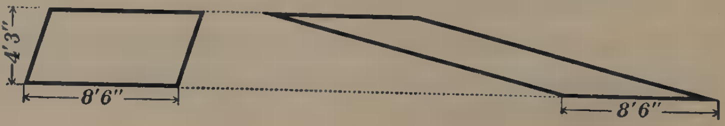
5. Here are four rectangles, drawn to the same scale. Which one has the largest area? Which one has the smallest area?



6. Here are three triangles, drawn to the same scale. Which one has the largest area? Find the area of each triangle.

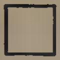


7. Here are two parallelograms, drawn to scale. Which, if either, has the larger area? Find the area of each one.

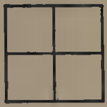


8. If the side of the square A here shown is 1″, what is the area of A? Find the side and the area of B; of C.

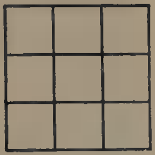
9. If we double the side of a square, what does this do to the perimeter? What does it do to the area?



A



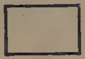
B



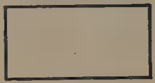
C

10. If we multiply the side of a square by 3, what does this do to the perimeter? to the area?

11. Here are two rectangles, in which the base of X is half that of Y. Does this tell us anything about the perimeter of X as compared with that of Y? the area of X as compared with that of Y? What more must we know in order to compare the perimeters? the areas? How does this case differ from the one discussed in Exs. 8–10?



X



Y

Copy the following statements, and complete each one by numbers or by words, drawing a figure to illustrate each case:

12. If we multiply the side of a square by 4, we multiply the perimeter by —— and the area by ——.

13. If we multiply the base of a rectangle by 2, but leave the height the same, we multiply the area by ——, but of the —— we can only say that it is greater.

14. If we multiply the height of a rectangle by 3, but leave the base the same, we multiply the area by ——, but we can only say that the perimeter is ——.

15. If we multiply both the base and height of a rectangle by 2, we multiply the perimeter by —— and the area by ——.

16. If we multiply both the base and height of a rectangle by 3, we multiply the perimeter by —— and the area by ——.

Copy the following statements and complete each one in the same manner as Exs. 12–16:

17. If we double the perimeter of a square, we multiply each side by —— and the area by ——.

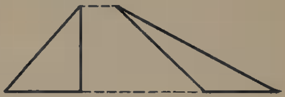
18. If we double the side of an equilateral (equal-sided) triangle, we multiply the perimeter by ——.

19. If we double the base of any triangle, leaving the height unchanged, we multiply the area by ——.

20. If we double the height of any triangle, leaving the base unchanged, we multiply the —— by ——.

21. If we double both the base and the height of any triangle, we multiply the —— of the triangle by ——.

22. If we increase the perimeter of a triangle, without changing either the base or the height, the area of the triangle ——.



23. If we multiply the perimeter of the first triangle in Ex. 22 by 10, and in so doing increase both the base and the height, the area of the triangle ——.

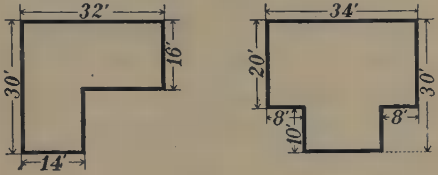
24. If the area of a square is 4 sq. in., the side is —— in. and the perimeter is —— in.

25. If the area of a triangle is 4 sq. in., the base may be 4 in. and the height —— in., or any other two numbers representing inches such that half their —— is ——; for example, the numbers may be —— and ——.

26. If we know only that the area of a rectangle is 4 sq. in., we —— tell what the perimeter is.

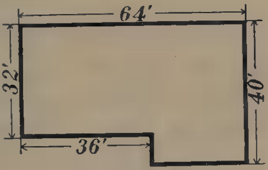
27. A rectangle 4 in. long and 1 in. high has an area equal to that of a square whose side is —— in., but the perimeter of the rectangle is —— than that of the square.

28. This picture shows the outlines of two cellars. Which cellar has the larger floor space, and how much more floor space has it?



29. In Ex. 28, what would be the answer if each of the sides were doubled?

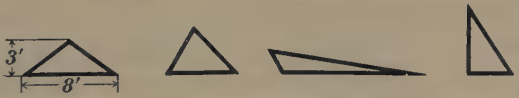
30. Here is the outline of the floor plan of a garage. How many feet is it around the inside of the walls of the garage?



31. In Ex. 30, how many square feet of concrete will be needed to floor the garage? How many square yards of concrete will be needed?

32. If a width of only 32′ can be allowed in building a garage, how long must it be built in order to have the same floor space as the one shown in Ex. 30?

33. These triangles are drawn to the scale of $\frac{1}{16}'' = 1'$.



Measure the figures carefully, find the lengths of the sides of the triangles which they represent, and then find the area of each triangle in square feet. Compare the results.

34. Draw figures to show that increasing the perimeter of a triangle may leave the area unchanged.

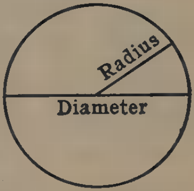
35. Draw figures to show that increasing the perimeter of a square must increase the area.

36. Draw figures to show that increasing the perimeter of a rectangle may decrease the area.

A SILENT READING LESSON

There is one kind of figure that we see very often but have not yet studied; namely, the *circle*. Familiar illustrations of circles are coins, wheels, the cross section of an orange, and similar objects. If we try to imagine what the world today would be without wheels, gears, wires, and all our machines that depend upon the circle, we can see how important the circle is to man.

A circle is a curve every point of which is at the same distance from a point within called the *center*. The distance around a circle is called the *circumference*. A straight line drawn through the center and terminated at each end by the circle is called a *diameter*, and a straight line drawn from the center to any point on the circle is called a *radius*.



A circle may be compared to the tire of a wheel; a circumference is like the length of such a tire; a radius is like a spoke of the wheel; a diameter is like two opposite spokes, and is, of course, twice the radius. The word "circumference" is often used for the line which forms the circle as well as for the length of the line, and the word "circle" is often used for the space inclosed by the curve line.

When we study geometry, we shall learn that a circumference of a circle is about $3\frac{1}{7}$ times the length of the diameter. It is more nearly 3.1416 times this length, but in our work we shall generally use $3\frac{1}{7}$, or $\frac{22}{7}$; that is, briefly stated,

$$\textit{Circumference} = \tfrac{22}{7} \times \textit{diameter}.$$

Thus, if the diameter of a circle is 10 in., the circumference is $3\frac{1}{7} \times 10$ in., or $31\frac{3}{7}$ in. Since in measuring we seldom use fractions like $\frac{3}{7}$ in., such a result means more if given decimally to the nearest 0.1 in.; that is, as 31.4 in.

Finding the Circumference. 1. If a boy's bicycle wheel has a diameter of 21 in., what is the circumference?

Since the circumference is $3\frac{1}{7}$ times the diameter, we have

$$\frac{22}{7} \times 21 \text{ in.} = 66 \text{ in.}, \text{ the circumference.}$$

2. Find the circumference of a circle whose radius is 7 in.

The diameter is 2×7 in., and so we have

$$\frac{22}{7} \times 2 \times 7 \text{ in.} = 44 \text{ in.}, \text{ the circumference.}$$

FINDING THE CIRCUMFERENCE

1. If your bicycle wheel has a diameter of 28 in., what is the circumference?

Find the circumference when the diameter is

2. 7 in.
3. 42 in.
4. 35 in.
5. 63 in.
6. 105 ft.
7. 119 yd.
8. 112 ft.
9. 133 yd.
10. 12 in.
11. 15 ft.
12. 62 in.
13. 51 ft.
14. $3\frac{1}{2}$ in.
15. 4.9 in.
16. 6.3 in.
17. 2.7 in.
18. 3 ft. 6 in.
19. 4 ft. 1 in.
20. 7 ft. 7 in.
21. 8 ft. 2 in.

22. If the radius of a circle is $3\frac{1}{2}$ in., what is the diameter? What is the circumference? If you are given the radius, how do you find the circumference?

Find the circumference when the radius is

23. 7 in.
24. 14 in.
25. 28 in.
26. 42 in.
27. 6 in.
28. 8 in.
29. $10\frac{1}{2}$ in.
30. 4.9 in.
31. 2 ft. 1 in.
32. 10 ft. 6 in.

33. How far will a bicycle wheel which is 2 ft. 3 in. in diameter go in 1 revolution? in 5000 revolutions?

34. If convenient, measure the diameter of a wheel on an automobile, find the circumference, compute how far the car will go while the wheel is revolving once, and then find how many revolutions the wheel makes in going 1 mi.

Finding the Diameter and the Radius. Sometimes it is easier to measure the circumference than the diameter of a circle. For example, a farmer who has a silo with a circular base can easily measure the circumference. From this, as shown in the following examples, he can then find the diameter, which he will need to know if he is computing the capacity:

1. A farmer measures the circumference of a silo and finds that it is 132 ft. What is the length of the diameter?

Since the circumference is $\frac{22}{7}$ times the diameter, we divide the circumference by $\frac{22}{7}$ to find the diameter, and have

$$132 \text{ ft.} \div \frac{22}{7} = \overset{6}{\cancel{132}} \text{ ft.} \times \frac{7}{\cancel{22}} = 42 \text{ ft., the diameter.}$$

2. The distance around a circular flower bed in a park is 286 ft. Find the radius.

We first find the diameter; that is, we first find that

$$286 \text{ ft.} \div \tfrac{22}{7} = 286 \text{ ft.} \times \tfrac{7}{22} = 91 \text{ ft., the diameter.}$$

The radius of the bed is then $\frac{1}{2}$ of 91 ft., or $45\frac{1}{2}$ ft.

FINDING THE DIAMETER AND THE RADIUS

1. The circumference of a round water tank is 75 ft. What is the diameter? What is the radius?

Find the diameter of a circle with a circumference of

2. 22 in.
3. 44 in.
4. 88 in.
5. 110 ft.
6. 154 ft.
7. 198 ft.
8. 30 in.
9. 80 in.
10. 150 ft.
11. 2.2 in.
12. 8.8 in.
13. 13.2 in.
14. 1 ft. 2 in.
15. 2 ft. 1 in.
16. 2 ft. 8 in.

Find the radius of a circle with a circumference of

17. 66 in.
18. 132 ft.
19. 4.4 in.
20. 165 ft.
21. 270 ft.
22. 176 ft.
23. 10 in.
24. 11.55 in.
25. 7 ft. 4 in.
26. 5 ft. 6 in.

MEASURING CIRCLES

1. Frank measures the diameter of the drive wheel of a locomotive and finds it to be 5 ft. 3 in. What is the circumference of the wheel?

2. What radius should be used in laying out a pattern for a pulley which is to have a circumference of 154 in.?

3. If an iron water pipe has a circumference of 4.2 in., what is the diameter?

Find the circumference of a circle with a diameter of

4. 427 ft.
5. 68.2 in.
6. 5.11 ft.
7. 6.37 in.
8. 4.69 ft.
9. 6.02 in.
10. 64.4 in.
11. 10 ft. 6 in.

Find the diameter of a circle with a circumference of

12. 132 ft.
13. 176 in.
14. 68.2 in.
15. 198 in.

Find the circumference of a circle with a radius of

16. 49 in.
17. 91 in.
18. 9.8 in.
19. 154 ft.

Using 3.1416 instead of $3\frac{1}{7}$, and giving the result to the nearest 0.1 in., find the circumference of a circle with a diameter of

20. 17 in.
21. 6.28 in.
22. 3.42 in.
23. 6 ft. 2 in.

24. The outside diameter of an iron water pipe is 4 in., and the iron is $\frac{1}{4}$ in. thick. Using $3\frac{1}{7}$, or $\frac{22}{7}$, as usual unless otherwise directed, find the inside circumference.

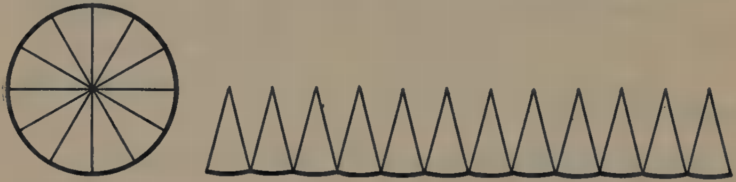
25. The globe in a schoolroom has a diameter of 30 in. How long is the equator on the globe?

26. If there is a globe in your schoolroom, measure the length of the equator and then find the radius of the circle of the equator.

27. If convenient, measure the circumference of the school flagpole or of some tree near the school and find the diameter.

A SILENT READING LESSON

As shown below, a circle can be separated into figures which are nearly triangles. The height of each triangle is the radius of the circle, and the sum of the bases is the circumference.



If they were exact triangles, the *area* of the circle, by which we mean the area inclosed by the circle, would be

$$\tfrac{1}{2} \times \text{radius} \times \text{circumference}.$$

If, later on, we study geometry, we shall prove that this is the true area of a circle. Since the circumference is $\frac{22}{7}$ times the diameter, we may now write "$\frac{22}{7} \times$ diameter" for "circumference" in the above expression, and have

$$\tfrac{1}{2} \times \text{radius} \times \tfrac{22}{7} \times \text{diameter}.$$

Then, writing "$2 \times$ radius" for "diameter," we have

$$\tfrac{1}{2} \times \text{radius} \times \tfrac{22}{7} \times 2 \times \text{radius},$$

or

$$\frac{\text{radius} \times 22 \times 2 \times \text{radius}}{2 \times 7};$$

from which, canceling the common factor 2, we have

$$\tfrac{22}{7} \times \text{radius} \times \text{radius}.$$

If we multiply a number by itself, the result is called the *square* of the number. We may therefore write "square of the radius" for "radius × radius"; that is, in finding the area of a circle, we have the following relation:

Area $= \frac{22}{7} \times$ *square of the radius.*

Finding the Areas of Circles. 1. Find the area of a circle which has a radius of 5 in.

Using the relation given on page 164, we have

$$\tfrac{22}{7} \times 5 \times 5 \text{ sq. in.} = \tfrac{550}{7} \text{ sq. in.} = 78\tfrac{4}{7} \text{ sq. in.}$$

That is, to the nearest 0.1 sq. in., which is a more sensible form for the answer, the area is 78.6 sq. in.

2. Find the area of a circle with a diameter of 8 in.

Since the diameter is 8 in., the radius is 4 in., and we have

$$\tfrac{22}{7} \times 4 \times 4 \text{ sq. in.} = \tfrac{352}{7} \text{ sq. in.} = 50.28 \cdots \text{ sq. in.},$$

the dots (· · ·) indicating that the division by 7 can be carried further. That is, to the nearest 0.1 sq. in. the area is 50.3 sq. in.

FINDING THE AREAS OF CIRCLES

1. In laying out a circular flower bed a man fastens two stakes to a string, 7 ft. apart. He drives one stake into the ground and marks the circle with the other as the string revolves about the first stake. What is the area of the bed?

Find the area of a circle with a radius of

2. 7 in.
3. 14 in.
4. 35 in.
5. $3\frac{1}{2}$ in.
6. 6.3 in.
7. $10\frac{1}{2}$ in.
8. 2 in.
9. 5 in.
10. 9 ft.
11. 7 ft. 7 in.
12. 5 ft. 6 in.
13. 2 yd. 6 in.

14. What is the radius of a circular table-top of which the diameter is 4 ft. 6 in.? What is the area?

Find the area of a circle with a diameter of

15. 42 in.
16. 28 in.
17. 56 in.
18. 14 ft.
19. 5.6 in.
20. 9.8 in.
21. 1 yd.
22. 2 yd.
23. 3 ft.
24. 4 ft. 8 in.
25. 3 ft. 6 in.
26. 10 ft. 6 in.

MEASURING CIRCLES

1. If we draw a circle with a radius half as long as that of a given circle, the area of the circle which we draw is what part of the area of the given circle?

2. If we draw a circle with double the radius of a given circle, the area of the circle which we draw is how many times as large as that of the given circle? Make and solve a problem to illustrate your answer.

3. Find the area of the base of a round water tank which is 42 ft. in diameter.

4. A horse is tethered by a rope fastened to a post in an open field. If the rope is 17 ft. 6 in. long, over approximately how many square feet of ground can the horse graze?

5. A flagpole has a circumference of 24.2 in. at the base. Find the diameter, the radius, and the area of the cross section of the flagpole at the base.

6. Using 3.1416 instead of $3\frac{1}{7}$, what is the area of the cross section of a circular iron shaft the circumference of which is 31.416 in.?

7. The outside diameter of an iron water pipe is 3 ft. 6 in. If the iron is 1 in. thick, how many square inches of iron are there in the cross section of the pipe?

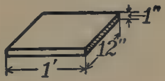
8. If Texas has 4,660,000 inhabitants, and 25% of them live within 100 mi. of Dallas, what, to the nearest unit, is the average population per square mile in this circle?

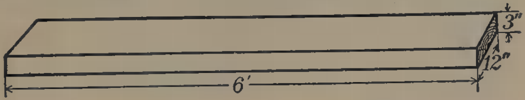
9. If the water pressure is 125 lb. to the square inch, what is the total pressure exerted on the end of a closed water pipe which has an inside diameter of 3.15 in.?

10. Measure the circumference of a baseball. From this find the diameter, the radius, and the cross-section area.

Measuring Lumber. In measuring lumber the unit used is the *board foot* (bd. ft.), which is a piece that is 1 sq. ft. on one surface and 1″ or less in thickness.

Thus, a board 2′ long, 1′ wide, and 1″ or less thick contains $2 \times 1 \times 1$ bd. ft., or 2 bd. ft., and one that is 12′ long, 8″ wide, and 1″ or less thick contains $12 \times \frac{8}{12} \times 1$ bd. ft., or 8 bd. ft.





The figure at the left shows a piece of board that contains exactly 1 bd. ft., while the plank shown at the right contains $6 \times \frac{12}{12} \times 3$ bd. ft., or 18 bd. ft.

In finding the number of feet, board measure (B.M.), in a plank that is $12' \times 6'' \times 2\frac{1}{2}''$, we may arrange the work as shown at the right, cancel, and find that the plank contains 15 bd. ft.

$$\boxed{12 \times \frac{1}{2} \times 2\frac{1}{2} = \frac{\overset{3}{\cancel{12}} \times 5}{\cancel{2} \times \cancel{2}} = 15}$$

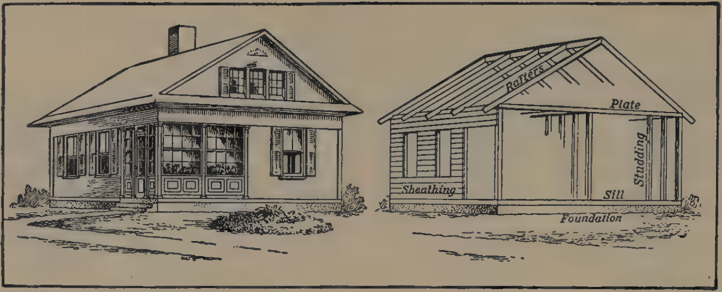
In billing lumber, a fraction of a board foot is always counted as equal to 1 bd. ft.

In finding the number of feet B.M. in several pieces we have the following convenient relation:

Number of feet B.M. = number of pieces × length in feet × width in feet × thickness in inches.

This relation is particularly convenient in finding the cost of an order for several pieces of lumber. For example, in finding the cost of 18 pieces, 16′ long, 4″ wide, and 3″ thick, at \$50 per M (that is, per 1000 bd. ft.), we have

$$18 \times 16 \times \frac{\cancel{4}}{\underset{4}{\cancel{12}}} \times \cancel{3} \times \frac{\$5\cancel{0}}{100\cancel{0}} = \$14.40, \text{ the cost.}$$



Building a House. In building the house shown in the picture above, a builder estimates the cost of materials and labor as stated in the following problems. The results of some of the computations that have to be made are given; the rest is left for the pupils to complete.

The right-hand figure in the picture shows the meaning of the terms used in the problems.

BUILDING A HOUSE

1. For sills and plates the builder needs the pieces indicated below. Fill in the parentheses and find the total number of feet B.M. required for the sills and plates:

3 pieces, 6″ × 8″, 16′ long	192 bd. ft.
4 pieces, 6″ × 8″, 14′ long	(?) bd. ft.
6 pieces, 6″ × 8″, 12′ long	(?) bd. ft.
4 pieces, 4″ × 6″, 14′ long	(?) bd. ft.
6 pieces, 4″ × 6″, 18′ long	(?) bd. ft.

2. Find the number of feet B.M. required for the floor timbers and rafters, for which the builder needs

30 pieces, 2″ × 10″, 16′ long
18 pieces, 2″ × 8″, 12′ long
13 pieces, 2″ × 8″, 6′ long
27 pieces, 2″ × 8″, 16′ long
27 pieces, 2″ × 8″, 12′ long
27 pieces, 2″ × 6″, 16′ long

3. Find the number of feet B.M. required for the studding, for which the builder needs

5 pieces, 4″ × 6″, 14′ long
45 pieces, 3″ × 4″, 12′ long
40 pieces, 3″ × 4″, 18′ long
100 pieces, 2″ × 4″, 14′ long
35 pieces, 2″ × 4″, 10′ long
25 pieces, 2″ × 4″, 18′ long
50 pieces, 2″ × 4″, 8′ long
50 pieces, 2″ × 4″, 14′ long

4. For the roof the builder needs 1050 bd. ft. of roofing boards. Find the total amount of lumber needed in Exs. 1–4. How much will this lumber cost the builder at an average price of $68 per M?

5. The builder needs also 2000 bd. ft. of sheathing at $60 per M; 1950 bd. ft. of flooring at $85 per M; 325 bd. ft. of porch flooring at $75 per M; 2500 clapboards at $14.25 per M; 7000 shingles at $12 per M. What will this cost?

6. He needs also gutters, cornice, corner boards, etc., $75; porch columns, $14; 16 doors at an average cost of $9.75 each; 16 windows at an average cost of $8.50 each; hardware, $115; chimney and plastering, $475; painting, $320; tin work, $64.50; labor, 6 workmen for 32 da. at $6 a day each; excavating and foundation, $300; grading, $65. What is the total cost of these items?

7. He must pay $485 for the plumbing and $645 for the heating plant. Taking all the material and labor as found in Exs. 4–7, what is the total cost of the house?

Find the number of feet B.M. in each of the following cases:

8. 32 planks, 3 in. thick, 8 in. wide, and 16 ft. long.

9. 36 joists, 3 in. by 4 in., and 16 ft. long.

10. 12 beams, 10 in. by 12 in., and 14 ft. long.

11. 120 boards, 6 in. wide, $\frac{3}{4}$ in. thick, and 16 ft. long.

12. 48 timbers, 6 in. by 8 in., and 18 ft. long.

PLASTERING AND PAPERING

1. Allowing 14 sq. yd. for openings, how many square yards of plastering are needed for the walls and ceiling of a room 21 ft. long, 15 ft. wide, and 9 ft. high?

2. How much will it cost, at 60¢ a square yard, to plaster a schoolroom $34\frac{1}{2}$ ft. long, 18 ft. wide, and 12 ft. high, making an allowance of 30% of the total surface to be plastered for blackboards, baseboard, ventilators, doors, and windows?

3. The walls of a room are 8 ft. 6 in. high above the baseboard, and the total length of the four walls, allowing for openings, is 72 ft. Allowing 6 in. on each strip for matching, how many rolls of paper, 18 in. wide and containing 16 yd. of paper, will be needed? Find the cost of the paper at 65¢ a roll.

4. Deducting 2 rolls for openings, how many rolls of 24-inch plain paper, 16 yd. long, will it take to cover the four walls of a room 25 ft. long, 20 ft. wide, and 12 ft. high? If, in buying the paper, parts of rolls cannot be bought, what will be the cost of the paper at 85¢ a roll?

5. The walls of a room are to be papered with plain paper costing 60¢ a roll. The walls are 9 ft. high above the baseboard, and their total length after deducting for windows and doors is 60 ft. Allowing $1\frac{1}{2}$ rolls to 100 sq. ft., what will be the cost of the paper?

6. Measure the length, width, and height of your schoolroom. Find the area of the ceiling in square yards, as you would have to do in estimating the cost of plastering.

7. Either by measuring or estimating the sizes of the openings or other unplastered parts of the walls of your schoolroom, find the approximate number of square yards of plastering in the walls, using the measurements which you made in Ex. 6.

WALLS AND FLOORS OF A HOUSE

1. A bathroom 9 ft. long and 6 ft. wide has a tile floor. The lower part of the walls also is tiled to a height of 3 ft. Deducting 12 sq. ft. for door and fixture openings, find the number of square feet of tiling used.

2. The rest of the walls and the ceiling of the room in Ex. 1, which is 9 ft. high, are plastered and painted. Find the number of square yards of painted surface, deducting 3 sq. yd. as the allowance for openings.

3. The walls of a cellar $24\frac{1}{2}$ ft. by $37\frac{1}{4}$ ft. by 8 ft. are to be whitewashed. Allowing 3 sq. yd. for door and window openings, find the number of square yards to be covered.

4. At $2.75 per square yard, including the cost of laying, find the cost of covering a kitchen floor, 15 ft. long and 12 ft. wide, with linoleum.

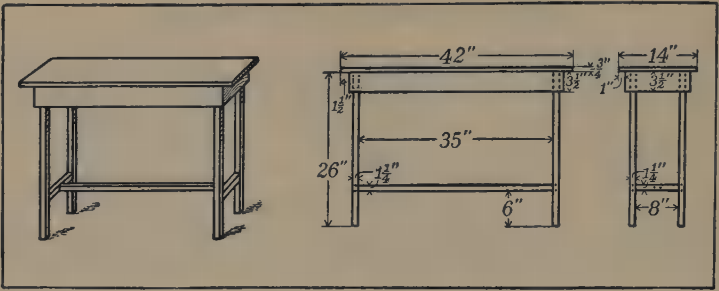
5. A rug 12 ft. square, placed in a dining room that is 16 ft. long and 14 ft. wide, will cover what per cent of the floor space?

6. How many strips of plain carpeting $\frac{3}{4}$ yd. wide are needed for a room 18 ft. wide?

7. A carpet has an 18-inch pattern. Is there any loss in matching the strips if they are 15 ft. long? if they are 14 ft. long? Illustrate each answer by a plan drawn to scale.

8. In allowing for waste in matching carpet patterns no allowance need be made on the first strip, since the other strips are cut to match that one. If a 27-inch carpet with a $\frac{1}{2}$-yard pattern is to be laid lengthwise of a room 17 ft. 6 in. long and 14 ft. wide, how much is wasted in matching on each strip after the first? Find the amount of carpet needed.

9. Would it be more economical if the strips in Ex. 8 ran across the room? If so, how many yards might be saved?



WORKING DRAWINGS

1. The above plan shows that a $\frac{3}{4}$-inch board, 14″ wide and 42″ long, is needed for the top of the stand. Not allowing for waste in cutting, how many board feet are needed?

2. Pieces $1\frac{1}{4}$″ square, all sawed out, can be bought for the legs. How long a piece is needed for each leg? Allowing 6″ for waste in cutting, how long a piece of this square stock is needed for the four legs?

3. The $1\frac{1}{4}$-inch square stock is also used for the braces at the bottom. Allowing another 6″ for waste in cutting, how many running feet of this stock are needed for the stand?

4. The strip around the top of the legs is $\frac{3}{4}$″ thick and $3\frac{1}{2}$″ wide. The side pieces are 3″ shorter than the top of the stand, and the end pieces are 2″ shorter than the width of the top. How many board feet are needed for this strip?

5. In making a tool cabinet, a boy uses a $\frac{3}{4}$-inch board 10″ wide. He cuts off 3 pieces 14″ long, 2 pieces 18″ long, and 5 pieces 20″ long. Allowing 6″ for waste in cutting, how many board feet did he use?

6. At $95 per M, what is the cost of the lumber that the boy uses, as described in Ex. 5?

Longitude and Time. In geography we learn about the longitude of places, and find that longitude is reckoned in degrees (°), minutes (′), and seconds (″) from the prime meridian, which passes through Greenwich, England.

Correspondence of Longitude to Time. Since the earth makes a complete revolution about its axis once in every 24 hr., the place in which we live will pass through 360° between now and this time tomorrow. That is, 360° of longitude corresponds to 24 hr., and so

1° of longitude corresponds to $\frac{1}{360}$ of 24 hr., or $\frac{1}{15}$ hr., or 4 min.;

1′ of longitude corresponds to $\frac{1}{60}$ of 4 min., or $\frac{1}{15}$ min., or 4 sec.;

1″ of longitude corresponds to $\frac{1}{60}$ of 4 sec., or $\frac{1}{15}$ sec.

In studying these relations, which belong partly to geography, it will help us to use the globe.

Earlier West, Later East. Because the earth revolves 360° in 24 hr., the sun appears to move 15° an hour from east to west. Therefore, when it is noon here it is an hour later, or 1 P.M., 15° east of here; an hour earlier, or 11 A.M., 15° west of here; and 6 hr. earlier, or 6 A.M., 90° west of here.

LONGITUDE AND TIME

All work oral

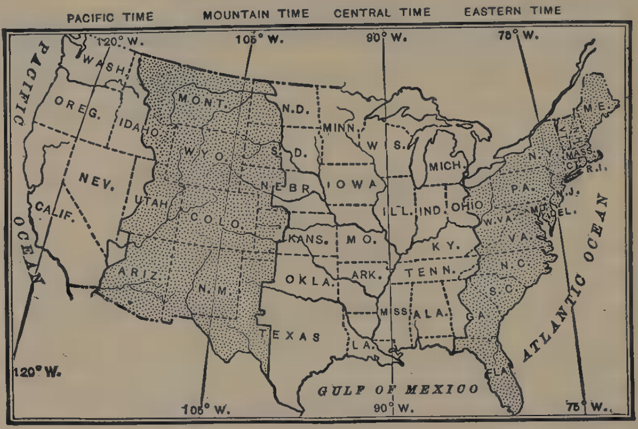
1. When it is 9 A.M. here, what time is it 15° east of here?

2. When it is noon here, what time is it directly north of here? 30° east of here? 90° west of here? 150° east of here?

3. What difference in time corresponds to a difference in longitude of $7\frac{1}{2}$°? 45°? 60°? 105°? 120°? 165°? 180°?

4. Since 1′ of longitude corresponds to 4 sec. of time, to what does 7′ correspond? 11′? 15′? 30′? 45′? 60′? 120′?

Standard Time. The map given below shows how the United States has been divided into four sections in order to regulate the time. Every place in each section uses *standard time,* which is the time of a certain meridian in that section.



In going from one section to the next, the time changes 1 hr. Thus, when it is noon by Eastern time (75° W.), it is 11 A.M. by Central time (90° W.), 10 A.M. by Mountain time (105° W.), 9 A.M. by Pacific time (120° W.), and 5 P.M. by Greenwich time (0°).

STANDARD TIME

1. What time is it now in your school? in California?

2. When it is 10 A.M. in Illinois, what time is it in Maine?

3. When it is 10 A.M. in New York, what time is it in Dallas? in Memphis? in Denver? in New Orleans?

4. When it is 9 A.M. in Seattle, what time is it in Savannah?

5. When it is noon in Maryland, what time is it in Wyoming?

6. When it is 9 A.M. in Albany, what time is it in Nevada?

Difference in Time. The relation of time to longitude is more apparent when we consider two places that are far apart. For example, if we wish to know the difference in local (not standard) time on two ships which are 65° 7′ 30″ of longitude apart, we may proceed as follows:

LONGITUDE	TIME		
65°	$65 \times \frac{1}{15}$ hr.	$= 4\frac{1}{3}$ hr.	= 4 hr. 20 min.
7′	$7 \times \frac{1}{15}$ min.	$= \frac{7}{15}$ min. =	28 sec.
30″	$30 \times \frac{1}{15}$ sec. =		2 sec.
	Difference in local time =		4 hr. 20 min. 30 sec.

DIFFERENCE IN TIME

1. The figures of the answer in the case above can be obtained by this division. Do you see why we divide by 15? Write the reason.

$$\boxed{\begin{array}{r} 4\ \ 20\ \ 30 \\ 15\overline{)65\ \ \ 7\ \ 30} \end{array}}$$

2. A difference of 27° 4′ 15″ in longitude corresponds to what difference in local time?

3. If two ships are 75° 30′ 30″ of longitude apart, what is the difference in local time?

4. The local time of Rome is that of 15° E. longitude, and that of Chicago is the time of 90° W. longitude. When it is 2 P.M. in Chicago, what time is it in Rome?

5. The local time of San Francisco is that of 120° W. longitude. When it is noon in London (on the prime meridian), what time is it in San Francisco?

6. At 8 30 A.M., local time, a ship at 68° 10′ 30″ W. longitude sends a wireless message that is picked up by a lightship at 70° W. longitude. At what hour, local time, is it received at the lightship? At what hour (local time, 75° W. longitude) is the message received in New York?

Difference in Longitude. Since 24 hr. of time corresponds to 360° of longitude, we have the following relations:

1 hr. corresponds to $\frac{1}{24}$ of 360°, or 15°, of longitude;
1 min. corresponds to $\frac{1}{60}$ of 15°, or $\frac{1}{4}$°, or 15′, of longitude;
1 sec. corresponds to $\frac{1}{60}$ of 15′, or $\frac{1}{4}$′, or 15″, of longitude.

Thus, in finding the difference in longitude between two ships on which the local time differs by 3 hr. 7 min. 3 sec., we may proceed as follows:

TIME	LONGITUDE	
3 hr.	3 × 15° =	45°
7 min.	7 × 15′ = 105′ =	1° 45′
3 sec.	3 × 15″ =	45″
	Difference in longitude =	46° 45′ 45″

DIFFERENCE IN LONGITUDE

1. The figures of the answer in the case above can be obtained by this multiplication. Do you see why in this solution we may multiply by 15? Write a statement of the reason involved.

$$\boxed{\begin{array}{rrr} 3 & 7 & 3 \\ & & 15 \\ \hline 46 & 45 & 45 \end{array}}$$

2. If the difference in local time between two steamships is 2 hr. 3 min. 10 sec., what is the difference in longitude?

3. When it is noon in Albany, it is 5 P.M. at Greenwich (0°). From what longitude does Albany take its time?

4. When it is noon at Greenwich, it is 6 A.M. in Iowa. From what longitude does Iowa take its time?

5. At 10 10 A.M., local time, a steamer at 26° 30′ W. longitude sends a wireless message to another steamer. If the message is received at 10 25 A.M., local time, what is the longitude of the second steamer?

IV. REVIEW AND DRILL

MINIMUM ESSENTIALS

1. Write the number of next year in Roman numerals.

2. Write the largest whole number that you can write in the usual way with six figures.

Copy the following examples and supply the missing numbers indicated by the stars:

3. 7.96	**4.** 9.96	**5.** $5.53	**6.** $4.78	**7.** 4 ft. 8 in.
3.75	7.28	***	3.96	3 ft. 9 in.
***	.63	2.46	.28	2 ft. 10 in.
8.82	4.07	5.90	5.00	10 in.
.77	2.70	2.75	2.75	7 ft. 3 in.
25.00	***	$25.00	***	***

8. From $3\frac{5}{8} + 6\frac{3}{4}$ subtract $2\frac{5}{32} + 1\frac{3}{16}$.

9. Multiply 9.46 by 0.38 and 17.245 by 0.07, giving each result to the nearest 0.01.

10. Divide 42.634 by 0.93 and 67.855 by 0.017, giving each result to the nearest 0.01.

11. Find the area of a circle having a diameter of 1.4 in.

12. The number 8.25 is 3% of what number? 30% of what number? 0.3% of what number?

13. At a discount of $12\frac{1}{2}\%$, what is the amount of the discount on an order amounting to $49.60 at list prices? What is the net amount of the order?

14. What is the interest on $275 for 9 mo. 15 da. at 6%?

15. On what sum is $14.40 the interest for 1 yr. at 5%?

16. Which is larger, the interest on $1500 for 6 mo. at $6\frac{1}{2}\%$ or that on $1498 for 6 mo. at 6%? How much larger is it?

PROBLEMS WITHOUT NUMBERS

All work oral

1. How do you find a per cent of a number?

2. If you know what a certain per cent of a number is, how do you find the number?

3. If you have two numbers, how do you find what per cent the first is of the second? what per cent the second is of the first? what per cent the first is of the sum of the numbers?

4. If you wish to borrow a certain sum of money, and can do so for a certain length of time at a certain rate of interest, how do you find the interest?

5. If you can buy a house for a certain cash payment and a certain number of equal payments at definite intervals, with interest on the unpaid balance at a certain rate, how do you find how much the house has cost you in all after you make the last payment?

6. If you know the result of adding two numbers, and one of the numbers, how do you find the other number?

7. Consider Ex. 6, supposing that you know the result of subtracting one number from another instead of the result of adding them. Does it make any difference whether the given number is the larger one or the smaller one?

8. Consider Ex. 6, supposing that you are given the result of multiplying one number by another.

9. Consider Ex. 6 for the result of dividing one number by another. Are there two cases, as in Ex. 7?

10. If you borrow money for a certain time at a certain rate, how will the interest compare with the interest on the same sum for half the time at double the rate? with the interest on double the sum for half the time at half the rate?

PROBLEMS FOR COMPLETION

1. Mr. Russell has borrowed $250 at 6% for 60 da. Complete the problem in any reasonable way and solve it.

Complete and solve the following problems:

2. Mr. Lodge has borrowed $500 for 8 mo. at 6%. After 2 mo. he borrows $250 more for 6 mo. at 5%.

3. At a sale Edward's father bought a post-card projector for showing colored post cards on a screen. The regular price was $8.50, but it was reduced 20% for the sale.

4. William has a small electric motor. It cost him $4.76 at a sale at which a discount of 15% was given.

5. The Stars played 16 games and lost a quarter of them.

6. Last season the Hillside team lost 7 games, which were $33\frac{1}{3}$% of the games on its schedule.

7. The Green Sox lost $33\frac{1}{3}$% of their games last season, but they won or tied 22 games.

8. Mr. Redfield wishes to borrow $7500 for 90 da. His bank will discount his note at $5\frac{1}{2}$%.

9. The diameter of the wheels on our automobile is 32 in. It is 1 mi. from our house to the post office.

10. Tom earns 75¢ every Saturday that he works. He has worked long enough to earn $8.25.

11. At 4 P.M. I sent a telegram from New York to Chicago. It was delivered 45 min. later. There is a difference of 1 hr. in time between New York and Chicago.

12. A cowboy outfit is marked $3.20; a small moving-picture machine, $12.30; and a small phonograph, $11.75. At a sale each of these articles is marked down 20%. A boy has $27 in the bank.

FOOD VALUES

1. In planning properly balanced meals we often estimate food values in *calories*; that is, in terms of their heat value to the body. A calorie is the amount of heat needed to raise the temperature of 1 pt. of water about 4°. If children of your age need 30 calories a day for each pound of weight, how many calories do you need?

2. If 1 lb. of dried beef yields 790 calories, and you have bought 25¢ worth of dried beef costing 50¢ a pound, how many calories have you bought?

3. If 1 lb. of granulated sugar yields 1750 calories, what part of an ounce do you need to eat so as to get 50 calories?

4. If there are 1620 calories to 1 lb. of rice, how many calories do you get when you eat 5¢ worth of rice at 15¢ a pound?

5. If there are 1250 calories in a loaf of bread costing 12¢, how much do 250 calories cost at this rate?

6. If there are 3410 calories to 1 lb. of butter, how many calories do you get from $\frac{1}{2}$ oz. of butter?

THE COST OF WASTEFULNESS

1. If a farm wagon that cost $90 is left out in the yard instead of being kept in the shed, it will last about 6 yr. If kept under cover, it will last about twice as long. How much does a farmer pay for his carelessness per year if he neglects to put his wagon under cover?

2. If farm machinery depreciates (loses value) about $8\frac{1}{2}\%$ a year, what is the annual loss on a machine costing $145? If due to carelessness the rate is 15%, what is the annual loss?

3. Each of two farmers bought a two-horse cultivator, and each paid $45. One of them took good care of his cultivator and made it last 15 yr. The other left his cultivator out during all kinds of weather and had to buy a new one at the end of every 5 yr. Not considering interest, how much did the careless farmer lose in the 15 yr.?

4. After threshing his wheat, a farmer had 16 T. of straw left. A ton of this straw contains 12 lb. of nitrogen, worth 30¢ a pound, and 10 lb. of potash and 2 lb. of phosphoric acid, each worth 5¢ a pound. If the farmer wastes the straw instead of using it as fertilizer, how much money does he waste?

HAULING BY MOTOR TRUCK

1. In a recent year the cost of a one-ton motor truck was \$1050 for the chassis and \$125 for the body. If the freight amounted to \$25, what was the total cost?

2. For the first year, the different items of upkeep of the motor truck in Ex. 1 were as follows: repairs, \$75; tires, \$480; insurance, \$61.05; depreciation due to wear, 10% of the original cost of the truck. Find the total upkeep.

3. The truck ran 14,400 mi. the first year, averaging 9 mi. to a gallon of gasoline. Find the total cost of the gasoline for the year at an average price of 22¢ a gallon.

4. The motor truck averaged 1 qt. of lubricating oil for each 45 mi. that it ran. Find the total cost of the oil for the first year at 75¢ a gallon.

5. The total cost of operating the truck for the first year was \$2957, including wages. Find to the nearest cent the average cost per mile.

6. In a recent year the cost of a two-ton motor truck was \$1875 for the chassis and \$150 for the body. If the freight amounted to \$40, what was the total cost?

7. In Ex. 6 the yearly depreciation due to wear was estimated at \$225. Find to the nearest 0.1 the per cent of the original cost charged as depreciation.

8. During a recent year 158,400 hogs were delivered to the Omaha market by motor truck. Allowing 80 hogs to a freight car, how many freight cars would it take to transport all these hogs to market?

9. In a recent year 138,000 cattle were delivered to the Cincinnati stockyards by motor truck. Allowing 30 cattle to a car, how many freight cars would it take to transport these cattle?

BUILDING A BARN

1. A farmer employed a carpenter to build a barn. As part of the materials needed, the carpenter gave the following order for lumber: 12 pieces 8″ × 8″ × 18′; 8 pieces 4″ × 6″ × 10′; 8 pieces 2″ × 8″ × 24′; 36 pieces 2″ × 10″ × 14′; 60 pieces 2″ × 8″ × 22′; 12 pieces 2″ × 8″ × 20′; 10 pieces 2″ × 8″ × 18′; 30 pieces 2″ × 8″ × 16′; 220 pieces 2″ × 8″ × 12′; 20 pieces 2″ × 4″ × 10′; 40 pieces 2″ × 4″ × 8′; 22 pieces 4″ × 4″ × 24′; 8 pieces 4″ × 4″ × 12′; 5000 bd. ft. of matched siding; 22,000 shingles. Find the number of feet of lumber B.M. ordered, its cost at an average price of $68 per M, and the cost of the shingles at $9.10 per M.

2. The carpenter figured on needing also 100 lb. of small nails, 140 lb. of board nails, and 650 lb. of spikes of various sizes. At an average price of 7¢ per pound, what was the cost of the nails and spikes that he needed?

3. The carpenter bought ten ready-made windows at $5.60 each, and allowed $50 for building the foundation of the barn. He bought also the necessary hardware, such as door hinges, latches, hangers, tracks for the large doors, and window fasteners, the bill amounting to $32.75. What was the cost of all these items?

4. The carpenter figured his work at $7.50 a day and the work of another man at $5.25 a day, both for a period of 37 da. How much was allowed for labor?

5. The farmer decided to put a concrete floor in the barn. He found that he would have to cover a space 58 ft. long and 36 ft. wide. At $3.60 per square yard, what was the cost of laying the floor?

6. The farmer also paid $185 for painting the barn. Using the results of Exs. 1–5, find the total cost of the barn.

V. LITTLE EXAMINATIONS

I. 1. $33\frac{1}{3}\% = \frac{\quad}{100} = \frac{\quad}{3}$.

2. $1\frac{2}{3} = 1 + \frac{\quad}{100} = (?)\%$.

3. $0.62\frac{1}{2} = \frac{\quad}{100} = (?)\%$.

4. $330\% = 3 + \frac{\quad}{10} = \frac{\quad}{10}$.

5. $125\% = 1 + \frac{\quad}{100} = 1\frac{1}{\quad}$.

6. 40% ÷ 20%.

7. $33\frac{1}{3}\%$ of $75.

8. $87\frac{1}{2}\%$ of $240.

9. 125% of $400.

10. 3.6% of $1260.

II. 1. Find $4\frac{1}{2}\%$ of $50, 9% of $25, and $2\frac{1}{4}\%$ of $100.

2. At 15%, what is the discount on a bill for $275?

3. If the discount on $85 is $4.25, what is the rate?

4. Find the circumference when the radius is 8 in.

5. In Ex. 4 find the area of the circle.

III. 1. Doubling the diameter does what to the area?

2. Find the diameter when the circumference is 12 in.

3. In Ex. 2, find the radius and the area.

4. Find the area of a rectangle 18 in. by 2 ft. 6 in.

5. An acre is what per cent of a square mile?

IV. 1. Find the interest on $1250 for 90 da. at 6%.

2. On what sum is $17.25 the interest for 30 da. at 6%?

3. Find the interest on $625 for 2 mo. 10 da. at 5%.

4. At $5\frac{1}{2}\%$, find the discount on a 90-day note for $450.

5. Find the interest on $1850 for 2 yr. 3 mo. at $4\frac{1}{2}\%$.

V. 1. Write a check for $5; for $10.37.

2. Write a promissory note for $150 due in 60 da.

3. Write a deposit slip of not more than five items.

4. Make up and solve a problem in square measure.

5. Make up and solve a problem in board measure.

CHAPTER III

I. REVIEW AND EFFICIENCY TESTS

A SILENT READING LESSON

This is our last year in the elementary school. The most important part of arithmetic is the work in addition, subtraction, multiplication, and division of whole numbers, common fractions, and decimals, including per cents. If we can do this work, we can usually solve any ordinary problem.

Why do people so often fail to solve correctly the ordinary problems that arise in daily life? *Because they cannot add, subtract, multiply, and divide correctly.*

Why do business men complain that the boys and girls they hire do not know arithmetic? *Because these boys and girls cannot add, subtract, multiply, and divide correctly.*

Why do so many people have trouble in doing simple examples like those which require finding the value of $3\frac{1}{2} \times 48¢$ or of $2\frac{7}{8} \times \$1.60$? *Because they did not learn these things thoroughly when they were in school.*

Do we wish to be counted among the failures when we begin our life's work? If not, it will be a good thing for us to begin this year with a thorough review of that part of arithmetic which we are most likely to need in everyday life. By so doing we can increase not only the speed at which we work but also the accuracy, because work that is done with a reasonable degree of rapidity is usually the most accurate. We should, however, never fail to check a computation to see that the result which we have obtained is correct.

ORAL SPEED TEST CHART IN ADDITION

Allowing 30 sec. to a row, add the following rows from left to right, and check by adding from right to left:

1. 3 8 4 9 5 6 3 6 8 7 3 7 5 8 4
2. 7 5 3 4 7 8 6 5 3 9 2 9 7 7 4
3. 5 6 4 5 7 8 2 6 9 7 6 5 6 4 5
4. 6 5 6 7 4 5 8 7 5 8 4 2 6 6 5
5. 4 6 5 6 5 6 9 6 6 7 5 1 7 5 6
6. 2 7 5 8 6 5 4 5 9 6 4 6 6 7 5
7. 1 4 4 3 8 7 7 4 4 8 3 8 8 6 5
8. 0 3 5 2 9 6 8 3 5 7 4 7 9 5 6
9. 8 2 6 1 9 6 9 2 6 6 5 8 8 4 7
10. 9 5 6 5 6 5 9 6 7 6 6 0 8 4 7

Allowing 15 sec. to a column, add these columns upward, and check by adding downward:

11.	**12.**	**13.**	**14.**	**15.**	**16.**	**17.**	**18.**
2	7	5	3	9	8	4	6
6	2	6	8	7	8	7	6
8	8	4	5	8	9	3	2
7	3	7	6	6	5	8	7
3	7	3	4	3	4	9	8
4	2	8	9	4	3	6	7
9	6	4	3	2	3	8	7
5	9	6	8	6	5	2	1
8	8	9	7	4	5	8	9
2	2	1	3	4	5	9	6
3	6	5	9	2	7	0	3
6	7	4	6	3	2	1	0

SELF-CHECKING ADDITION

1. Copy the following table; add each horizontal line and write the result in the right-hand column; add each column and write the result below it; then add the sums of the lines and also of the columns. The last two results should be the same, and the number should be written in the space marked *:

4	3	4	2	8	6	9	4	8	3	8	5	
3	9	7	3	2	1	7	2	6	5	2	4	
7	2	2	9	7	4	5	9	4	7	9	3	
6	8	8	6	5	2	6	8	2	9	7	2	
4	6	3	4	3	5	3	1	7	6	4	6	
3	7	2	1	8	6	2	9	5	4	3	8	
												*

2. A traveling man's expense account for a week is given in the table below. Perform the additions as directed in Ex. 1:

DATE	FARES		BOARD		TELEPHONE		POSTAGE		MISCELLANEOUS		TOTAL	
1	2	20	3	90		20				55		
2	1	40	3	85		30		12		60		
3		60	4	20		75		08	1	10		
4	1	95	4	10	1	20		16		75		
5	2	60	3	95		45		24		45		
6	1	46	4	25		25		12	1	00		
7	2	32	4	35		20		18		75		
TOTAL												

ORAL DRILL CHART IN RAPID ADDITION

1. Add 39 and 26 as a business man would; that is, think of 39 + 26 as 39 + 20 and 59 + 6.

Do these additions by the method suggested in Ex. 1:

2.	22	22	22	22	22	42	65	47
	10	16	18	28	29	39	38	18
3.	27	34	45	55	78	58	47	68
	36	27	35	26	12	14	27	53
4.	38	68	27	47	57	68	78	85
	18	18	19	19	29	12	13	68
5.	90	95	95	96	98	57	67	72
	10	10	15	15	64	13	13	37
6.	85	85	86	88	64	89	99	33
	20	25	25	37	36	42	56	81
7.	58	58	58	58	65	65	67	83
	50	52	56	57	40	45	48	68
8.	77	77	78	78	78	78	78	92
	20	23	30	32	35	56	68	61
9.	64	64	64	74	89	89	89	75
	30	49	39	59	40	44	74	39
10.	37	37	37	47	47	67	78	97
	66	50	56	66	69	79	56	99
11.	85	92	32	86	27	38	94	68
	27	38	59	44	59	46	87	39

REVIEW DRILL CHART IN COLUMN ADDITION

Copy the following, add, and check each result:

1. $2.75
1.46
3.90
.75
1.00

3. $4.80
2.69
1.47
3.29
4.16

5. $5.00
2.70
.69
3.82
4.01

7. $2.88
1.49
3.76
4.05
3.71

9. $1.47
2.93
4.12
.76
1.47

2. $3.27
.42
1.27
2.68
.39
4.00
7.68
1.29
3.00

4. $5.75
2.86
.92
3.00
4.05
.75
2.10
3.76
4.24

6. $5.80
2.60
3.07
4.26
.39
.57
2.61
1.75
6.00

8. $7.28
2.96
.39
8.72
.48
6.75
3.29
4.36
9.64

10. $5.96
6.82
3.48
7.56
8.29
3.07
6.40
3.36
3.00

Copy the following, add, check each result, and write the number of minutes taken for all the columns:

11. $2864.75
3904.08
628.37
5072.99
832.87
6271.03
876.53
273.06
81.27
4269.34

12. $5873.96
2981.44
3837.62
897.68
3278.48
687.49
853.61
3400.07
3.25
289.63

13. 5738.2
698.3
4000.0
783.6
3408.8
629.3
4426.3
3900.0
15.6
862.3

14. 473.82
698.87
438.37
507.96
296.81
300.09
41.76
391.63
.72
512.19

ORAL DRILL CHART IN SUBTRACTION

1. How do you subtract 28¢ from 54¢? Do you have to take pencil and paper to do what every business man does in his head? If it were 54¢ − 30¢, you would give the result at once as 24¢; but 28¢ lacks only 2¢ of being 30¢, and so the result is 24¢ + 2¢. So think simply "28¢ + 2¢ = 30¢, 54¢ + 2¢ = 56¢, and 56¢ − 30¢ = 26¢." Try this in subtracting 38¢ from 54¢.

2. In 61 − 34, subtract by thinking first "34 + 6 = 40" and then "61 + 6 = 67." What is the result?

Do these subtractions orally:

3.	47	47	47	57	56	5.4	0.92
	$\underline{12}$	$\underline{27}$	$\underline{28}$	$\underline{38}$	$\underline{38}$	$\underline{3.7}$	$\underline{0.49}$
4.	60	61	61	71	82	8.3	0.85
	$\underline{32}$	$\underline{32}$	$\underline{35}$	$\underline{37}$	$\underline{36}$	$\underline{4.5}$	$\underline{0.58}$
5.	72	71	70	73	73	8.4	2.92
	$\underline{38}$	$\underline{42}$	$\underline{42}$	$\underline{47}$	$\underline{48}$	$\underline{5.8}$	$\underline{1.96}$
6.	98	92	90	90	92	9.2	4.93
	$\underline{52}$	$\underline{52}$	$\underline{52}$	$\underline{61}$	$\underline{63}$	$\underline{6.8}$	$\underline{2.95}$
7.	87	87	86	84	84	9.2	5.93
	$\underline{28}$	$\underline{37}$	$\underline{37}$	$\underline{37}$	$\underline{39}$	$\underline{3.9}$	$\underline{1.75}$
8.	96	96	95	93	92	9.1	6.90
	$\underline{36}$	$\underline{48}$	$\underline{48}$	$\underline{58}$	$\underline{67}$	$\underline{7.6}$	$\underline{2.89}$
9.	58	58	98	84	73	8.8	8.75
	$\underline{27}$	$\underline{29}$	$\underline{69}$	$\underline{46}$	$\underline{27}$	$\underline{6.9}$	$\underline{6.85}$

RAPID SUBTRACTION

Numbers 1 to 8, oral

1. Suppose that you make a purchase amounting to $2.46 and give the dealer a 5-dollar bill. Think as the dealer does: "$2.46 + **4¢** = $2.50, $2.50 + **50¢** = $3, $3 + **$2** = $5." Then add mentally 4¢, 50¢, and $2, and state the amount of change that is due you.

Make change from $5 when you buy something that costs

2. $2.57. **3.** $1.72. **4.** $2.61. **5.** $0.36. **6.** $3.52.

7. In subtracting 4726 from 9000, some business men begin at the left, thinking of 9000 as 8990 + 10, written as here shown. Using this method of subtraction, how much is 9000 − 3268?

$$\begin{array}{cccc} 8 & 9 & 9 & 10 \\ 4 & 7 & 2 & 6 \\ \hline 4 & 2 & 7 & 4 \end{array}$$

8. How do you check your work in subtraction? Illustrate by the case of 42 − 17 = 25; by the case of 100 − 72.

See if you can do all these subtractions in 5 min.:

9. 3000 1111	**13.** 3000 1786	**17.** 5726 3987	**21.** 52,123 18,068	**25.** 12.8300 6.9426
10. 4000 2222	**14.** 4000 2938	**18.** 8209 6138	**22.** 61,023 52,836	**26.** 301.220 49.726
11. 6000 3999	**15.** 5000 2673	**19.** 7012 3648	**23.** 72,133 29,239	**27.** 8023.08 4768.29
12. 8000 4852	**16.** 6000 2673	**20.** 7000 4728	**24.** 70,000 34,820	**28.** 8000.00 5290.76

29. From $1,000,000 subtract $750,920 + $124,480.

30. From $2,500,000 subtract $425,000 + $75,000 + $16,250.

REVIEW DRILL CHART IN SUBTRACTION

1. Beginning with 50, subtract orally the following numbers, each from the preceding result: 5, 2, 3, 4, 6, 4, 2, 4, 5, 7, 6, 2. That is, say "45, 43, 40," and so on.

In each of the following cases begin at the left and subtract orally from 50, as in Ex. 1, until you reach zero:

2. 5 7 3 5 4 6 5 2 3 4 2 4
3. 3 7 2 8 5 6 5 4 3 2 2 3
4. 7 5 2 3 8 1 8 6 5 4 0 1
5. 5 8 7 6 5 1 2 3 4 2 4 3

Copy and subtract, checking each result:

6. $1768
492

7. $4107
298

8. $5123
1768

9. $8234
3968

10. $9230
4768

11. $2089.70
379.69

12. $8231.21
4681.96

13. $9210.82
7876.98

14. $8023.81
6789.96

15. $7333.03
5768.79

16. $8273.86
4969.98

17. $7234.32
2989.75

18. $6000.00
4783.81

19. $7000.00
4383.69

20. $8000.00
3270.08

21. $12,782.68
698.98

22. $23,072.08
16,789.79

23. $30,292.72
17,693.49

24. $52,293.37
16,820.98

25. $75,000.00
968.39

26. From $432.86 + $981.08 subtract $42.38 + $681.96.

27. Subtract $292.07 + $57.36 from $75.82 + $378.42.

28. From $1,500,000 subtract $32,750.25 + $125,875.50.

REVIEW DRILL CHART IN MULTIPLICATION

After copying the examples, see if you can do these ten multiplications in 6 min.:

1. 32,236	**3.** 45,008	**5.** 41,433	**7.** 77,088	**9.** 40,756
24	37	53	87	86
2. 21,408	**4.** 48,808	**6.** 30,098	**8.** 59,960	**10.** 27,824
46	49	72	36	58

Multiply as follows, and check each result:

11. 10 × $0.35.
12. 10 × $7251.
13. 100 × $2.87½.
14. 1000 × $0.25.
15. 50 × $64,224.
16. 401 × 3664.
17. 503 × 2763.
18. 630 × 4876.
19. 580 × 6924.
20. 847 × 6476.
21. 972 × 3502.
22. 707 × 8763.
23. 5002 × 7983.
24. 6107 × 6107.
25. 4004 × 4004.
26. 25 × $160.
27. 25 × $640.
28. 25 × $75.25.
29. 12½ × $6400.
30. 125 × $640.
31. 283 × $48.25.
32. 342 × $29.75.
33. 276 × $49.42.
34. 428 × $75.19.
35. 3574 × 0.05.
36. 0.25 × 536.
37. 0.25 × 5.36.
38. 0.25 × 0.536.
39. 0.75 × 576.
40. 7.5 × 57.6.
41. 33⅓ × 66.
42. 33⅓ × 48.
43. 16⅔ × $636.
44. 5 × 47,664.
45. 125 × $480.
46. 704 × 6.25.
47. 172 × 31.7.
48. 776 × 0.125.
49. 0.175 × 424.
50. 1.75 × 424.
51. 175 × 424.
52. 175 × 4.24.
53. 0.425 × 584.
54. 375 × 504.
55. 3.75 × 5.04.

56. Find the product of 24.7 + 16.8 times 12.07 − 10.69.

REVIEW DRILL CHART IN DIVISION

After copying the examples, see if you can do all these divisions in 7 min.:

1. $4\overline{)3368}$ $5\overline{)1725}$ $6\overline{)3942}$ $7\overline{)5264}$ $8\overline{)4456}$

2. $3\overline{)7779}$ $2\overline{)5550}$ $9\overline{)4446}$ $8\overline{)7464}$ $7\overline{)6566}$

3. $4\overline{)9232}$ $8\overline{)9048}$ $7\overline{)4844}$ $9\overline{)6669}$ $5\overline{)4405}$

4. $2\overline{)3425}$ $3\overline{)4177}$ $4\overline{)6283}$ $5\overline{)7291}$ $6\overline{)3337}$

5. $7\overline{)6441}$ $8\overline{)9649}$ $9\overline{)3470}$ $7\overline{)8507}$ $5\overline{)9643}$

6. $6\overline{)4445}$ $8\overline{)7327}$ $9\overline{)5555}$ $7\overline{)1004}$ $7\overline{)3005}$

Perform the following divisions, finding the results in Exs. 27–36 to the nearest cent:

7. 482 ÷ 5.
8. 507 ÷ 4.
9. 732 ÷ 6.
10. 810 ÷ 3.
11. 477 ÷ 9.
12. 743 ÷ 2.
13. 347 ÷ 3.
14. 607 ÷ 7.
15. 555 ÷ 8.
16. 702 ÷ 9.
17. 50,568 ÷ 602.
18. 13,020 ÷ 620.
19. 19,251 ÷ 629.
20. 76,109 ÷ 629.
21. 33,750 ÷ 125.
22. 26,288 ÷ 106.
23. 153,186 ÷ 422.
24. 237,975 ÷ 501.
25. 972,360 ÷ 222.
26. 520,251 ÷ 303.
27. $48.72 ÷ 16.
28. $39.54 ÷ 12.
29. $57.81 ÷ 15.
30. $34.51 ÷ 17.
31. $208.65 ÷ 65.
32. $282,127 ÷ 75.
33. $359,333 ÷ 78.
34. $463,481 ÷ 55.
35. $582,603 ÷ 82.
36. $4283.25 ÷ 25.

37. Divide 98 × 2871 × 0.628 by 49 × 33 × 3.14.

REVIEW DRILL CHART IN DIVISION

See if you can do the following six divisions in 12 min., finding each result to the nearest 0.01:

1. $177\overline{)3427.81}$

2. $42.3\overline{)286.94}$

3. $239\overline{)7341.68}$

4. $63.8\overline{)427.52}$

5. $125\overline{)3492.50}$

6. $39.7\overline{)797.97}$

Divide as indicated:

7. 62.9 ÷ 37.
8. 6.29 ÷ 37.
9. 62.9 ÷ 3.7.
10. 6.29 ÷ 3.7.
11. 80.5 ÷ 35.
12. 80.5 ÷ 3.5.
13. 8.05 ÷ 3.5.
14. 46.2 ÷ 22.
15. 46.2 ÷ 2.2.
16. 4.62 ÷ 2.2.
17. 600 ÷ 0.6.
18. 600 ÷ 0.06.
19. 6 ÷ 0.0006.
20. 627 ÷ 0.3.
21. 6.27 ÷ 0.3.
22. 7.14 ÷ 0.42.
23. 62.32 ÷ 7.6.
24. 6.232 ÷ 0.76.
25. 8190 ÷ 0.9.
26. 61.20 ÷ 1.2.
27. 7.200 ÷ 1.2.
28. 78.4 ÷ 0.08.
29. 3.12 ÷ 0.04.
30. 7.81 ÷ 1.1.
31. 8.91 ÷ 0.11.
32. 4.55 ÷ 0.07.
33. 2.73 ÷ 0.03.
34. 8.96 ÷ 12.8.
35. 4.41 ÷ 0.21.
36. 17.28 ÷ 14.4.
37. 1231.2 ÷ 2.4.
38. 1.2312 ÷ 2.4.
39. 62.75 ÷ 123.
40. 6.275 ÷ 1.23.
41. 36.08 ÷ 32.8.
42. 3.608 ÷ 0.328.
43. 1.036 ÷ 0.560.
44. 882.32 ÷ 0.82.
45. 4.4116 ÷ 0.41.
46. 11,300 ÷ 5.65.
47. 6.51021 ÷ 3.207.
48. 651.021 ÷ 320.7.
49. 176.4 ÷ 0.0072.
50. 5.44 ÷ 0.0068.
51. 717.75 ÷ 4.125.
52. 88.232 ÷ 8.2.
53. 633.775 ÷ 12.3.
54. 24.5970 ÷ 0.027.
55. 928.444 ÷ 0.44.
56. 1.24792 ÷ 1.54.
57. 0.56943 ÷ 33.3.
58. 0.78210 ÷ 68.7.
59. 62.351 ÷ 31.678.
60. 489.246 ÷ 24.335.

REVIEW DRILL CHART IN FRACTIONS

Add the following:

1. $\frac{1}{4}$, $\frac{3}{8}$.
2. $\frac{1}{4}$, $\frac{3}{16}$.
3. $\frac{1}{4}$, $\frac{5}{16}$.
4. $\frac{1}{4}$, $\frac{9}{16}$.
5. $\frac{3}{4}$, $\frac{13}{16}$.
6. $\frac{1}{8}$, $\frac{3}{16}$.
7. $\frac{3}{8}$, $\frac{7}{16}$.
8. $\frac{3}{8}$, $\frac{15}{16}$.
9. $\frac{5}{8}$, $\frac{15}{16}$.
10. $\frac{7}{8}$, $\frac{13}{16}$.
11. $\frac{1}{2}$, $\frac{7}{32}$.
12. $\frac{1}{2}$, $\frac{15}{32}$.
13. $\frac{1}{4}$, $\frac{25}{32}$.
14. $\frac{3}{4}$, $\frac{31}{32}$.
15. $\frac{3}{8}$, $\frac{5}{32}$.
16. $\frac{3}{4}$, $\frac{7}{24}$.
17. $\frac{3}{4}$, $\frac{17}{24}$.
18. $\frac{5}{8}$, $\frac{5}{24}$.
19. $\frac{7}{8}$, $\frac{23}{24}$.
20. $\frac{5}{16}$, $\frac{1}{64}$.

21. Add the fractions $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{3}{4}$, $\frac{7}{16}$, $\frac{1}{8}$, $\frac{5}{8}$, and $\frac{1}{2}$.
22. Add the fractions $\frac{3}{4}$, $\frac{5}{8}$, $\frac{3}{16}$, $\frac{1}{2}$, $\frac{7}{8}$, $\frac{1}{4}$, $\frac{1}{16}$, $\frac{3}{16}$, and $\frac{9}{16}$.

Add the following:

23. $8\frac{3}{4}$, $5\frac{1}{3}$.
24. $6\frac{3}{4}$, $8\frac{2}{3}$.
25. $2\frac{1}{3}$, $7\frac{1}{8}$.
26. $6\frac{1}{3}$, $5\frac{3}{8}$.
27. $8\frac{2}{3}$, $9\frac{5}{8}$.
28. $7\frac{2}{3}$, $8\frac{7}{8}$.
29. $62\frac{5}{6}$, $29\frac{3}{5}$.
30. $75\frac{7}{8}$, $62\frac{15}{16}$.
31. $92\frac{1}{3}$, $75\frac{4}{15}$.
32. $73\frac{2}{3}$, $68\frac{7}{15}$.
33. $52\frac{4}{5}$, $75\frac{13}{15}$.
34. $48\frac{9}{10}$, $73\frac{11}{12}$.
35. $5\frac{4}{5}$, $6\frac{2}{3}$, $4\frac{2}{15}$.
36. $6\frac{7}{8}$, $3\frac{1}{2}$, $5\frac{15}{16}$.
37. $4\frac{1}{3}$, $3\frac{3}{5}$, $5\frac{1}{30}$.
38. $4\frac{7}{8}$, $3\frac{2}{3}$, $5\frac{11}{24}$.
39. $2\frac{7}{8}$, $5\frac{13}{16}$, $6\frac{5}{32}$.
40. $3\frac{5}{6}$, $2\frac{5}{12}$, $6\frac{7}{24}$.

Subtract as follows:

41. $\frac{3}{4} - \frac{1}{8}$.
42. $\frac{5}{8} - \frac{1}{4}$.
43. $\frac{5}{8} - \frac{3}{16}$.
44. $\frac{7}{8} - \frac{5}{16}$.
45. $\frac{1}{2} - \frac{5}{32}$.
46. $\frac{7}{8} - \frac{5}{32}$.
47. $\frac{7}{8} - \frac{11}{32}$.
48. $\frac{1}{2} - \frac{5}{16}$.
49. $\frac{5}{32} - \frac{1}{8}$.
50. $\frac{5}{8} - \frac{7}{32}$.
51. $\frac{1}{2} - \frac{11}{32}$.
52. $\frac{15}{32} - \frac{3}{64}$.
53. $\frac{13}{16} - \frac{5}{8}$.
54. $\frac{29}{64} - \frac{1}{4}$.
55. $32 - 4\frac{7}{8}$.
56. $63 - 5\frac{3}{4}$.
57. $72 - 8\frac{9}{10}$.
58. $48 - 3\frac{7}{32}$.
59. $67 - 5\frac{3}{16}$.
60. $71 - 6\frac{5}{24}$.
61. $80 - 9\frac{5}{48}$.
62. $9\frac{7}{16} - 7\frac{3}{8}$.
63. $8\frac{9}{16} - 3\frac{3}{8}$.
64. $6\frac{11}{16} - 4\frac{5}{8}$.
65. $7\frac{15}{16} - 3\frac{7}{8}$.
66. $9\frac{31}{32} - 6\frac{3}{4}$.
67. $8\frac{29}{32} - 5\frac{3}{8}$.
68. $23\frac{1}{8} - 8\frac{3}{16}$.

REVIEW DRILL CHART IN FRACTIONS

See if you can do these additions in $1\frac{1}{4}$ *min.:*

1. $\frac{1}{2} + \frac{1}{4}$. 3. $\frac{1}{2} + \frac{1}{8}$. 5. $\frac{1}{2} + \frac{5}{8}$. 7. $\frac{1}{4} + \frac{1}{8}$. 9. $\frac{3}{4} + \frac{3}{8}$.
2. $\frac{1}{2} + \frac{3}{4}$. 4. $\frac{1}{2} + \frac{3}{8}$. 6. $\frac{1}{2} + \frac{7}{8}$. 8. $\frac{1}{4} + \frac{7}{8}$. 10. $\frac{3}{4} + \frac{5}{8}$.

See if you can do these subtractions in $1\frac{1}{4}$ *min.:*

11. $\frac{1}{2} - \frac{1}{4}$. 13. $\frac{1}{2} - \frac{1}{8}$. 15. $\frac{7}{8} - \frac{1}{2}$. 17. $\frac{1}{4} - \frac{1}{8}$. 19. $\frac{3}{4} - \frac{3}{8}$.
12. $\frac{3}{4} - \frac{1}{2}$. 14. $\frac{1}{2} - \frac{3}{8}$. 16. $\frac{5}{8} - \frac{1}{2}$. 18. $\frac{7}{8} - \frac{1}{4}$. 20. $\frac{3}{4} - \frac{5}{8}$.

See if you can do these multiplications in $1\frac{1}{4}$ *min.:*

21. $\frac{1}{2} \times \frac{1}{4}$. 23. $\frac{1}{2} \times \frac{1}{8}$. 25. $\frac{5}{8} \times \frac{1}{2}$. 27. $\frac{1}{4} \times \frac{1}{8}$. 29. $\frac{3}{4} \times \frac{3}{8}$.
22. $\frac{1}{2} \times \frac{3}{4}$. 24. $\frac{1}{2} \times \frac{3}{8}$. 26. $\frac{7}{8} \times \frac{1}{2}$. 28. $\frac{7}{8} \times \frac{3}{4}$. 30. $\frac{3}{4} \times \frac{5}{8}$.

See if you can do these divisions in $1\frac{1}{4}$ *min.:*

31. $\frac{3}{4} \div \frac{3}{8}$. 33. $\frac{3}{4} \div \frac{5}{8}$. 35. $\frac{3}{4} \div \frac{7}{8}$. 37. $\frac{2}{3} \div \frac{3}{4}$. 39. $\frac{2}{3} \div \frac{7}{8}$.
32. $\frac{3}{8} \div \frac{3}{4}$. 34. $\frac{5}{8} \div \frac{3}{4}$. 36. $\frac{7}{8} \div \frac{3}{4}$. 38. $\frac{3}{4} \div \frac{2}{3}$. 40. $\frac{3}{5} \div \frac{2}{3}$.

Multiply as follows:

41. $\begin{array}{r} 34 \\ \underline{3\frac{1}{2}} \end{array}$
42. $\begin{array}{r} 48 \\ \underline{23\frac{1}{4}} \end{array}$
43. $\begin{array}{r} 56 \\ \underline{38\frac{3}{4}} \end{array}$
44. $\begin{array}{r} 96 \\ \underline{15\frac{7}{8}} \end{array}$
45. $\begin{array}{r} 64 \\ \underline{35\frac{5}{8}} \end{array}$
46. $\begin{array}{r} 84 \\ \underline{22\frac{2}{3}} \end{array}$
47. $\begin{array}{r} 29\frac{1}{2} \\ \underline{36} \end{array}$
48. $\begin{array}{r} 38\frac{2}{3} \\ \underline{45} \end{array}$
49. $\begin{array}{r} 52\frac{7}{8} \\ \underline{25} \end{array}$
50. $\begin{array}{r} 5\frac{1}{2} \\ \underline{2\frac{1}{2}} \end{array}$
51. $\begin{array}{r} 7\frac{3}{4} \\ \underline{3\frac{1}{2}} \end{array}$
52. $\begin{array}{r} 9\frac{7}{8} \\ \underline{6\frac{1}{2}} \end{array}$
53. $\begin{array}{r} 8\frac{1}{4} \\ \underline{2\frac{3}{8}} \end{array}$
54. $\begin{array}{r} 6\frac{3}{4} \\ \underline{5\frac{5}{8}} \end{array}$
55. $\begin{array}{r} 7\frac{2}{3} \\ \underline{3\frac{3}{8}} \end{array}$

Divide as follows:

56. $7\frac{3}{4} \div 2\frac{1}{2}$. 58. $8\frac{2}{3} \div 3\frac{1}{4}$. 60. $2\frac{1}{2} \div 1\frac{1}{4}$. 62. $8\frac{5}{8} \div 2\frac{7}{8}$.
57. $9\frac{7}{8} \div 3\frac{3}{4}$. 59. $3\frac{1}{4} \div 8\frac{2}{3}$. 61. $1\frac{1}{4} \div 2\frac{1}{2}$. 63. $2\frac{7}{8} \div 8\frac{5}{8}$.

LADDER TEST DRILL CHART

Copy each of these three ladders, and, beginning at the foot, see if you can climb rapidly to the top without making a single mistake:

I	II	III
20. $\frac{7}{8}$ mi. = (?) ft.	20. $8\frac{1}{2}$ gal. = (?) qt.	20. 27 in. ÷ 3 yd.
19. $\frac{3}{4}$ mi. = (?) ft.	19. $8\frac{1}{2}$ qt. = (?) pt.	19. $3\frac{5}{8}$ yd. + 7 in.
18. $\frac{1}{2}$ mi. = (?) ft.	18. $2\frac{1}{2}$ bu. = (?) pk.	18. $3\frac{1}{2}$ yd. ÷ 8 in.
17. 68 ft. = (?) in.	17. $3\frac{3}{4}$ ft. = (?) in.	17. $2\frac{1}{2}$ ft. ÷ 6 in.
16. 68 in. = (?) ft.	16. $3\frac{3}{4}$ rd. = (?) ft.	16. 6 yd. − 6 in.
15. $\frac{3}{4}$ yd. = (?) ft.	15. $3\frac{3}{4}$ yd. = (?) in.	15. 6 yd. ÷ 6 ft.
14. $\frac{3}{4}$ ft. = (?) in.	14. $3\frac{7}{8} \div 4\frac{1}{2}$.	14. 0.03 % of 2.5.
13. $2\frac{3}{4} \div 3\frac{7}{8}$.	13. $4\frac{1}{2} \div 3\frac{7}{8}$.	13. 0.03×2.5.
12. $3\frac{7}{8} \div 2\frac{3}{4}$.	12. $4\frac{1}{2} \times 3\frac{7}{8}$.	12. 3 % of 16.8.
11. $3\frac{7}{8} \times 2\frac{3}{4}$.	11. $4\frac{1}{2} - 3\frac{7}{8}$.	11. $3\frac{3}{4} \div 6\frac{7}{8}$.
10. $3\frac{7}{8} - 2\frac{3}{4}$.	10. $4\frac{1}{2} + 3\frac{7}{8}$.	10. $6\frac{7}{8} \div 3\frac{3}{4}$.
9. $3\frac{7}{8} + 2\frac{3}{4}$.	9. $5.94 \div 198$.	9. $6\frac{7}{8} - 3\frac{3}{4}$.
8. $38.4 \div 19.2$.	8. $594 \div 1.98$.	8. $3\frac{3}{4} \times 6\frac{7}{8}$.
7. 38.4×19.2.	7. $59.4 \div 19.8$.	7. $\frac{3}{4} \times \frac{1}{2} \times \frac{7}{8}$.
6. $38.4 - 19.2$.	6. 5.94 % of 198.	6. $\frac{3}{4} - \frac{1}{2} + \frac{7}{8}$.
5. $38.4 + 19.2$.	5. 5.94×19.8.	5. $\frac{3}{4} + \frac{1}{2} - \frac{7}{8}$.
4. $484 \div 363$.	4. 5.94×1.98.	4. $\frac{3}{4} + \frac{1}{2} + \frac{7}{8}$.
3. 484×363.	3. 59.4×19.8.	3. $\frac{3}{4} + \frac{1}{2} + \frac{1}{8}$.
2. $484 - 363$.	2. $59.4 - 19.8$.	2. $\frac{1}{4} + \frac{1}{2} + \frac{1}{8}$.
1. $484 + 363$.	1. $59.4 + 19.8$.	1. $\frac{1}{2} + \frac{5}{8} + \frac{1}{4}$.

COMPLETING PROBLEMS

Copy and complete the following problems:

1. From a piece of silk containing $42\frac{1}{8}$ yd. a clerk sold me $8\frac{3}{4}$ yd. The clerk had —— yd. left.

2. At $4.20 a yard, I paid —— for the $8\frac{3}{4}$ yd. of silk which I bought.

3. After the holidays the silk in Ex. 2 was marked down 15%. The price of the silk was then —— a yard.

4. If my scores on four arithmetic tests were 60, 72, 80, and 84, my average score was ——.

5. In an arithmetic test Fred tried 48 problems and solved 36 correctly, while Frank tried 52 problems and solved 39 correctly. Of the problems tried, Fred solved ——% correctly, and Frank solved ——% correctly.

6. On another arithmetic test Fred solved 48 problems correctly, which was 80% of the total number given. If Frank solved 70% correctly, he solved —— problems.

7. The players on the Stars' basketball team weighed 124 lb., 132 lb., 118 lb., 120 lb., and 131 lb., and those on the Crescents' team weighed 115 lb., 125 lb., 128 lb., 132 lb., and 130 lb. The average weight of the —— team was —— lb. greater than that of the —— team.

8. A man uses 8 gal. of gasoline in making a trip of 100 mi. in his car. He estimates that on a trip of 375 mi. over much the same kind of road he will use —— gal.

9. A salesman who is paid a commission of $12\frac{1}{2}$% on goods that he sells, earns $—— on a sale amounting to $140.

10. A commission merchant sells a consignment of farm produce for $480. If he charges a commission of 8%, the farmer receives $——.

SHORT CUTS IN MULTIPLICATION AND DIVISION

All work oral

1. How do you multiply by $\frac{1}{2}$? How do you divide by $\frac{1}{2}$?

2. How do you multiply by $\frac{3}{4}$? How do you divide by $\frac{3}{4}$?

3. Where do you place the decimal point in multiplying $15.20 by 0.05? by 5%? by 5? by 50?

4. How do you multiply by 10? by 100? by 1000?

5. How do you divide by 100? by 1000? by 2000?

6. How do you multiply by 25? Have you given the shortest method that you know?

7. What is the shortest method that you know of finding 25% of a number?

8. How do you find 75% of 88? Is this the shortest way?

9. How do you multiply by 50? Illustrate by multiplying 22 by 50; 122 by 50; 4800 by 50.

10. What is the short method of multiplying by $33\frac{1}{3}$%? Illustrate by finding $33\frac{1}{3}$% of $36.

11. State the shortest method you know of finding $16\frac{2}{3}$% of a number. Illustrate by finding $16\frac{2}{3}$% of $480.

12. Which is the shorter method of multiplying by 125, to multiply by $\frac{1000}{8}$ or by $\frac{500}{4}$? Explain your answer.

13. What short cut do you use in dividing by 125? Illustrate by dividing 2000 by 125.

14. Find $12\frac{1}{2}$% of $160 by the shortest method you know.

15. If you buy $3\frac{1}{2}$ yd. of silk at $4 a yard, how much does it cost? How much change do you receive from $20?

16. If you buy $5\frac{3}{8}$ yd. of velvet at $3.20 a yard, must you take a pencil and paper to find the cost? If you find it mentally, what steps do you take? What is the cost?

REVIEW DRILL CHART ON SHORT CUTS

State rapidly the value of 1 % of

1. 35.
2. 75.
3. 90.
4. 325.
5. 320.
6. 305.
7. 1575.
8. 1570.
9. 1500.
10. 3000.
11. 3050.
12. 3005.
13. 15,000.
14. 15,100.
15. 15,125.

By first taking 1 % of each of the following numbers, find the value of

16. 5 % of 800.
17. 8 % of 500.
18. 7 % of 700.
19. 8 % of 800.
20. 3 % of 500.
21. 5 % of 300.
22. 6 % of 500.
23. 5 % of 600.
24. 3 % of 750.
25. 4 % of 725.
26. 5 % of 850.
27. 6 % of 975.
28. 6 % of 2000.
29. 6 % of 2225.
30. 7 % of 4280.

Find the value in each of the following cases:

31. $33\frac{1}{3}\%$ of 900.
32. $12\frac{1}{2}\%$ of 800.
33. $12\frac{1}{2}\%$ of 960.
34. $8\frac{1}{3}\%$ of 600.
35. $16\frac{2}{3}\%$ of 960.
36. $6\frac{1}{4}\%$ of 176.
37. $2\frac{1}{2}\%$ of 336.
38. $1\frac{1}{4}\%$ of 672.
39. $37\frac{1}{2}\%$ of 96.
40. $87\frac{1}{2}\%$ of 88.
41. $62\frac{1}{2}\%$ of 72.
42. 250 % of 40.
43. 10 % of 2750.
44. 100 % of 275.
45. $62\frac{1}{2}\%$ of 800.
46. $66\frac{2}{3}\%$ of 720.
47. 125 % of 96.
48. $87\frac{1}{2}\%$ of 968.

Using short cuts, multiply the following numbers by 25:

49. 160.
50. 180.
51. 280.
52. 310.
53. 720.
54. 900.
55. 940.
56. 860.
57. 540.
58. 680.
59. 360.
60. 220.
61. 170.
62. 110.
63. 330.
64. 320.
65. 70.
66. 700.
67. 530.
68. 640.

REVIEW DRILL CHART ON SHORT CUTS

By first multiplying by 10 and then dividing by 2, multiply each of the following numbers by 5:

1. 36.
2. 72.
3. 326.
4. 724.
5. 316.
6. 254.
7. 1122.
8. 1096.
9. 1896.
10. 1752.

By first multiplying by 100 and then dividing by 2, multiply each of the following numbers by 50:

11. 34.
12. 28.
13. 842.
14. 788.
15. 1122.
16. 1238.
17. 1762.
18. 1848.
19. 1824.
20. 1426.

Using short cuts, multiply each of the following numbers in turn by 125 and 500:

21. 48.
22. 64.
23. 32.
24. 720.
25. 736.
26. 232.
27. 130.
28. 133.
29. 637.
30. 642.
31. 961.
32. 165.
33. 3972.
34. 7234.
35. 4610.

Using short cuts, divide each of the following numbers in turn by 5, 25, and 125:

36. 38.
37. 47.
38. 98.
39. 160.
40. 220.
41. 940.
42. 888.
43. 270.
44. 385.
45. 362.
46. 588.
47. 934.
48. 2750.
49. 5635.
50. 9999.

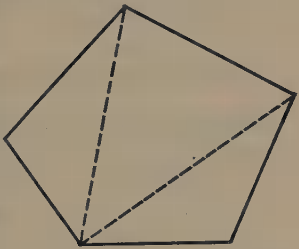
Using a short method, find the interest when the principal, time, and rate are as follows:

51. \$500, 60 da., 6%.
52. \$320, 30 da., 4%.
53. \$650, 90 da., 6%.
54. \$1200, 24 da., 6%.
55. \$2500, 66 da., 6%.
56. \$4750, 83 da., 5%.

II. MEASUREMENTS

A SILENT READING LESSON

We already know how to measure lengths, time, and weight and how to find the areas of rectangles, triangles, parallelograms, and circles. Thus, we know how to find the number of square yards of linoleum needed to cover a kitchen floor, or the number of square feet of surface to be covered in painting the triangular gable end of a house, or the cost of resilvering the back of a circular mirror at so much per square inch, and so on for similar cases.



We can also find the area of any figure that we can divide into triangles; for example, we can find in this way the area of an irregular field such as is represented by the figure above.

We have not yet learned, however, how to find the volumes of solid figures, a thing that it is often necessary to do. For example, on the farm we might need to know the volume of an oat bin or of a silo used for storing feed for the cattle, or in the city we might need to know how much earth must be removed in excavating for the foundation of a building, or we might wish to know the capacity of a water tank on the top of one of the apartment houses.

Likewise, we have not yet learned how people find the distances across wide rivers, or how they measure the heights of trees, flagpoles, or mountains without climbing them. We may never have to measure the width of a river or the height of a mountain, but we should like to know how such things are done, because they are matters of general information among educated people.

REVIEW OF SURFACE MEASUREMENTS

Numbers 1 to 32, oral

1. How do you find the area of a square?

State the areas of squares with sides of

2. 2 in.
3. 5 in.
4. 6 ft.
5. 9 ft.
6. $\frac{1}{2}$ in.
7. $\frac{1}{4}$ in.
8. 0.5 in.
9. 0.7 in.
10. 11 in.
11. 12 in.

12. How do you find the area of a rectangle?

State the areas of rectangles with sides of

13. 2 in., 4 in.
14. 3 ft., 6 ft.
15. 1 yd., 3 yd.
16. 2 yd., 5 yd.
17. 10 rd., 16 rd.
18. 4 rd., 12 rd.

19. How do you find the area of a parallelogram?

State the areas of parallelograms with bases and heights respectively as follows:

20. 5 in., 2 in.
21. 2 in., 5 in.
22. 6 ft., 3 ft.
23. 3 ft., 6 ft.
24. 14 yd., 2 yd.
25. 20 yd., 8 yd.

26. How do you find the area of a triangle?

State the areas of triangles with bases and heights respectively as follows:

27. 1 ft., 1 ft.
28. 2 ft., 2 ft.
29. 3 ft., 3 ft.
30. 1 yd., 1 yd.
31. 2 in., 5 in.
32. 12 in., 11 in.

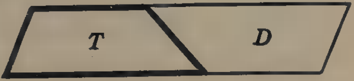
33. If we double the sides of a square, how does this affect the area? Draw a figure to illustrate your answer.

34. Find the area of a rectangle $2\frac{1}{4}$ ft. long and $1\frac{1}{2}$ ft. wide.

35. Find the area of a parallelogram whose base is 6 ft. 8 in. and whose height is half as much.

Area of a Trapezoid. There is another four-sided figure, the *trapezoid,* of which we have not yet learned how to find the area. To know how to do this is important, because many fields and building lots have the shape of a trapezoid.

Suppose that we have the trapezoid T shown in this figure. We may draw and cut out a trapezoid D exactly the same size and shape as T, and then fit the two together to form a parallelogram, as shown. That is, the area of a trapezoid is half the area of a parallelogram with the same height and with a base equal to the total length of the two parallel sides of the trapezoid, or, briefly stated,



$$\mathit{Area} = \tfrac{1}{2} \times \mathit{height} \times (\mathit{lower\ base} + \mathit{upper\ base}),$$

the parentheses meaning that we add the lengths of the two parallel sides before we multiply.

For example, in finding the area of a trapezoid with a height of 4 in. and parallel sides of 8 in. and 10 in., we see that the area is $\frac{1}{2} \times 4 \times (8 + 10)$ sq. in.; that is, it is $\frac{1}{2} \times 4 \times 18$ sq. in. Arranging the work as here shown, the result is found to be 36; that is, the area of the trapezoid is 36 sq. in.

$$\boxed{\frac{4 \times \overset{9}{\cancel{18}}}{\cancel{2}} = 36}$$

AREA OF A TRAPEZOID

1. A farmer has a field in the form of a trapezoid. The height of the trapezoid is 10 rd., and the parallel sides are $31\frac{1}{4}$ rd. and $16\frac{3}{4}$ rd. What is the area of the field?

2. A building lot in a village is in the form of a trapezoid. The height of the trapezoid is 66 ft., and the parallel sides are 78 ft. 6 in. and 86 ft. 8 in. What is the area of the lot?

Find the areas of the following trapezoids, the height in each case being given first and then the two parallel sides:

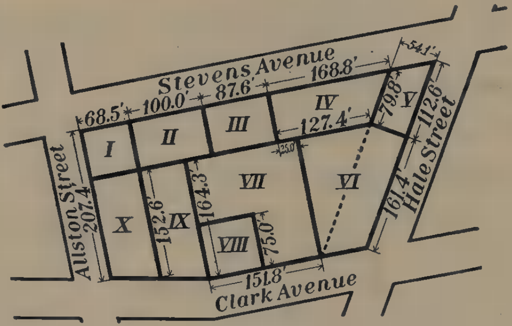
3. $16\frac{1}{2}$ in.; $8\frac{1}{4}$ in. and 12 in.

4. 3.5 in.; 3.5 in. and 5.4 in.

5. $4\frac{1}{2}$ ft.; 3 ft. and $4\frac{1}{2}$ ft.

6. 16 rd.; 18 rd. and 24 rd.

7. An irregular city block is divided into ten lots as shown in the figure below. Each measurement is given to the nearest 0.1 ft. (0.1′), and the areas of the lots are to be found to the nearest 0.1 sq. ft. Find the area of the corner lot marked I.



8. Find the area of each of lots II, III, IV, and V.

9. Find the area of each of lots IX and X, finding the length of any side that you need to use by adding or subtracting the measurements given on the plan.

10. By dividing lot VI into two parts, as suggested by the dotted line, find the area of the lot.

11. Find the combined area of lots VII and VIII.

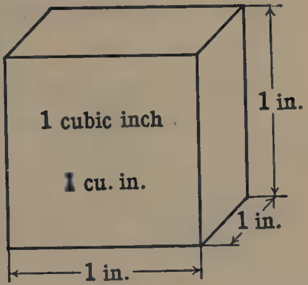
12. Find the area of lot VIII and then subtract it from the result of Ex. 11, thus finding the area of lot VII.

13. By dividing lot VII into two parts, find the area directly, thus checking the result found in Ex. 12.

Table of Cubic Measure. Before learning about volumes, we shall need to review the following table of cubic measure:

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)

If a block in the form of a cube is 1 in. on an edge, it is a a 1-inch cube, and its *volume* is 1 cu. in. If the block is 1 ft. on an edge, it is a 1-foot cube, and its volume is 1 cu. ft., and similarly for yards.



Another measure of volume is the *cord*. The standard cord of wood is a pile 8 ft. long, 4 ft. wide, and 4 ft. high, which is equivalent to 128 cu. ft. In practice, however, a pile of wood 8 ft. long, 4 ft. high, and 2 ft. wide is often called "a cord of 2-foot wood," and similarly for other lengths into which the sticks may be sawed.

CUBIC MEASURE

1. In excavating a cellar, a contractor removed 8100 cu. ft. of earth. Express the amount as cubic yards.

2. On one section of a new state road 8000 cu. yd. of crushed rock were used. How many cubic feet were used?

3. A box has a volume of $2\frac{1}{2}$ cu. ft. Express the volume of the box in cubic inches.

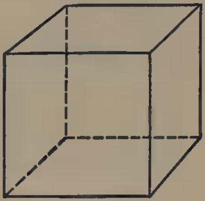
Copy these statements, supplying the missing numbers:

4. 2 cu. yd. = —— cu. ft.
5. 81 cu. ft. = —— cu. yd.
6. 2 cu. ft. = —— cu. in.
7. $\frac{1}{2}$ cu. ft. = —— cu. in.
8. $\frac{2}{3}$ cu. yd. = —— cu. in.
9. $3\frac{1}{2}$ cu. yd. = —— cu. ft.

STUDYING THE CUBE

Copy and complete the following statements:

1. A cube has —— square faces. These faces are the base, the top, and the —— sides.



2. A cube has —— edges, which are the straight lines where any two faces meet.

3. A cube has —— corners or vertices.

4. If the edge of a cube is 1 in., the area of each face is —— sq. in., and the area of the entire surface of the cube is therefore —— sq. in.

5. If the edge of a cube is 2 in., the area of each face is —— sq. in., and the area of the entire surface is —— sq. in.

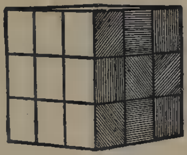
6. If the edge of a cube is 1 in., the volume is —— cu. in. If the edge is 2 in., the volume is —— cu. in.

7. The number which multiplied by itself gives 100 is ——. If the area of the entire surface of a cube is 600 sq. in., the edge is —— in.

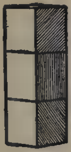
Copy the following table and fill the blank spaces:

	Edge of Cube	Total Length of Edges	Area of a Face	Area of Entire Surface	Volume of Cube
8.	1 in.	12 in.	1 sq. in.	6 sq. in.	1 cu. in.
9.	2 in.		4 sq. in.		8 cu. in.
10.	3 in.			54 sq. in.	
11.		60 in.			125 cu. in.
12.		84 in.			
13.			36 sq. in.		
14.			144 sq. in.		

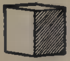
Volume of a Rectangular Solid. A solid with six faces, each of which is a rectangle, is called a *rectangular solid*. Just as a square is a special kind of rectangle (one with four equal sides), the cube is a special kind of rectangular solid. These figures are all rectangular solids, and A and C are cubes.



A



B



C

If the cube C contains 1 cu. in., how many cubic inches does B contain? If A has 9 cu. in. in the bottom layer of cubes, how many cubic inches does A contain?

If A were a box 4 in. long and 3 in. wide, how many cubic inches would there be in the bottom layer of cubes? If it were 5 in. high, how many cubic inches would it contain?

If a box is 5 in. long, 4 in. wide, and 3 in. high, how many cubic inches does it contain?

In finding the amount of earth removed from an excavation 20 yd. long, 10 yd. wide, and 2 yd. deep, we have

$$20 \times 10 \times 2 = 400, \text{ the number of cubic yards.}$$

Instead of this we may use the same plan that we do in finding areas; that is, we may write

$$20 \times 10 \times 2 \text{ cu. yd.} = 400 \text{ cu. yd.}$$

Thus, to find the volume of a rectangular solid, we multiply together the length, width, and height, or, briefly stated,

$$\textit{Volume} = \textit{length} \times \textit{width} \times \textit{height}.$$

A rectangular solid was formerly called a *rectangular parallelepiped*, a name that we still hear occasionally.

VOLUME OF A RECTANGULAR SOLID

1. In our school there is an aquarium 14 in. long, $9\frac{1}{2}$ in. wide, and $8\frac{1}{2}$ in. deep. How many square inches are there in the bottom? How many cubic inches of water does the aquarium contain when level full?

2. The interior of a freight car is 34 ft. long, 8 ft. wide, and 7 ft. high. How many square feet are there in the floor of the car? How many cubic feet of space are there in the car?

3. Near our school they have excavated a cellar 34 ft. long, 16 ft. wide, and 9 ft. deep. How many cubic feet of earth were removed? How many cubic yards?

4. At $1.40 a cubic yard, how much will it cost to excavate a cellar 28 ft. by 38 ft. by 6 ft.?

5. Near our school they are building a wall which is to be 60 ft. long, 6 ft. high, and $1\frac{1}{2}$ ft. thick. How many cubic feet will the wall contain?

6. How much will it cost to excavate a ditch 240 ft. by 2 ft. by 5 ft. at $1.20 per cubic yard? at $1.50 per cubic yard?

7. How many cords of 4-foot wood (128 cu. ft.) are there in a pile 68 ft. long, 4 ft. wide, and 6 ft. high?

8. Suppose that your schoolroom is 36 ft. 8 in. long, 24 ft. wide, and 12 ft. high. What is the number of cubic feet of space in the room? the number of cubic yards?

9. A tunnel 940 ft. long has a cross-section area of 180 sq. ft. How much earth and rock were excavated?

10. How many cubic feet of storage space are there in a storeroom 40 ft. long, 32 ft. wide, and 10 ft. high?

11. In preparing a flower bed in a park it was necessary to fill in a space 80 ft. by 120 ft. to an average depth of $1\frac{1}{2}$ ft. At 75¢ a cubic yard, what was the cost of the earth used?

12. A box is 4 ft. long, 3 ft. wide, and $2\frac{1}{2}$ ft. deep. How many cubic feet does it contain?

13. If the box in Ex. 12 were half as long, but the width and depth were the same, what would be its volume?

14. If each measurement in Ex. 12 were doubled, what would be the volume of the box?

15. From Ex. 14 we may infer that doubling each edge of a rectangular box multiplies the volume by what number?

Copy and complete the following statements:

16. If we double the edge of a cube, we multiply the area of each face by ——, and we multiply the area of the entire surface of the cube by ——.

17. If we double the edge of a cube, we multiply the volume of the cube by ——.

18. Doubling each edge of a rectangular box multiplies the volume of the box by ——.

19. Doubling each edge of a rectangular block, such as a brick, multiplies the area of each face by ——.

20. Dividing each edge of a rectangular box by 2 divides the volume of the box by ——.

21. At $62\frac{1}{2}$ lb. per cubic foot, the weight of water that will just fill a cubic box 6 in. on an edge is —— lb.

22. If a block of stone 2 ft. long, 1 ft. wide, and 6 in. thick weighs 190 lb., a block of the same kind of stone 4 ft. long, 2 ft. wide, and 1 ft. thick weighs —— lb.

23. A block of granite 3 ft. long, 2 ft. wide, and $1\frac{1}{2}$ ft. thick is to be polished on all the faces except the one on which it stands. If the base is 3 ft. long and 2 ft. wide, the number of square feet to be polished is ——.

24. Find the number of cubic inches in a box 3 ft. 6 in. long, 18 in. wide, and 1 ft. deep.

25. The bottom of an ordinary farm-wagon box is usually 3 ft. by 10 ft., and the depth of the box varies from 24 in. to 26 in. Find the capacity in cubic feet for a depth of 24 in.; of 25 in.; of 26 in.

26. If a wagon box 3 ft. by 10 ft. by 24 in. will hold 50 bu. of shelled corn, how many bushels of shelled corn can be put into a wagon box 3 ft. by 10 ft. by 26 in.?

27. Some farmers estimate that $2\frac{2}{5}$ cu. ft. of corn in the ear will produce 1 bu. of shelled corn. How many bushels of shelled corn would there be in a crib 20 ft. long, 12 ft. wide, and 8 ft. high, filled with corn in the ear?

28. How many cubic yards of earth must be removed in excavating a tunnel that is 492 ft. long and has a rectangular cross section 39 ft. wide and 19 ft. 6 in. high?

29. An arithmetic class measured the excavation for the cellar of a new house near the school and found it to be 34 ft. 6 in. long, 24 ft. 6 in. wide, and 7 ft. deep. How many cubic yards of earth were taken out? How much did the excavation cost at $1.50 per cubic yard?

30. The ice chamber of a refrigerator is 16 in. by 24 in. by 18 in. Ice weighs 8% less than water, and 1 cu. ft. of water weighs $62\frac{1}{2}$ lb. If the ice comes in pieces weighing 10 lb., 25 lb., 50 lb., 100 lb., 150 lb., and 200 lb., which one or which two of these pieces is nearest to the amount of ice required to fill the chamber?

31. In building a section of a state highway 4 mi. long and 45 ft. wide, a layer of crushed rock was spread over the road to an average depth of 6 in. At $1.50 per cubic yard, what was the cost of the crushed rock?

REVIEW OF MEASUREMENTS

Copy the following tables, which show measurements of squares, and fill the blank spaces:

	SIDE	PERIMETER	AREA
1.	2 in.		
2.		20 in.	
3.			9 sq. in.
4.		60 in.	

	SIDE	PERIMETER	AREA
5.			16 sq. in.
6.		22 in.	
7.	2.4 in.		
8.		16.4 in.	

Copy the following table, which shows measurements of rectangles, and fill the blank columns:

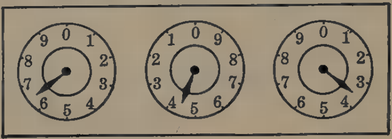
	LENGTH	WIDTH	PERIMETER	AREA
9.	2 ft. 6 in.	1 ft. 8 in.		
10.	3 ft. 9 in.	2 ft. 4 in.		
11.	4 ft. 2 in.	3 ft. 10 in.		
12.	1 ft. 10 in.	18 in.		

Copy the following table, which shows measurements of rectangular boxes, and fill the blank columns:

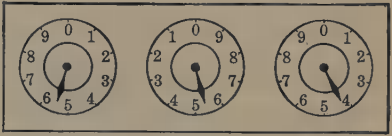
	LENGTH	WIDTH	DEPTH	VOLUME
13.	2½ ft.	2 ft.	1 ft.	
14.	3 ft.	2½ ft.	2½ ft.	
15.	3½ ft.	3 ft.	2 ft.	
16.	4 ft.	3½ ft.	2¼ ft.	
17.	6 in.	2½ in.	2¼ in.	
18.	8½ in.	4 in.	3 in.	
19.	4½ in.	2½ in.	1¼ in.	

READING THE GAS METER

1. Here is a picture of three dials on a gas meter. If the left-hand dial indicates ten thousands, the middle dial thousands, and the right-hand dial hundreds, the hands on the dials show that 64,300 cu. ft. of gas has passed through the meter. While more than 64,300 cu. ft. of gas has been used, we shall consider only the hundred last passed by the hand, although some gas companies read meters only to the thousand last passed. Notice that the middle dial is read in the opposite direction from the others. Suppose that this is the meter reading for May 1, that the reading for April 1 was 62,300, and that the gas company charges its customers $1.20 per M (that is, $1.20 per 1000 cu. ft.) for gas. How much is the bill for the gas that was used during the month of April?



2. If the reading of this gas meter was 52,700 a month ago, how much is the bill for the month at $1.10 per M?



3. If the reading in Ex. 2 was 51,500 a month ago, how much is the bill?

4. If 3700 cu. ft. of gas was used in January and 3300 cu. ft. in February, how much was the gas bill each of these months at $1.35 per M?

5. If 4500 cu. ft. of gas was used in January, and 20% less in June, how much was the June gas bill at $1.20 per M?

6. If Mrs. Jones were more careful to turn down the burners on her gas stove, she could reduce her gas bill about 8%. If she used 5600 cu. ft. of gas last month at $1.10 per M, how much could she have saved?

7. Some gas companies allow a cash discount of 5% on bills that are paid before the 10th of a month. If the meter readings are 83,400 and 85,200 on the first days of two consecutive months, and the rate is $1.25 per M, how much will be saved by paying the bill before the 10th?

Copy the following table and fill in the blank spaces:

	Present Reading of Meter	Previous Reading of Meter	Cubic Feet Consumed	Rate per M before Discount	Amount of Bill Rendered	Cash Discount 5%	Net Amount Due
8.	32,400	31,600		$1.10			
9.	26,700	23,900		.90			
10.	43,700	42,600		1.20			
11.	38,500	33,900		1.00			
12.	67,100	63,800		1.40			
13.	72,300	68,400		.85			
14.	52,200	48,600		1.35			
15.	74,300	69,900		1.05			

16. In factories where large amounts of gas are used, or in places where there is natural gas, the meters are generally read only to the thousand last passed. In a factory where the meter readings are 671,000 today and 479,000 a month ago, how much is the gas bill at 90¢ per M?

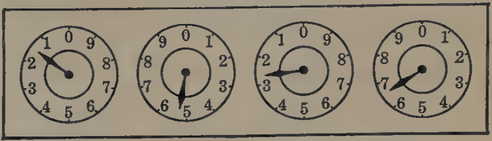
17. In a factory which used natural gas, the meter read 225,000 on March 1 and 412,000 on April 1. How much was the gas bill for March at 45¢ per M?

18. Our school gas meter read 685,000 today and 637,000 a month ago. How much is the bill at 55¢ per M?

19. If possible, read the gas meter in your school or home and find the cost of the gas used in a month.

READING THE ELECTRIC METER

1. Mr. Rogers lights his store by electricity. The meter is read in a way similar to that in which a gas meter is read, and shows thousands, hundreds, tens, and units of *kilowatt hours* (K. W. H.), which is the unit by which electricity is measured. Notice that the second and fourth dials from the left are read one way and that the first and third are read the other way. Read the meter.



2. If the reading in Ex. 1 was 1208 K. W. H. a month ago, how much electricity was used during the month?

3. If a meter reads 3704 K. W. H. on Jan. 1 and 3820 K. W. H. on Feb. 1, how much is the electric-light bill for January at $10\frac{1}{2}$¢ per K. W. H.?

4. Companies usually make lower rates to large users than to small users, and also make lower rates where the electricity is used for power or cooking instead of for lighting. Separate meters record the electricity used for the different purposes. If, in a month, a machine shop uses 375 K. W. H. at 8¢ per K. W. H. for lighting and 6285 K. W. H. at $2\frac{1}{2}$¢ for running the machines, how much is the bill for the month?

5. Mr. Brown used 22 K. W. H. at 12¢ for lighting his house last May and 240 K. W. H. at 3¢ for cooking. If he took advantage of a discount of 5% for payment of the bill within 10 da., how much did he pay?

6. Find the amount paid in settling a bill for 32 K. W. H. at $14\frac{1}{2}$¢ and 119 K. W. H. at $4\frac{1}{2}$¢, less a discount of 5%.

7. If possible, read the electric meter in the school or at home and compute the amount of the bill for a month.

Practical Equivalents. In solving problems, we often need to know the relations between certain measures. For example, if we know the number of cubic feet in a tank, we may wish to know how many gallons the tank will hold. The following tables of equivalents need not be memorized:

EXACT VALUES	APPROXIMATE VALUES
2150.42 cu. in. = 1 bu.	$1\frac{1}{4}$ cu. ft. = 1 bu.
231 cu. in. = 1 gal.	$7\frac{1}{2}$ gal. = 1 cu. ft.
$31\frac{1}{2}$ gal. = 1 bbl.	$4\frac{1}{5}$ cu. ft. = 1 bbl.

The values on the left are exact equivalents, while those on the right are the approximations that are often used. Thus, while 1 bu. = 2150.42 cu in., the value frequently taken in practical cases is $1\frac{1}{4}$ cu. ft. or, less often, 2150 cu. in.

Since 1 bu. = $1\frac{1}{4}$ cu. ft., we can find the number of bushels in a bin by dividing the number of cubic feet by $1\frac{1}{4}$, or by $\frac{5}{4}$; still more conveniently, we may multiply by $\frac{4}{5}$. That is, expressing the measurements in feet, we have

$$\textit{Number of bushels} = \frac{4 \times \textit{length} \times \textit{width} \times \textit{height}}{5}.$$

For example, suppose that we wish to find the capacity in bushels of a bin which is 20 ft. long, 16 ft. wide, and 7 ft. deep.

$$\boxed{\frac{4 \times \overset{4}{\cancel{20}} \times 16 \times 7}{\cancel{5}} = 1792}$$

Using the relation given above, and canceling, we find, as shown at the right, that the number of bushels is 1792; that is, the capacity of the bin is 1792 bu.

Similarly, to find the capacity in gallons of a rectangular tank which is 10 ft. × 8 ft. × 6 ft., we have

$$10 \times 8 \times 6 \times 7\tfrac{1}{2} \text{ gal.} = 3600 \text{ gal.}$$

Other Equivalents. The following relations pertaining to volumes need not be memorized:

1. A ton of hay varies in volume from about 450 cu. ft. to 550 cu. ft., depending on the quality and on how much the hay has settled. In our work 500 cu. ft. may be taken as the volume of a ton.

2. The volume of a ton of hard coal may be taken as 35 cu. ft. and that of a ton of soft coal as 42 cu. ft.

3. In brickwork, 22 bricks with the mortar may be considered to occupy a volume of 1 cu. ft.

The following examples show how we may apply the facts given in the above statements:

1. A coal bin 20 ft. long and 16 ft. wide is filled to a depth of 7 ft. with hard coal. How many tons of coal are there in the bin?

Referring to the second statement above, as we always do in such cases, we proceed as here shown and find that the number of tons is 64.

$$\frac{\overset{4}{\cancel{20}} \times 16 \times \cancel{7}}{\underset{\cancel{5}}{\cancel{35}}} = 64$$

2. A mason is to build a wall 25 ft. long, 8 ft. high, and 1 ft. thick. How many thousand bricks will be needed?

Referring to the third statement, we find, as here shown, that 4.4 is the number of thousand bricks needed for the wall.

$$\frac{\cancel{25} \times \cancel{8} \times 1 \times 22}{\underset{\underset{5}{\cancel{40}}}{\cancel{1000}}} = \frac{22}{5} = 4.4$$

The mason would probably say that he needed $4\frac{1}{2}$ M to allow for breakage and because the estimate of 22 bricks to 1 cu. ft. is only approximate. Problems in real life do not often "come out even," and we should learn in school always to use common sense in interpreting a result.

PRACTICAL EQUIVALENTS

1. What is the capacity in bushels of a bin which is 8 ft. by 6 ft. by 4 ft.? which is 6 ft. by 9 ft. by 3 ft.?

2. Using the exact equivalent, what is the capacity in bushels of a box which is 18 in. by 3 ft. by 2 ft.?

3. Using the approximate equivalent, what is the capacity in gallons of a tank which is 5 ft. by 6 ft. by 4 ft.?

4. What is the capacity in gallons of a tank which is $3\frac{1}{2}$ ft. by $6\frac{1}{4}$ ft. by $2\frac{2}{3}$ ft.?

5. A cistern is 6 ft. square and $4\frac{2}{3}$ ft. deep. How many barrels of water will it hold?

6. An iron tank is $7\frac{1}{2}$ ft. long, $6\frac{3}{4}$ ft. wide, and $4\frac{2}{3}$ ft. deep. How many barrels of water will it hold?

Find the number of bushels of grain in a rectangular bin

7. 12 ft. by 16 ft., filled to a depth of 6 ft.

8. 14 ft. by 18 ft., filled to a depth of 9 ft.

9. 16 ft. by 24 ft., filled to a depth of 10 ft.

Find the number of bushels of grain in a freight car

10. $7\frac{1}{2}$ ft. by 28 ft., filled to a depth of 7 ft.

11. $7\frac{1}{2}$ ft. by 30 ft., filled to a depth of $5\frac{1}{2}$ ft.

12. A freight car 8 ft. by $26\frac{3}{4}$ ft. is filled with corn to a depth of 6 ft. At 75¢ per bushel, find the value of the corn.

13. How many tons of hay are there in a stack containing 4750 cu. ft.? in a mow containing 8250 cu. ft.?

14. Find the number of tons of hard coal that can be put into a bin which is 21 ft. long, and 15 ft. wide, and 6 ft. high.

15. A coal bin is 21 ft. long, 13 ft. wide, and 8 ft. deep. How many tons of soft coal will it hold?

16. A coal bin is 14 ft. long, 8½ ft. wide, and 6¼ ft. deep. How many tons of soft coal will it hold?

17. A haymow is 22 ft. long, 14 ft. wide, and 9 ft. high. How many tons of hay will it hold?

18. A barn has two haymows, each 24 ft. long, 15 ft. 6 in. wide, and 10 ft. high. How many tons of hay will each mow hold? How many tons will the barn hold?

19. How many cords of 4-foot wood (128 cu. ft.) are there in a pile 4 ft. wide, 8 ft. high, and 32½ ft. long?

20. How many cords of 4-foot wood are there in a pile 4 ft. wide, 6½ ft. high, and 44 ft. long?

21. A schoolroom is 30 ft. long, 23 ft. wide, and 12 ft. high. If an average of 200 cu. ft. is allowed for each pupil, as is often done, what is the greatest number of pupils that should be assigned to this room?

22. A gasoline storage tank holds 82,500 gal. What is the capacity of the tank in cubic feet?

23. An aquarium contains 3.8 cu. ft. How many gallons of water will it hold when full?

24. A tank is 18 in. long, 9¼ in. wide, and 10⅛ in. deep. How many gallons of water can be put into the tank?

25. The volume of a can is found to be 1732.5 cu. in. If the can is filled with milk, what is the value, at 12¢ a quart, of the contents of the can?

26. A wall is 32½ ft. long, 2 ft. thick, and 32½ ft. high. How many cubic feet of masonry does it contain?

27. A brick wall is to be built 42 ft. long, 1½ ft. thick, and 6¼ ft. high. How many bricks will be needed?

28. If possible, measure your school lot and find the number of bricks needed for a 6-foot wall around it.

SHIPPING FRUIT

1. The standard box for shipping Florida oranges is 12 in. by 12 in. by 33 in. How many cubic inches does it contain?

2. In finding the volume of a New Jersey bushel cranberry box, which is $8\frac{3}{4}$ in. by 12 in. by 22 in., a pupil wrote:

$$8\tfrac{3}{4} \times 12 \times 22 = \frac{35 \times \overset{3}{\cancel{12}} \times 22}{\cancel{4}} = 2310 \text{ (cu. in.)}.$$

How does this box differ from the standard bushel?

3. In shipping Georgia peaches a crate 8 in. by $12\frac{1}{2}$ in. by 22 in. is used. This crate contains how much more than 1 bu.?

4. California apples and pears for local use are shipped in boxes which are 10 in. by 22 in. by 12 in. By how much does one of these boxes differ from the standard bushel?

5. In California the orange box is sometimes $11\frac{1}{4}$ in. by $26\frac{1}{2}$ in. by $11\frac{1}{4}$ in. and sometimes $7\frac{3}{4}$ in. by 22 in. by $17\frac{1}{2}$ in. Which box holds the more, and how much more does it hold?

6. Dried fruits are shipped in 50-pound or 25-pound boxes. The 50-pound box is 9 in. by $15\frac{1}{4}$ in. by 9 in. Find the volume.

7. The 25-pound box mentioned in Ex. 6 is sometimes $9\frac{3}{4}$ in. by $13\frac{3}{4}$ in. by $5\frac{3}{4}$ in., and sometimes 9 in. by 16 in. by 5 in. Which box holds the more, and how much more does it hold?

8. For shipping fruit out of the state the sizes of some of the boxes used in California are as follows:

Plums	$11\frac{3}{4}$ in. by $18\frac{1}{2}$ in. by 4 in.
Cherries	$10\frac{3}{8}$ in. by $16\frac{1}{2}$ in. by $2\frac{1}{2}$ in.
Pears	$11\frac{3}{4}$ in. by $18\frac{1}{2}$ in. by $8\frac{1}{2}$ in.
Peaches	$11\frac{3}{4}$ in. by $18\frac{1}{2}$ in. by $4\frac{7}{8}$ in.

Find the capacity in cubic inches of each of these boxes.

Other Problems Relating to Volumes. Suppose that a storage bin for potatoes is built into the ground. We can measure the length and width; but if the bin is full of potatoes, we cannot conveniently measure the depth. If we know the capacity of the bin, however, we can compute the depth.

That is, in such problems we are given the volume and certain measurements and have to find other measurements. Although not so common as those relating to the finding of volumes, such problems have many uses. The method of solving these problems is illustrated by the following cases:

1. If a box 5 ft. long and 4 ft. wide contains 60 cu. ft., how deep is it?

Since the box is 5 ft. long and 4 ft. wide, the area of the base is 5 × 4 sq. ft., or 20 sq. ft.

If the box were 1 ft. deep, the volume would be 20 cu. ft. Therefore the required depth must be as many times 1 ft. as 20 cu. ft. is contained in 60 cu. ft., or 3. That is, the box is 3 ft. deep.

$$\boxed{\frac{\overset{\overset{3}{\cancel{12}}}{\cancel{60}}}{\cancel{5}\times\cancel{4}} = 3}$$

2. If a block $2\frac{1}{2}$ in. high contains 70 cu. in., what is the area of the base?

If the block were 1 in. high, the base would contain 70 sq. in., and so the required area of the base is 70 sq. in. ÷ $2\frac{1}{2}$; that is, as here shown, it is 28 sq. in.

$$\boxed{70 \div 2\tfrac{1}{2} = \frac{\overset{14}{\cancel{70}} \times 2}{\cancel{5}} = 28}$$

3. How thick is a block which has a volume of 100 cu. in. and a base with an area of 20 sq. in.?

If the block were 1 in. thick, it would contain 20 cu. in. Hence the required thickness is as many times 1 in. as 20 cu. in. is contained in 100 cu. in., or 5; that is, it is 5 in.

$$\boxed{\begin{array}{r} 5 \\ 2\cancel{0}\overline{)10\cancel{0}} \end{array}}$$

PROBLEMS RELATING TO VOLUMES

1. In planning to place a rectangular tank, which is to hold 10 bbl., in the attic of his house, a man estimates that the tank must contain 42 cu. ft. If he can make the tank only 3 ft. 6 in. high on account of the roof, what is the smallest area that the base of the tank will cover?

2. A rectangular tank with base 9 ft. by $16\frac{2}{3}$ ft. contains 900 cu. ft. How deep is the tank?

3. A box with base 13 in. by 20 in. contains 3900 cu. in. How deep is the box?

4. A rectangular tank 4 ft. deep contains 450 cu. ft. What is the area of the base of the tank?

5. A hall 22 ft. high and 30 ft. wide contains 28,600 cu. ft. Find the length of the hall.

6. The area of the base of a block containing 700 cu. ft. is 56 sq. ft. How high is the block?

7. A hall which contains 41,341.3 cu. ft. is 41.3 ft. long and 36.4 ft. wide. How high is the hall?

8. The area of the bottom of a box containing 42,159 cu. in. is 1081 sq. in. How deep is the box?

9. A block of granite is 14 in. long and 8.3 in. wide, and contains 836.64 cu. in. How thick is the block?

10. The area of the floor of an assembly hall which contains 15,246 cu. ft. is 1386 sq. ft. How high is the hall?

11. A storeroom 14.3 ft. long and 13 ft. 6 in. wide has a capacity of 2316.6 cu. ft. How high is the room?

12. The floor of a square bedroom contains 144 sq. ft., and the room contains 1368 cu. ft. Find the length, width, and height of the room, first finding the number which multiplied by itself gives 144.

USING WHAT YOU HAVE LEARNED

1. How many thousand bricks, each 4 in. by 8 in. by 2 in., can be packed in a kiln 25 ft. by 60 ft. by 9 ft.?

2. A rectangular pile of cut stones is 27 ft. long, 5 ft. wide, and 6 ft. high. Find the value of the pile at $6.40 a cord, a cord of stone having the same volume (128 cu. ft.) as a cord of 4-foot wood.

3. Express in gallons the capacity of a tank which is 6 ft. long, 4 ft. wide, and 4 ft. deep.

4. If it takes 550 cu. ft. of clover hay to make a ton, how many tons of clover hay can be put into a mow 32 ft. long, half as wide as it is long, and half as deep as it is wide?

5. How many gallons of water are there in a watering trough that is 6 ft. long, 16 in. wide, and filled with water to a depth of three fourths of the width?

6. How many bars of soap, each 4 in. long, $2\frac{3}{4}$ in. wide, and $1\frac{1}{2}$ in. thick, can be packed in a box 16 in. long, $13\frac{3}{4}$ in. wide, and 1 ft. deep? Will the bars completely fill the box?

7. When Arthur visited his uncle, who owned a large farm, he told his uncle about the arithmetic he was studying. His uncle told him that in the barn there was an oat bin 8 ft. long and 5 ft. wide, and asked Arthur to find how many bushels of oats there were in the bin. Arthur took a stick and measured the depth of the oats, finding it to be 3 ft. 8 in. If he allowed $1\frac{1}{4}$ cu. ft. to the bushel, what answer should he get?

8. Express in bushels the capacity of a bin which is 4 ft. by 6 ft. by 3 ft. $6\frac{1}{2}$ in.

9. A bin 21 ft. 6 in. long and 10 ft. wide is filled with wheat to the depth of 5 ft. Allowing 2150 cu. in. to the bushel, what is the value of the wheat at $1.20 a bushel?

Capacity and Weight. The New York State laws specify that a bushel of each of various grains and vegetables shall weigh a certain amount. The following table of typical weights need not be memorized, but should be used for reference:

Apples	48 lb.	Onions	57 lb.
Barley	48 lb.	Peas	60 lb.
Beans	60 lb.	Potatoes, sweet	54 lb.
Buckwheat	48 lb.	Potatoes, white	60 lb.
Corn, shelled	56 lb.	Rye	56 lb.
Oats	32 lb.	Wheat	60 lb.

CAPACITY AND WEIGHT

1. How many bushels of beans are there to a ton?

2. How many bushels of onions are there to a ton?

3. How many bushels are there in a ton of shelled corn?

Find the number of bushels to a ton of

4. Oats.

5. Barley.

6. Buckwheat.

7. Peas.

8. White potatoes.

9. Sweet potatoes.

10. A bushel of rye weighs how many times as much as a bushel of oats?

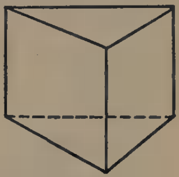
11. A bushel of sweet potatoes is what part as heavy as a bushel of white potatoes?

12. A bushel of buckwheat is what part as heavy as a bushel of shelled corn?

13. If a farmer sells 15 T. of white potatoes to a starch factory, how many bushels does he sell?

14. Find the weight in tons of 1000 bu. of wheat; 1000 bu. of oats; 1000 bu. of rye; 1000 bu. of shelled corn.

Prism. In the rectangular solids which we have been studying, the base was either a square or a rectangle. In the solid here shown the base and the face opposite and parallel to it are triangles which are exactly alike, and the other faces are rectangles. A solid like this is called a *prism,* and this particular one is called a *triangular prism.* The distance between the two parallel faces is called the *height,* or *altitude,* of the prism.

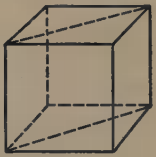


As long as they are exactly alike, the base and opposite face of a prism may have any number of sides greater than two. Thus, a rectangular solid, such as a crayon box, may be called a *rectangular prism.*

Volume of a Prism. Contractors, builders, and mechanics frequently have to find the volumes of prisms of various kinds.

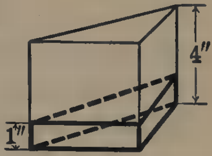
In learning how to find the volume of a prism, it will help us first to consider the following questions:

If the kind of prism here shown, which, as we know, is a cube, is 4 in. on an edge, what is the area of the base?



The figure below represents a triangular prism which is half of the cube above. What is the area of the base of this half? What is the volume of the section, 1 in. high, which is indicated by the heavy lines? What is the volume of the entire prism?

We have thus found the volume of a triangular prism. The volume of any other kind of prism is found in the same way; that is, the volume of a prism is the product of the area of the base and the height, or, briefly stated,



Volume = area of base × height.

VOLUME OF A PRISM

1. A part for a machine is cast in the form of a prism whose cross section is a trapezoid. The cross-section area is 26 sq. in., and the length of the part is 3′ 2″. Sketch the figure roughly and then find the volume of the part.

2. At 490 lb. per cubic foot, find the weight of the machine part described in Ex. 1.

Find the volume of a prism when the area of the base and the height are as follows:

3. 375 sq. in., 29 in.
4. 48 sq. in., 3 in.
5. 62 sq. ft., 3 ft.
6. $1\frac{1}{2}$ sq. ft., 4 in.
7. $64\frac{1}{2}$ sq. in., 8 in.
8. 62.8 sq. in., 3.1 in.

Find the height of a prism when the volume and the area of the base are as follows:

9. 243 cu. in., 29 sq. in.
10. 140 cu. in., $17\frac{1}{2}$ sq. in.
11. 783 cu. in., 87 sq. in.
12. $178\frac{1}{4}$ cu. in., $19\frac{1}{4}$ sq. in.

Find the volumes of rectangular prisms (rectangular solids) with dimensions as follows:

13. 42 in. long by $6\frac{1}{2}$ in. wide by $3\frac{1}{2}$ in. high.

14. $3\frac{1}{2}$ in. wide by $3\frac{3}{4}$ in. long by 8 in. high.

15. $6\frac{1}{2}$ ft. long by $4\frac{1}{2}$ ft. wide by 4 ft. high.

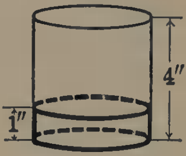
16. 21.3 in. wide by 29.2 in. long by 3.7 in. high.

17. 33.7 in. wide by 36.3 in. long by 12.4 in. high.

18. A rectangular solid with a base $12\frac{1}{2}$ in. square is twice as high as it is wide. Find its volume.

19. A prism has a square base, a volume of 1100 cu. in., and a height of 11 in. Find the lengths of the edges of the base.

Volume of a Cylinder. We all know what a *cylinder* is. An ordinary tin can is a good illustration. For example, suppose that we have a tomato can in which the area of the base is 12 sq. in. If, as here shown, we first consider a section 1 in. high, we have a cylinder 1 in. high with a volume of 12 cu. in. Then if the can is 4 in. high, its volume is 4 × 12 cu. in., or 48 cu. in. That is, just as in the case of the prism, the volume of a cylinder is the product of the area of the base and the height.



Since the common cylinder has a circular base, the area of the base is $\frac{22}{7}$ times the square of the radius. Therefore, in such a cylinder we have

$$\textit{Volume} = \tfrac{22}{7} \times \textit{square of the radius} \times \textit{height}.$$

VOLUME OF A CYLINDER

1. Find the volume of a cylinder whose base has an area of 600 sq. ft. and whose height is 3 ft. 6 in.

2. A cylindric water tank is 40 ft. high and 40 ft. in diameter. What is the radius of the tank (the radius of the base)? What is the capacity of the tank in cubic feet?

3. How many cubic feet are there in a cylindric stone shaft whose height is 16 ft. and whose diameter is 1 ft. 11 in.?

4. How many gallons (1 cu. ft. = $7\frac{1}{2}$ gal.) does a cylindric tank hold if the height is 32 ft. and the diameter is 30 ft.?

5. Find the volume of a boiler 15 ft. long, with an inside diameter of 3 ft. 6 in.

6. Find the number of cubic inches in a water pipe of which the length is 18 ft., the outside diameter is 4 in., and the thickness of the metal is $\frac{1}{4}$ in.

7. The boys in the picture are measuring the circumference and the height of the water tank. They find them to be 44 ft. and 20 ft. respectively. Find the diameter of the tank in feet, the volume in cubic feet, and the capacity in gallons.

Find the volume of a cylinder when the area of the base and the height are respectively

8. 9 sq. in. and 4 in.

9. 8 sq. ft. and 9 ft.

10. 27 sq. in. and 13 in.

11. $6\frac{1}{2}$ sq. in. and 3 in.

12. 7 sq. in. and $2\frac{1}{2}$ in.

13. 88 sq. in. and 15 in.

14. $16\frac{1}{2}$ sq. in. and $3\frac{1}{2}$ in.

15. $18\frac{2}{3}$ sq. ft. and 27 in.

16. $48\frac{1}{2}$ sq. in. and 43 in.

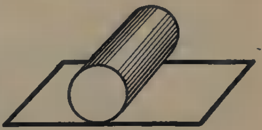
17. 62.2 sq. in. and 7.8 in.

18. Find the height of a cylinder when the volume and the area of the base are respectively $22\frac{1}{2}$ cu. ft. and $7\frac{1}{2}$ sq. ft.; when they are 197.6 cu. ft. and 30.4 sq. ft.

19. Find the area of the base of a cylinder when the volume and the height are respectively 324 cu. ft. and 18 ft.; when they are 33.15 cu. ft. and 2 ft. 2 in.

20. Find the volume of a cylinder when the height and the radius of the base are respectively 27 in. and 4 in.; when they are 32 in. and 8 in.; when they are 2 ft. 6 in. and 2 in.

Curve Surface of a Cylinder. If we should slit the curve surface of a cylinder and spread it out flat, what would the surface become? Try this yourself with a paper cylinder.



How can we find the area of this surface? What is the area of a rectangle 8 in. long and 6 in. wide? What is the area of the curve surface of a cylinder with a circumference of 8 in. and a height of 6 in.? We thus see that the area of the curve surface of a cylinder is the product of the circumference and the height.

Since the circumference of the common cylinder is $\frac{22}{7}$ times the diameter, in such a cylinder we have

$$\textit{Area of curve surface} = \tfrac{22}{7} \times \textit{diameter} \times \textit{height}.$$

SURFACE OF A CYLINDER

Numbers 1 to 12, oral

State the areas of the curve surfaces of cylinders with heights and circumferences as follows:

1. 6 in., 10 in.
2. 60 in., 70 in.
3. 25 in., 40 in.
4. 50 in., 100 in.
5. 40 in., 15 in.
6. 20 in., 20 in.
7. 25 in., 30 in.
8. 60 in., 50 in.
9. 66 in., 48 in.
10. $12\frac{1}{2}$ in., 24 in.
11. $16\frac{2}{3}$ in., 36 in.
12. $33\frac{1}{3}$ in., 33 in.

13. How much sheet iron is needed for a stovepipe 8 ft. long and 6 in. in diameter, allowing 1 sq. ft. for overlapping?

14. Solve Ex. 13, supposing that the length is 6 ft. and the diameter is 8 in.

15. A tin cup is to be 3 in. in diameter and 3 in. high. Allowing 1 sq. in. for overlapping, how many square inches of tin are needed for the curve surface?

MEASURING CYLINDERS

1. Write a statement telling how you find the circumference and the area of a circle when you know the radius.

2. How many square inches are there in the curve surface of a wire 60 ft. long and $\frac{7}{8}$ in. in circumference?

3. If the height and diameter of a solid cylinder are each 8 in., what is the area of the entire surface; that is, what is the area of the curve surface plus the areas of the two circles which form the bases?

4. Allowing 7 sq. in. for waste, how many square inches of tin are needed to make a cylindric cup which shall be 4 in. high and 4 in. in diameter?

5. How many square feet are there in the curve surface of a water tank that is 40 ft. high and 127.6 ft. in circumference?

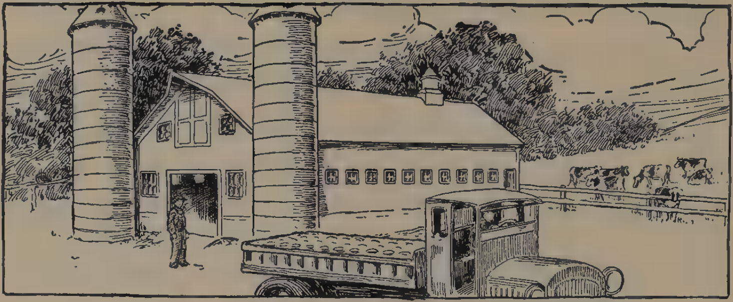
6. How large a surface is covered in painting a smokestack 30 ft. high and $2\frac{1}{2}$ ft. in diameter, outside measurements?

7. In polishing the curve surface of a granite cylindric shaft 3 ft. 6 in. in diameter and 22 ft. 3 in. long, how many square feet of surface are polished?

8. One of the rooms in a factory is heated by 210 ft. of steam pipe 2 in. in diameter. Find the radiating surface; that is, find the area of the curve surface which gives out the heat.

9. A suspension bridge has four cables, each 1872 ft. long and 1 ft. 2 in. in diameter. In order to bid on a contract for painting the bridge, a contractor had to know the surface area of these cables. Find this area.

10. In making metal bedsteads a factory uses iron rods 0.7 in. in diameter covered with thin rolled brass. In filling an order for 6000 ft. of these rods, how many square feet of rolled brass will be needed, not allowing for waste?



PROBLEMS ABOUT SILOS

1. A farmer finds that to feed a herd of ten cows for 6 mo. he needs a cylindric silo 10 ft. in diameter and 25 ft. high. How many cubic feet of silage does such a silo hold?

2. For a herd twice as large as in Ex. 1 a cylindric silo 12 ft. in diameter and 32 ft. high is needed. Has this twice the volume of the other silo? If not, how much does it lack?

3. What would the height of the first silo mentioned in Ex. 2 have to be in order to have twice the volume of the other? What effect would the packing down of the silage, due to the increased height, have upon the amount that could be put into the silo? Can you see why a capacity twice that in Ex. 1 is not needed?

4. The internal diameter of a silo is 12 ft. and the height is 25 ft. How many cubic feet does the silo hold?

5. The outside diameter of a silo is 14 ft. 7 in. Allowing 8 in. on each hoop for joining, how many feet of band iron are needed to make 12 hoops for the silo?

6. How many feet, board measure, of 1-inch lumber will it take to line the inner wall of a silo that is 21 ft. in diameter and 28 ft. high?

7. Ten years ago it cost about $3.10 per ton of capacity to build concrete silos that would hold about 75 T. How much did it cost then to build a concrete silo holding 71 T.? If the cost today has increased 80%, what is now the cost of building such a silo?

8. At $1.90 per square yard, how much will it cost to put a cement floor in a silo that is 21 ft. in diameter?

9. How many cubic feet of silage can be put into a silo that is 17 ft. 6 in. in diameter and 20 ft. high?

10. At 85¢ per square yard, how much will it cost to paint the curve surface of a silo that is 25 ft. high and 18 ft. 6 in. in diameter, outside measurements?

11. While corn silage weighs from 25 lb. to 50 lb. per cubic foot, depending upon the place in the silo from which it is taken and upon the amount of moisture it contains, a fair average is 40 lb. Allowing 40 lb. to the cubic foot, how many tons of silage can be put into a silo that is 18 ft. 8 in. in diameter and 32 ft. high?

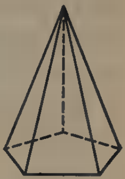
12. Allowing 40 lb. daily per cow, how many tons of silage will be needed for a herd of 32 cows for 188 da.?

13. Find the total number of tons of silage that are needed, during a period of 180 da., in order to feed 32 cows 50 lb. daily per cow, 16 cows 40 lb. daily per cow, and 22 calves 16 lb. daily per calf.

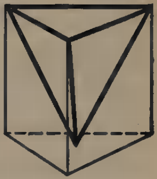
14. A silo that is 40 ft. high and 10 ft. in diameter will hold approximately 70 T. of silage. How many tons can be put into a silo 40 ft. high and 20 ft. in diameter?

15. A silo 40 ft. high and 10 ft. in diameter will hold silage enough for 15 dairy cows for 240 da. How many dairy cows can be fed for 240 da. from a silo of the same height and of twice the diameter?

Pyramid. Another solid, about which it is interesting to know, has the shape shown in this figure. The base of this solid, which is called a *pyramid*, may have three or more sides, and the other faces are triangles which meet in a point. This point is called the *vertex* of the pyramid, and the distance from the vertex to the base is called the *height*, or *altitude*, of the pyramid.



Volume of a Pyramid. Suppose that we have a hollow prism and a hollow pyramid of the same base and the same height. If we fill the prism with water and then pour the water into the pyramid, we shall find that the pyramid can be exactly filled three times with the water that fills the prism, — a relationship that is suggested by the figure at the right. That is, from our study of the prism we see that the volume of a pyramid is one third the product of the area of the base and the height, or, briefly stated,



$$Volume = \frac{1}{3} \times area\ of\ base \times height.$$

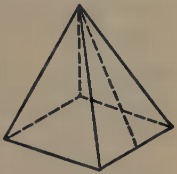
VOLUME OF A PYRAMID

Find the volume of a pyramid when the area of the base and the height are respectively

1. 36 sq. in. and 7 in.
2. 187 sq. ft. and 10 ft.
3. 202 sq. ft. and 8 ft.
4. 89.6 sq. in. and 12.8 in.
5. 400 sq. in. and 30 in.
6. 129.3 sq. in. and 3.7 in.
7. 702.3 sq. in. and 16.8 in.
8. 140.4 sq. in. and 23.4 in.

9. The Great Pyramid in Egypt was originally 482 ft. high and had a square base 768 ft. on a side. What was the original volume? the weight in tons at 168 lb. per cubic foot?

Lateral Area of a Pyramid. The problem of finding the volume of a pyramid is one that arises more often in practical work than that of finding the area of its surface. For our purposes we shall consider only the kind of pyramid in which the vertex is exactly over the center of an equal-sided and equal-angled base, the so-called *regular pyramid,* illustrated here and on page 234.

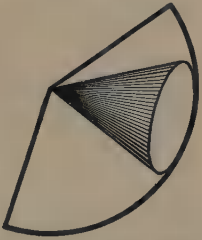


The altitude of any triangular face of this kind of pyramid is called the *slant height,* and the sum of the areas of the triangular faces is called the *lateral area* of the pyramid.

Since the area of each triangular face is half the product of its base and height, we see that the lateral area of the pyramid is half the product of the perimeter of the base and the slant height, or, briefly stated,

Lateral area $= \frac{1}{2} \times$ *perimeter of base* $\times$ *height.*

Cone. The solid called the *cone* is so familiar to every one of us as to need no definition. As in the case of the pyramid, we shall consider only the kind of cone illustrated here and on page 236.



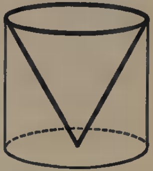
If we slit the curve surface of such a cone and flatten it out, as here shown, it is evident that we have part of a circle. From our study of the pyramid, we may infer that the area of this curve surface is half the product of the circumference of the base and the slant height, or, briefly stated,

Area of curve surface $= \frac{1}{2} \times$ *circumference* $\times$ *slant height.*

Thus, in a cone in which the circumference of the base is 15 in. and the slant height is 4 in., we have

$\frac{1}{2} \times 15 \times 4$ sq. in. $= 30$ sq. in., the area of the curve surface.

Volume of a Cone. The cone has the same relation to the cylinder that the pyramid has to the prism (page 234); that is, the volume of a cone is one third the product of the area of the base and the height.



As in the case of the cylinder, we shall consider only the kind of cone which has a circular base. Then, since the area of this base is $\frac{22}{7}\times$ the square of the radius, in finding the volume of a cone we have

$$Volume = \tfrac{1}{3} \times \tfrac{22}{7} \times \textit{square of the radius} \times \textit{height}.$$

Thus, for a cone whose height is 8 in. and whose base has a radius of 2 in., we have

$$\tfrac{1}{3} \times \tfrac{22}{7} \times 2 \times 2 \times 8 \text{ cu. in.} = 33\tfrac{11}{21} \text{ cu. in., the volume.}$$

Such a result is better given decimally as 33.5 cu. in.

PYRAMIDS AND CONES

In each case find the lateral area of a pyramid when the perimeter of the base and the slant height are as follows:

1. 24 in., 16 in. **2.** 4 ft. 3 in., 9 in. **3.** 4 ft. 9 in., 12 in.

In each case find the area of the curve surface of a cone when the circumference of the base and the slant height are as follows:

4. $15\frac{3}{4}$ in., $4\frac{3}{4}$ in. **5.** 7 ft. 6 in., 4 ft. **6.** 3 ft. 6 in., 15 in.

In each case find the volume of a cone when the radius of the base and the height are as follows:

7. 15.4 in., 6 in. **8.** 6.3 in., 15 in. **9.** 42 in., 18 in.

10. In the picture at the top of the page, suppose that the cylinder is 14 in. high and 14 in. in diameter. How many cubic inches are there inside the cylinder that are not occupied by the cone?

11. How much slating is used on a pyramidal steeple whose slant height is 40 ft. and the perimeter of whose base is 48 ft.?

12. A dome is surmounted by a gilded cone whose slant height is 16 ft. 8 in. and the radius of whose base is 3 ft. 6 in. How many square feet must be gilded?

13. A pyramid with a lateral area of 400 sq. in. has a slant height of 10 in. Find the perimeter of the base.

14. A conic spire on a church has a slant height of 68 ft., and the circumference of the base is 60 ft. What is the area of the curve surface?

15. What is the area of the entire surface of a cone whose slant height is 7 ft. and whose base is 7 ft. in diameter?

16. A circular cornucopia is $3\frac{1}{8}$ in. across and $8\frac{3}{4}$ in. deep. How many cubic inches does it contain?

17. A conical strainer is 5 in. across the top and the slant height is 6 in. Find the area of the curve surface.

18. What is the lateral area of a pyramid with a slant height of 10 in. and a square base 2 in. on a side?

19. What is the volume of a pyramid with a height of 12 in. and a square base 4 in. on a side?

20. The diameter of the base of a cone is 14 in. and the slant height is 20 in. Find the area of the curve surface.

21. Find the volume of a cone whose base has an area of 16 sq. in. and whose height is 8 in.

22. Find the volume of a cone whose base has a diameter of 28 in. and whose height is 12 in.

23. In Ex. 22 find the volume of a cone of twice the diameter.

24. The Great Pyramid originally had a slant height of 616 ft. and a square base which was 768 ft. on a side. Find the original lateral area.

Area and Volume of a Sphere. The last solid that we shall consider is the *sphere*. If we wind half the surface of a sphere with a cord, and then wind this length of cord around a cylinder whose height and diameter are each equal to the diameter of the sphere, we shall find that the cord covers half the curve surface of the cylinder.

As we already know, the area of the curve surface of the cylinder is $\frac{22}{7} \times$ diameter $\times$ height, and the height of the cylinder is twice the radius of the sphere. Hence the area of the surface of the sphere is $\frac{22}{7} \times 2 \times$ radius $\times 2 \times$ radius, or, writing "square of the radius" for "radius $\times$ radius,"

$$\textit{Area} = 4 \times \tfrac{22}{7} \times \textit{square of the radius.}$$

For the volume of a sphere it is proved in geometry that

$$\textit{Volume} = \tfrac{4}{3} \times \tfrac{22}{7} \times \textit{the cube of the radius,}$$

where "cube of the radius" means "radius $\times$ radius $\times$ radius."

Thus, in finding the area of the surface and the volume of a ball which has a radius of 4 in., we have

$$4 \times \tfrac{22}{7} \times 4 \times 4 \text{ sq. in.} = 201\tfrac{1}{7} \text{ sq. in., the area;}$$

and $\frac{4}{3} \times \frac{22}{7} \times 4 \times 4 \times 4$ cu. in. $= 268\frac{4}{21}$ cu. in., the volume.

That is, the area of the surface is about 201.1 sq. in., and the volume is about 268.2 cu. in.

AREAS AND VOLUMES OF SPHERES

1. If a sphere has a radius of 7 in., what is the area of the surface of the sphere?

2. Find the area of the surface of a hemispherical dome which has a diameter of 49 ft.

3. The sun's diameter is 866,500 mi. Find the area of the sun's surface to the nearest 1000 sq. mi.

4. Given that 1 cu. ft. of granite weighs 165 lb., find the weight of a sphere of granite 1 ft. in diameter.

5. A bowl is in the form of a hemisphere which is 4.2 in. in diameter. How many cubic inches does it contain?

6. Marble is 2.7 times as heavy as water, and 1 cu. ft. of water weighs 1000 oz. Find the weight of a sphere of marble 3 ft. in circumference.

7. Lead is 11.35 times as heavy as water, and 1 cu. ft. of water weighs 1000 oz. Find, to the nearest ounce, the weight of a lead ball $1\frac{1}{2}$ in. in diameter.

8. Taking the earth as an exact sphere with a radius of 4000 mi., find the volume to the nearest 1000 cu. mi.

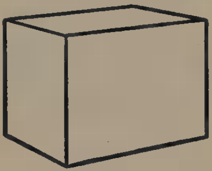
9. Considering the moon as a sphere 2160 mi. in diameter, find the area of its surface to the nearest 1000 sq. mi. and the volume to the nearest 1000 cu. mi.

10. A ball with a diameter of 6 in. is placed in a cubic box which is 6 in. on an edge, inside measurements. How many cubic inches inside the box are not occupied by the ball?

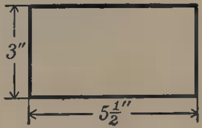
11. Find the volume of a sphere 1 ft. in diameter, of a cylinder of the same diameter and height, and of a cone of the same diameter and height. The volume of the sphere is how many times the volume of the cone? The volume of the cylinder is how many times the volume of the cone?

MEASURING SOLIDS FROM WORKING DRAWINGS

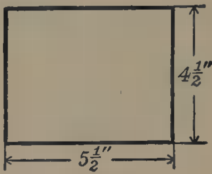
1. Below is shown the kind of drawing which a workman might use in proceeding to cut out a rectangular block of the size given. Find the volume of the solid represented.



The Block



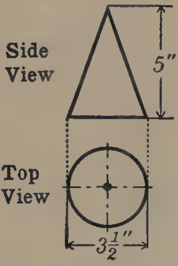
Top View



Side View

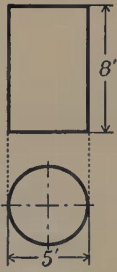
Find the volume of each of the solids represented below:

2.



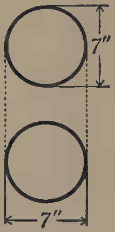
Cone

3.



Cylinder

4.

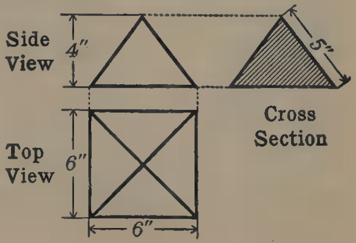


Sphere

5. This figure represents a pyramid with a square base. Find the area of the base.

6. In Ex. 5 find the volume of the pyramid.

7. In Ex. 5 find the area of each of the four triangular faces and the area of the entire surface of the pyramid.



REVIEW OF THE RULES OF MEASUREMENT

Copy and complete the following statements:

1. The fraction $\frac{22}{7}$ is the number by which we multiply the —— of a circle to obtain the ——.

2. The diameter of a circle is $\frac{7}{22}$ times the ——.

3. The area of a circle is —— times the square of the radius.

4. The area of a —— is half the base times the height.

5. The area of a rectangle is the —— times the ——.

6. The volume of a rectangular solid is the —— times the —— times the ——.

7. The volume of a prism is the —— of the —— times the ——.

8. The —— of a circular cylinder is $\frac{22}{7}$ times the —— of the —— times the height.

9. The area of the curve surface of a circular cylinder is $\frac{22}{7}$ times the —— times the height.

10. The —— of a pyramid or a cone is $\frac{1}{3}$ the —— of the —— times the ——.

11. The area of a sphere is 4 times $\frac{22}{7}$ times the —— of the ——.

12. The volume of a sphere is $\frac{4}{3}$ times —— times the —— of the radius.

13. If we multiply the diameter of a circle by $\frac{22}{7}$, the result is the ——.

14. If we multiply the square of the radius of a circle by ——, the result is the ——.

15. The volume of a cube whose edge is 7 in. is ——.

16. The volume of a sphere whose radius is 7 in. is ——.

REVIEW OF MEASUREMENTS

1. A steamfitter cuts off 68 in. of pipe. Express this as feet and inches; as feet and a fraction; as yards and inches.

2. A milk dealer sells 465 qt. of milk. Express this as gallons and quarts; as gallons and a fraction.

3. A milk dealer sells 85 pt. of cream. Express this as quarts and pints; as quarts and a fraction; as gallons and pints; as gallons and a fraction.

4. If you cut off $\frac{7}{8}$ of a piece of tin that contains 5 sq. ft., how many square inches do you cut off?

5. Counting 64 sheets to a pound, what is the weight of 5000 sheets of paper?

6. If evaporation from a tank is estimated at $8\frac{1}{3}\%$ in a week, how many gallons will be lost in a week from an open tank which contains 4800 gal.?

7. What is the reduction in length of a 240-yard roll of cloth that loses $2\frac{1}{2}\%$ of its length by shrinkage?

8. A sign in a loft says that the maximum weight should not exceed 60 lb. per square foot. If the loft floor is $48' \times 62'$, and if there are already $35\frac{1}{4}$ T. of merchandise on the floor, how much more weight may be safely added?

9. While sailing out to the fishing grounds, Albert heard the skipper say that the water at that point was 36 fathoms deep. Taking the fathom as 6 ft., how many feet is this? How many yards?

10. How many feet are there in 42 fathoms?

11. How many fathoms are there in 42 ft.?

12. Which of the years 1975, 1988, 2000, and 2004 will be leap years? Write the reason for each of your answers.

13. Find to the nearest 100 cu. yd. the amount of earth which must be removed in digging a canal $2\frac{3}{8}$ mi. long, 200 ft. wide, and 16 ft. deep.

14. Taking the weight of 1 cu. ft. of marble as 173 lb., find the weight of a cylindric marble column that is 12 ft. high and 14 in. in diameter.

15. A prism 12 in. high has a base whose area is 154 sq. in. How does the volume compare with that of a cone which is 12 in. high and whose base has a radius of 7 in.?

16. The amount of water that flows over Niagara Falls averages 264,000 cu. ft. per second. Find the average number of gallons carried over the falls per minute.

17. To the nearest 100 cu. yd., how many cubic yards of gravel are required to grade $2\frac{3}{4}$ mi. of road if the gravel is laid 6 in. deep over a width of 24 ft.?

18. How many boards, each 12 ft. long, are required to build a four-board fence about a rectangular field which is 40 rd. wide and contains 20 A.?

19. A steel rod 6 ft. 10 in. long has a triangular cross section whose area is 3 sq. in. Given that 1 cu. ft. of water weighs 62.5 lb., and that steel is 7.83 times as heavy as water, find to the nearest pound the weight of the rod.

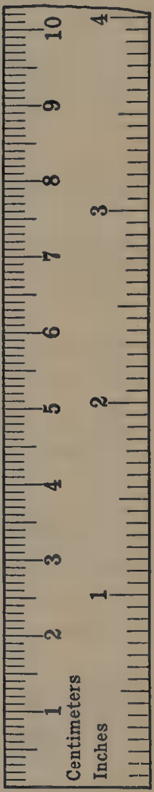
20. Find the circumference of a circle whose diameter is 22.6 in. by multiplying the diameter by 3.1416; by $3\frac{1}{7}$; by $\frac{355}{113}$. Compare the results, stating which is the largest and how much larger it is than each of the others.

21. Find the diameter of a circle whose circumference is 471.24 in. by dividing the circumference by 3.1416; by $3\frac{1}{7}$; by $\frac{355}{113}$. Compare the results.

22. What effect does doubling both the diameter and the height have upon the volume of a cylinder?

A SILENT READING LESSON

In place of our common system of measures many foreign countries use what is known as the *metric system*. We often use this system in this country, as when we speak of 450-meter wave lengths in connection with radio.



The practical part of the system is very easily learned, and since it is used in all kinds of scientific work, everyone ought to know something about it.

The principal units of this system are the *meter*, which is equivalent to about $3\frac{1}{4}$ ft. or 1.1 yd.; the *kilogram*, which is equivalent to about 2.2 lb.; and the *liter*, which is equivalent to about 1 qt. In measuring long distances the unit used is the *kilometer*, which is 1000 meters and is equivalent to about 1100 yd., or nearly $\frac{5}{8}$ mi.

Just as 1 mill is $\frac{1}{1000}$ of a dollar,

so 1 millimeter is $\frac{1}{1000}$ of a meter,

and just as 1 cent is $\frac{1}{100}$ of a dollar,

so 1 centimeter is $\frac{1}{100}$ of a meter.

If we know these facts, we know about all of the metric system that we shall really need for our ordinary reading. Then, when we read about a 400-meter wave length, we shall think of a wave that is about 440 yd. long, and when we read about a 10-kilometer race, we shall think of it as a 6-mile race.

One great advantage of the metric system is that, like our system of money, it runs by 10's, 100's, 1000's, and so on.

Metric Length. The part of the table of metric length that we shall be most likely to need is as follows:

1 millimeter (mm.) = 0.001 meter (m.)
1 centimeter (cm.) = 0.01 meter
1 decimeter (dm.) = 0.1 meter
1 kilometer (km.) = 1000 meters

The meter is equivalent to 39.37 in., or about $3\frac{1}{4}$ ft., or 1.1 yd., and the kilometer is equivalent to about 0.6 mi.

MEASURING LENGTH

Taking 1 m. as 39.37 in., express as inches:

1. 47 m. 3. 52 m. 5. 324 cm. 7. 4680 mm.
2. 34.5 m. 4. 67.5 m. 6. 2.83 cm. 8. 3000 mm.

9. Express in inches the diameter of a 75-millimeter gun.

10. A hill is 203 m. high. Express this height in feet.

11. A tower is 37.5 m. high. Express this height in feet.

12. A wheel is 1.3 m. in diameter. Express this in inches.

13. The distance between two European cities is 78 km. Express this distance in miles.

14. When Calvin was in France, he took a trip of 120 km. on the river Seine. Express this distance in miles.

15. If it takes a European train 7 hr. to make a trip of 326 km., what is the average rate in miles per hour?

16. How many inches are there in 1 rd.? How many centimeters are there in 1 km.? It may take you 3 min. to find the first answer, but you can find the other one mentally, for you have simply to think of 100 × 1000.

17. How many thirty-seconds of an inch are there in a yard? A similar question in the metric system would ask for the number of millimeters in a meter. What is it?

Metric Square Measure. The table of square measure, in which the **square meter** ($m.^2$) is the unit, is as follows:

1 square millimeter ($mm.^2$) = 0.000001 square meter ($m.^2$)
1 square centimeter ($cm.^2$) = 0.0001 square meter
1 square decimeter ($dm.^2$) = 0.01 square meter
1 square kilometer ($km.^2$) = 1,000,000 square meters

Metric Cubic Measure. The table of cubic measure, in which the unit is the **cubic meter** ($m.^3$), is as follows:

1 cubic millimeter ($mm.^3$) = 0.000000001 cubic meter ($m.^3$)
1 cubic centimeter ($cm.^3$) = 0.000001 cubic meter
1 cubic decimeter ($dm.^3$) = 0.001 cubic meter

These tables need not be memorized as they are easily derived from the table of length. That is, 1 cm. = 0.01 m., and so 1 $cm.^2 = 0.01 \times 0.01$ $m.^2 = 0.0001$ $m.^2$

We often find 1 $cm.^3$ written as 1 cc. or 1 ccm.

METRIC SQUARE AND CUBIC MEASURES

Express the following as square centimeters:

1. 750 $km.^2$ **2.** 37 $m.^2$ **3.** 4296 $mm.^2$ **4.** 6.25 $m.^2$

Express the following as cubic meters:

5. 19,750,000 $cm.^3$ **7.** 7000 $cm.^3$ **9.** 937 $cm.^3$
6. 826,427,653.84 $mm.^3$ **8.** 3500 $dm.^3$ **10.** 486 $dm.^3$

11. How many square inches are there in 3 sq. ft.? How many square centimeters are there in 3 $m.^2$? Which is the easier to find?

12. How many cubic sixteenths of an inch are there in one cubic half inch? A similar question in the metric system would ask for the number of cubic millimeters in 1 $cm.^3$ What is it? Which is the easier to find?

Metric Weight. The important part of the table of weight, in which the unit is the **gram** (g.), is as follows:

1 centigram (cg.) = 0.01 gram (g.)
1 decigram (dg.) = 0.1 gram
1 kilogram (kg.) = 1000 grams
1 metric ton (t.) = 1000 kilograms

A kilogram, commonly called a *kilo,* is approximately equivalent to 2.2 lb., and a metric ton is equivalent to about 2205 lb.

Metric Capacity. The important part of the table of capacity, in which the unit is the **liter** (l.), is as follows:

1 centiliter (cl.) = 0.01 liter (l.)
1 hectoliter (hl.) = 100 liters

The liter is the capacity of a cube which is 10 cm., or 0.1 m., on an edge (that is, the cube contains 1000 cm.^{3}, or 0.001 m.^{3}) and is practically equivalent to one quart.

METRIC WEIGHT AND CAPACITY

Express the following as kilograms:

1. 374 lb.
2. 428 lb.
3. 204 lb.
4. 352 oz.
5. 48.4 lb.
6. 300 T.
7. 3270 g.
8. 7290 g.
9. 173.8 lb.
10. 7380 g.
11. 4900 g.
12. 6350 g.

Express the following as pounds:

13. 2 kg.
14. 20 kg.
15. 300 kg.
16. 3000 kg.

Express the following as liters:

17. 7.5 hl.
18. 8 pt.
19. 2.25 hl.
20. $12\frac{1}{2}$ gal.
21. 2500 cl.
22. 500 gal.
23. 15 hl.
24. 6.83 hl.

25. A man weighs 82 kg. How many pounds is this?

26. A girl weighs 110 lb. How many kilos does she weigh?

USING THE METRIC SYSTEM

1. An importer wishes to order 1200 yd. of silk from a French silk manufacturer. How many meters should he order?

2. The distance from Paris to a certain other French city is 209 km. Express this distance in miles.

3. A cylindric gasoline tank on a foreign-made automobile is 80 cm. long and 35 cm. in diameter. Taking 1000 cm.^{3}, or 1 l., as equivalent to one quart, how many gallons of gasoline will the tank hold?

4. A cubic centimeter of water weighs a gram. What is the metric weight of a cubic meter of water? What is the weight in pounds?

5. In computing the duty on a shipment of 8 t. of merchandise from abroad, an importing house had to know the weight of the shipment in pounds. What was this weight?

6. Find the capacity in liters of a rectangular tank 130 cm. long, 85 cm. wide, and 95 cm. deep.

7. Using the facts given in Ex. 4, find the weight of 1 l. of water, and then find the weight of water needed to fill the tank in Ex. 6.

8. Taking 231 cu. in. to the gallon, find the capacity in gallons of a tank 51 in. long, 34 in. wide, and 37 in. deep, which is practically the same size as the tank in Ex. 6. Taking 8.3 lb. as the weight of 1 gal. of water, find the weight of water needed to fill the tank.

9. Compare the work in Exs. 6–8, and write a statement about which system you found the easiest to use.

10. In repairing a foreign-made automobile a mechanic needed some bolts that had 5 threads per centimeter of length. This was equivalent to about how many threads per inch?

III. RATIO AND PROPORTION

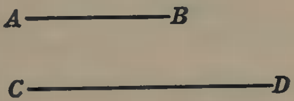
A SILENT READING LESSON

We often have occasion to compare the size of objects. For example, we frequently say that one building is twice as wide as another; that one field has half the area of another; that one hill is one and a half times as high as another; or that one tree is three times as large round as another, and so on.

In the same way we can compare numbers. For example, we know that 4 is twice as large as 2, half as large as 8, and two thirds as large as 6. Division is simply a comparison of numbers, for when we say that $12 \div 2 = 6$, we say in effect that 12 is six times as large as 2.

A fraction is an expressed division. For example, the fraction $\frac{2}{5}$ means that 2 is to be divided by 5, and the fraction $\frac{6}{3}$ means that 6 is to be divided by 3.

A fraction like $\frac{2}{5}$ or $\frac{6}{3}$ is also called a *ratio*, and we say that $\frac{2}{5}$ shows the ratio of 2 to 5 and that $\frac{6}{3}$ shows the ratio of 6 to 3. Sometimes we express the ratio of 2 to 5 by writing $2:5$, which we read as "the ratio of 2 to 5," but the fractional form is more familiar and so is generally used. In fact, $1 \div 2$, $1:2$, and $\frac{1}{2}$ are merely three different ways of indicating the division of 1 by 2.



We shall use ratios chiefly in comparing lines. Here are two lines, *AB* and *CD*. If we measure the lengths of the lines, we shall find that *AB* is $\frac{3}{4}$ in. long and *CD* is $1\frac{1}{4}$ in., or $\frac{5}{4}$ in., long. Hence to express the ratio of *AB* to *CD*, we have

$$\frac{3}{4} \div \frac{5}{4} = \frac{3}{4} \times \frac{4}{5} = \frac{3}{5};$$

that is, in simplest form the ratio of *AB* to *CD* is $\frac{3}{5}$. Similarly, if we wish to express the ratio of *CD* to *AB*, we have

$$\frac{5}{4} \div \frac{3}{4} = \frac{5}{4} \times \frac{4}{3} = \frac{5}{3}.$$

Terms of a Ratio. Just as in division we do not divide 5 ft. by 5 yd., so in expressing ratios, the two numbers, or *terms*, must be of the same kind.

For example, the ratio of \$2 to \$4 is $\frac{2}{4}$, or $\frac{1}{2}$, but the ratio of 10¢ to \$1 is $\frac{10}{100}$, or $\frac{1}{10}$.

Similarly, the ratio of 6 ft. to 5 ft. is $\frac{6}{5}$, or $1\frac{1}{5}$, but the ratio of 6 yd. to 5 ft. is $\frac{18}{5}$, or $3\frac{3}{5}$.

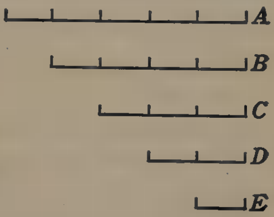
In a ratio the first term is sometimes called the *antecedent*, and the second term, the *consequent*.

EXPRESSING RATIOS

All work oral

1. Looking at the lines below, state the ratio of E to D; of E to C; of E to B.

Referring to these lines, state the values of the following ratios:



2. E to A.
3. D to A.
4. $2E$ to D.
5. $2D$ to B.
6. D to C.
7. C to A.
8. $4E$ to A.
9. $5E$ to A.
10. What is the ratio of any number to itself? to half itself?

Referring to the above lines, state the values of these ratios:

11. A to $B + E$.
12. A to $B + C$.
13. B to $C + E$.
14. B to $A + E$.
15. A to E.
16. C to B.
17. $3E$ to C.
18. $2D$ to A.
19. B to $A + D + E$.
20. A to $B + C + D$.
21. $B + E$ to $C + D$.
22. $A + E$ to $B + D$.
23. What is the simplest fraction that expresses the ratio of 2 to 4? of 4 to 8? of 4 to 16? of 6 to 18? of 10 to 8? of 12 to 10? of 25 to 15?

Writing Ratios. If David is 4 ft. tall and his brother Will is 5 ft. tall, David is $\frac{4}{5}$ as tall as Will; that is, the ratio of David's height to Will's height is $\frac{4}{5}$.

The ratio of 4 ft. to 5 ft. may be written in any one of the following forms:

$$\frac{4\text{ ft.}}{5\text{ ft.}} \qquad \frac{4}{5} \qquad 4\text{ ft.} : 5\text{ ft.} \qquad 4:5 \qquad 4\text{ ft.} \div 5\text{ ft.} \qquad 4 \div 5$$

We usually, however, write simply the fraction $\frac{4}{5}$, because this form is the one with which we are most familiar.

READING AND WRITING RATIOS

Numbers 1 to 23, oral

1. The ratio of 1 to 2 is $\frac{1}{2}$. What is the ratio of 3 to 4?

2. The ratio of 3 to 9 is $\frac{3}{9}$, but we express it as simply as possible and say that it is $\frac{1}{3}$. What is the ratio of 4 to 20?

3. The ratio of 24 to 4 is $\frac{24}{4}$, but we express it as 6. What is the ratio of 30 to 10?

State as simply as you can the following ratios:

4. 3 to 6.
5. 6 to 3.
6. 4 to 8.
7. 8 to 4.
8. 8 to 1.
9. 1 to 1.
10. 3 to 3.
11. 9 to 9.
12. 9 to 1.
13. 1 to 9.
14. 3 to 15.
15. 15 to 3.
16. 20 to 4.
17. 4 to 20.
18. 5 to 25.
19. 10 to 20.
20. 10 to 5.
21. 20 to 10.
22. 5 to 10.
23. 17 to 34.

24. Of the ratios 3 to 18, 5 to 30, and 8 to 42, which is the largest? Which two are equal?

25. Write two ratios, each equal to the ratio of 7 to 42; to the ratio of $3\frac{1}{2}$ to 21.

26. State which of the following ratios is equal to $\frac{5}{8}$:

$$\frac{15}{24} \qquad \frac{25}{80} \qquad \frac{30}{48} \qquad \frac{49}{56} \qquad \frac{40}{62} \qquad \frac{42}{64} \qquad \frac{40}{64} \qquad \frac{45}{81}$$

27. State which of these ratios is equal to the ratio of 7 to 9:

14 to 18 21 to 30 21 to 27 63 to 81

28. When divided by 2, what number becomes 3? What is the missing numerator in the statement that $\frac{}{2} = 3$?

29. In $\frac{x}{5} = 2$, what number is represented by x?

30. If the ratio of x to 7 is 3, what is the value of x?

31. If Fred is half as old as his brother Jack, what is the ratio of Fred's age to Jack's? of Jack's age to Fred's?

Copy and complete the following statements:

32. 27 in. is —— times 9 in., and so the ratio of 27 to 9 is ——.

33. 12 in. is —— of 36 in., and so the ratio of 12 to 36 is ——.

34. 64 lb. is —— times as heavy as 4 lb. and —— as heavy as 128 lb.; and so the ratio of 64 lb. to 4 lb. is ——, and the ratio of 64 lb. to —— is $\frac{1}{2}$.

Express the following ratios as per cents:

35. 4 to 100. **36.** 4 to 16. **37.** 2 to 50. **38.** 75 to 200.

Find the number represented by x in each of the following cases:

39. $\frac{x}{2} = 5$. **40.** $\frac{x}{4} = 3$. **41.** $\frac{x}{4} = \frac{1}{2}$. **42.** $\frac{2}{x} = \frac{1}{3}$.

43. The ratio of the height of Rob's father to Rob's height is $\frac{3}{2}$. Rob's father is 6 ft. tall. How tall is Rob?

44. If, out of 12 games of basket ball, your team has won 6 and lost 4, and the other 2 were tied, what is the ratio of games won to games lost? of games won to games played? of games tied to games lost? of games tied to the sum of games won and lost?

PROBLEMS ABOUT MIXTURES

1. Suppose that we add a cup of pure vinegar to a cup of water. Find the ratio of the vinegar to the mixture, the phrase "ratio of the vinegar to the mixture" meaning, of course, the ratio with respect to the given unit, the cupful. What is the ratio of the water to the mixture? of the vinegar to the water?

2. If a farmer mixes 1 lb. of timothy seed with 1 lb. of redtop and $\frac{1}{2}$ lb. of clover, what is the ratio of the timothy to the mixture? The timothy is what per cent of the mixture?

3. In Ex. 2, what is the ratio of the clover to the mixture? The clover is what per cent of the mixture?

4. In Ex. 2, what is the ratio of the weight of the clover to the combined weight of the timothy and the redtop?

Copy and complete the following statements:

5. If we mix 1 lb. of clover and 3 lb. of timothy, the ratio of the clover to the mixture is ——.

6. In Ex. 5 we have 4 lb. of mixture. If we wish 32 lb., we should take —— lb. of clover and —— lb. of timothy.

7. In making molasses candy Nora uses $\frac{1}{2}$ cup of butter, 2 cups of sugar, 1 cup of molasses, and $1\frac{1}{2}$ cups of water. The ratio of butter to sugar is ——; of sugar to molasses is ——; of butter to water is ——. If Nora doubled the recipe, she would use —— cup of butter, —— cups of sugar, —— cups of molasses, and 3 cups of ——, and the three ratios would then be ——, ——, and ——.

8. In making fudge Emily uses 5 oz. of chocolate, $1\frac{1}{2}$ cups of milk, and 6 cups of sugar. If she doubles the recipe, she takes —— oz. of chocolate, —— cups of milk, and —— cups of sugar. In each case the ratio of milk to sugar is ——.

Separating a Number in a Given Ratio. Suppose that we wish to separate 25 marbles into two groups having the ratio 2 to 3.

Since one group has the value 2, and the other has the value 3, the whole has the value 5.

Then, since one group is $\frac{2}{5}$ of the whole, it contains $\frac{2}{5}$ of 25, or 10, and the other contains $\frac{3}{5}$ of 25, or 15.

Check. $10 + 15 = 25$, and $\frac{10}{15} = \frac{2}{3}$.

Take the sum of the terms of the ratio as the denominator of each of two fractions and the terms of the ratios as the numerators, and multiply by these fractions the number to be separated.

That is, applying this rule to the above case, we have

$$2 + 3 = 5; \quad \tfrac{2}{5} \text{ of } 25 = 10; \quad \tfrac{3}{5} \text{ of } 25 = 15.$$

SEPARATING IN A GIVEN RATIO

Numbers 1 to 6, oral

1. Separate 9 in the ratio 2 to 1; 1 to 1.

2. Separate 8 hr. in the ratio 1 to 3; 7 to 1; 1 to 1.

3. Separate 44 in the ratio 5 to 6; 1 to 10; 9 to 2.

4. The ratio of the length of summer to the length of the year is 1 to 4. How many months are there in the summer?

5. The ratio of clear to cloudy days one April was $\frac{1}{5}$. How many days were clear and how many were cloudy?

6. In playing 33 games of ringtoss the ratio of Will's score to Mollie's was $\frac{5}{6}$. How many games did each win?

7. If in November the ratio of clear to cloudy days was 3:2, how many days were there of each kind?

8. Four boys arrange a three-weeks' camping trip. The first boy stays 18 da.; the second comes late and stays 12 da.; the third comes for the last 2 wk.; the fourth stays 3 wk. If the total expense is $32.50, what is each boy's share?

Proportion. An expression of equality between two ratios is called a *proportion*. For example, we see that the ratio \$2 : \$3 is equal to the ratio 10 ft. : 15 ft., and so we may write

$$\$2 : \$3 = 10 \text{ ft.} : 15 \text{ ft.},$$

which is a proportion that is read "\$2 is to \$3 as 10 ft. is to 15 ft." It may be written also as

$$2 : 3 = 10 : 15, \quad \text{or as} \quad 2 \div 3 = 10 \div 15,$$

or, still better, as $\frac{2}{3} = \frac{10}{15}$.

The first and fourth terms of a proportion are called the *extremes*, and the second and third are called the *means*. For example, in the above case, 2 and 15 are the extremes, and 3 and 10 are the means. In any proportion

The product of the means equals the product of the extremes.

For example, in the above proportion we have

$$3 \times 10 = 30, \quad \text{and} \quad 2 \times 15 = 30.$$

Using Proportion. 1. What number bears the same ratio to 7 that 10 does to 35?

Writing x in place of the number to be found, we have

$$\frac{x}{7} = \frac{10}{35}.$$

Since the product of the extremes equals the product of the means, in this case we have

$$35 \times x = 7 \times 10,$$

or, what is the same thing, we may write

$$x = \frac{\cancel{7} \times \overset{2}{\cancel{10}}}{\underset{\cancel{5}}{\cancel{35}}} = 2.$$

Check. Writing 2 for x, $\frac{2}{7} = \frac{10}{35}$, and $2 \times 35 = 7 \times 10$; that is, since these two products are equal, our result is correct.

2. If a miller can make 4 bbl. of flour from 18 bu. of wheat, how many barrels of flour can he make from 207 bu.?

The amounts of flour produced depend upon (have the same ratio as) the amounts of wheat used; that is,

$$\frac{x \text{ bbl. of flour}}{4 \text{ bbl. of flour}} = \frac{207 \text{ bu. of wheat}}{18 \text{ bu. of wheat}},$$

or

$$\frac{x}{4} = \frac{207}{18}.$$

By what shall we multiply these equals in order to leave x by itself? Multiplying by this number, we find that

$$x = \frac{4 \times 207}{18} = 46, \text{ the number of barrels.}$$

USING PROPORTION

1. Suppose that a boy can ride 2 mi. in 20 min. on his bicycle. In finding how long it will take the boy to go $\frac{3}{4}$ mi. at the same rate, a pupil writes

$$\frac{x}{20} = \frac{\frac{3}{4}}{2}.$$

He then multiplies by 20, and finds that

$$x = \frac{20 \times \frac{3}{4}}{2} = (?).$$

2. At the rate of $55\frac{1}{2}$ mi. in 1 hr. 30 min., how long will it take a train to go 148 mi.?

3. If a man can skate 75 yd. in $8\frac{1}{5}$ sec., how long will it take him to skate 200 yd. at this rate?

4. If a man can skate 220 yd. in 19 sec., how long will it take him to skate 5 mi. if he can maintain this rate?

5. A map is drawn so that 1 in. represents 16 mi. on the surface of the earth. What length represents 40 mi.?

6. A map is drawn so that 1 in. represents 8 mi. How many square miles of land are represented by a portion of the map 3 in. square?

7. If a man can swim 45 yd. in 23 sec., how long will it take him to swim 2 mi. if he can maintain this rate?

8. If a farmer needs 26 lb. of seed for $\frac{1}{2}$ A., how many pounds does he need for 19.9 A.?

9. Find the circumference of a circle whose radius is to the radius of a circle 366.66 in. in circumference as 3.40 : 5.82; that is, find x in the proportion

$$\frac{x}{366.66} = \frac{3.40}{5.82}.$$

10. If a shipment weighing 750 lb. can be transported 100 mi. for a certain amount, would a heavier shipment be transported a shorter or a longer distance for the same amount? How far should a shipment weighing $\frac{1}{2}$ T. be transported for the same amount as 750 lb. for 100 mi.?

11. One man can do a piece of work in 10 da., and another can do the same work in 9 da. If the men are paid in proportion to the work they do, and if the wages of the first are \$5.40 a day, what are the wages of the second?

12. Walter is thinking of a number which has the same ratio to 17 that 35 has to 119. What is the number?

Find the number represented by x in each of these proportions:

13. $\frac{x}{51} = \frac{12}{17}$.

14. $\frac{x}{17} = \frac{54}{51}$.

15. $\frac{41}{11} = \frac{x}{22}$.

16. $\frac{x}{250} = \frac{35}{50}$.

17. $\frac{x}{18.2} = \frac{9.1}{7.3}$.

18. $\frac{82.7}{82} = \frac{x}{4.1}$.

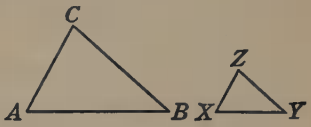
19. $\frac{x}{21.7} = \frac{2.4}{3.3}$.

20. $\frac{x}{1.37} = \frac{2.4}{13.7}$.

21. $\frac{x}{0.5} = \frac{12.25}{1.75}$.

Similar Figures. Figures that have exactly the same shape, like the two triangles here shown, are called *similar figures.*

The corresponding sides of similar figures are proportional.



In such a statement we, of course, mean that it is the numbers which represent the lengths of the sides that are proportional; that is, the numbers form a proportion.

Thus, in the two similar triangles here shown, we can form proportions with the sides AB, BC, XY, and YZ; for example, we may write

$$\frac{AB}{BC} = \frac{XY}{YZ}, \quad \text{or} \quad \frac{AB}{XY} = \frac{BC}{YZ}.$$

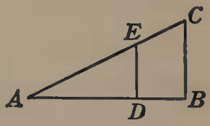
PROPORTION IN TRIANGLES

All work oral

1. In the two similar triangles shown above, if AB is twice XY, how does AC compare with XZ?

2. In the triangles shown above, if the ratio $AC : AB$ is equal to $\frac{3}{4}$, what is the value of the ratio $XZ : XY$? $XY : YZ$?

3. Using this figure, write a proportion involving AD, AB, AE, and AC. If $AD = \frac{2}{3} AB$, what is the ratio of AE to AC? of DE to BC? of $\frac{1}{3} BC$ to $2\,DE$?



4. In the figure of Ex. 3, if $AB = \frac{13}{16}$ in., $AD = \frac{9}{16}$ in., and $AE = \frac{5}{8}$ in. $= \frac{10}{16}$ in., what is the length of AC?

5. In the figure of Ex. 3, suppose that BC represents the height of a man 6 ft. tall, DE the height of a boy, AB the man's shadow, and AD the boy's shadow. Show how you would proceed to find the height of the boy after you have measured the shadows.

Measuring Heights and Distances. The height of a tree, of a high building, or of a tower can be found by using proportion.

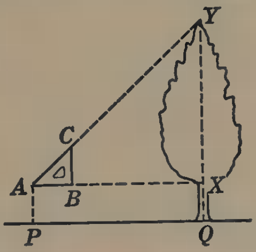
This figure shows how a man can measure the height of a tree by using a right triangle in which $AB = BC$. Holding the triangle so that AB is level, he finds a point P from which he can just see the top of the tree along AC when the triangle is at the height of his eye (the distance AP) above the ground. Then, because

$$AB = BC$$

and

$$\frac{AB}{BC} = \frac{AX}{XY},$$

the height of the tree above X is equal to the distance AX, or PQ, and the height QY is then $QX + XY$.



The figure is not drawn to scale, as the size of the triangle is exaggerated, and so is its height above the ground.

MEASURING HEIGHTS AND DISTANCES

1. What is the height of the tree in the above case, if the triangle ABC is held at a height of 5 ft. 4 in. above the ground and $PQ = 23$ ft. 8 in.?

2. In Ex. 1, suppose that the man uses a triangle in which $AB = 2\,BC$, that $PQ = 75$ ft., and that the point A is 5 ft. 8 in. above the ground. Find the height of the tree.

3. In Ex. 1, suppose that the man uses a triangle in which $AB = 8$ in. and $BC = 6$ in. If $PQ = 66$ ft. and $AP = 5$ ft. 6 in., what is the height of the tree?

Solve Ex. 3 when

4. $PQ = 40$ ft., $AP = 5$ ft.

5. $PQ = 42$ ft., $AP = 5$ ft.

6. $PQ = 45$ ft., $AP = 6$ ft.

7. $PQ = 39$ ft., $AP = 5$ ft. 4 in.

8. $PQ = 70$ ft., $AP = 5$ ft. 2 in.

9. $PQ = 61$ ft., $AP = 5$ ft. 3 in.

SHADOW RECKONING

1. The boys in the picture above find that the fence post is 3 ft. 2 in. high and casts a shadow 3 ft. 6 in. long. They find that the shadow of the flagpole is 50 ft. 8 in. long at the same time. What is the height of the flagpole?

2. If a tree casts a shadow 60 ft. 6 in. long when a post 3 ft. 3 in. high casts one 5 ft. 6 in. long, how high is the tree?

3. How high is a flagpole which casts a shadow 100 ft. long when a gatepost 9 ft. high casts a shadow 11 ft. 3 in. long?

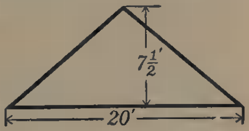
4. How high is a building which casts a shadow 75 ft. long when a post 6 ft. 2 in. high casts a shadow 12 ft. 6 in. long?

5. How high is a tree which casts a shadow 30 ft. long when a post 6 ft. 3 in. high casts a shadow 3 ft. 9 in. long?

6. Martin and George wished to find the height of the school building. Martin stood a yardstick upright and measured its shadow, which he found to be 4 ft. long. George measured the shadow of the building and found it to be 36 ft. long. How high was the building? Draw a figure to illustrate.

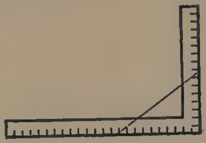
7. Using either the method given above or the one given on page 259, find the approximate height of your school building or of some object or building near the school.

OTHER USES OF PROPORTION

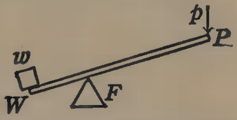


1. To a scale of 8 in. to 20 ft., a pupil drew a plan for the gable end of a roof here shown. What length represented the $7\frac{1}{2}$-foot "rise" of the roof? By using proportion, how could the pupil find from the plan the real length of the "slope" of the roof?

2. The teacher said that the plan in Ex. 1 was not as convenient as one showing only half the gable, with the 10-foot "run" represented by 10 in. Draw such a plan and measure the slope. How long is the slope of the roof?



3. In finding the slope in a case like Ex. 1 a carpenter would use his square. On the tongue, or short arm, he would take a point $7\frac{1}{2}$ in. from the corner; then on the blade, or long arm, he would take a point 10 in. from the corner. He would then take the number of inches between these points as the number of feet in the slope of the roof. Is this correct? Draw the figure and write the reasons involved.



4. In a lever supported at a fulcrum F, if sufficient power (p) is applied downward at P, a weight (w) can be lifted upward at W. In problems involving levers, we have the relation

$$\frac{p}{w}=\frac{FW}{FP}.$$

If $w = 90$ lb., $FW = 1\frac{1}{2}$ ft., and $FP = 4\frac{1}{2}$ ft., what power (p) is needed to lift the weight?

5. Paul and Carl just balance a seesaw. Paul, who weighs 100 lb., sits 4 ft. from the fulcrum which supports the plank. If the plank is 9 ft. long, how much does Carl, who sits at the other end, weigh?

Proportion in Business. While proportion is chiefly valuable in connection with measurements, it can also be used in solving business problems, although there are often easier ways. Simple ratio, however, is sometimes used, as in dividing a sum of money into two parts having a given ratio.

In solving by proportion a problem which asks, "If 3 books cost \$4.20, how much will 5 books cost?" we, of course, assume that the books are all sold at the same price, and write

$$\frac{x}{\$4.20} = \frac{5}{3}.$$

Then, as in the case on page 255, we may write

$$x = \frac{5 \times \$\overset{1.40}{\cancel{4.20}}}{\cancel{3}} = \$7.00, \text{ the required cost.}$$

PROPORTION IN BUSINESS

1. If 7 lb. of meat cost \$2.24, how much will 5 lb. cost?

2. If 6 yd. of cloth cost \$8.28, how much will 7 yd. cost?

3. If 5 books cost \$8.25, how much will 9 books cost?

Copy this table and fill the blank spaces:

	AMOUNT	COST	AMOUNT	COST
4.	5 yd.	\$9	14 yd.	
5.		\$21	3 yd.	\$9
6.	9 books	\$18		\$10
7.	6 hats		7 hats	\$31.50
8.		42¢	17 pencils	\$1.02

9. Make up and solve three problems like Exs. 1–3.

IV. POWERS AND ROOTS

A SILENT READING LESSON

When we studied the area of a circle, we used the *square* of the radius, and found that the square of 3 means 3×3, or 9. We often write 3^2 (read "3 square") for the square of 3, which is also called the *second power* of 3.

We may also have the *third power* (often called the *cube*) of a number. For example, the third power of 2 is $2 \times 2 \times 2$, or 8, which shows that 2 is taken three times as a *factor*. We often write 2^3 (read "2 cube") for the third power of 2.

In the same way we may have the fourth power of a number, and other powers. The power is always indicated by the *exponent*; for example,

$3^2 = 3 \times 3 = 9$, where the exponent is 2;

$4^3 = 4 \times 4 \times 4 = 64$, where the exponent is 3;

$5^4 = 5 \times 5 \times 5 \times 5 = 625$, where the exponent is 4.

We may also have powers of fractions; for example,

$(\frac{1}{2})^2 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$, the square of $\frac{1}{2}$;

$(\frac{1}{2})^3 = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$, the cube of $\frac{1}{2}$.

Similarly, we may have powers of decimals; for example,

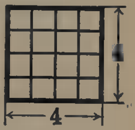
$0.1^2 = 0.1 \times 0.1 = 0.01$, the square of 0.1;

$1.2^3 = 1.2 \times 1.2 \times 1.2 = 1.728$, the cube of 1.2.

From what we have learned, we see that the area of a square may be obtained by taking the second power of its side. For example, if the side of a square is 5 in., the area is 5^2 sq. in., or 25 sq. in.

Similarly, the volume of a cube may be obtained by taking the third power of its edge. For example, if the edge of a cube is 3 in., the volume is 3^3 cu. in., or 27 cu. in.

Square Root. If a square is 4 units on a side, it has an area of 4 × 4, or 16 square units. Since 16 is the square of 4, we can easily see why 4 is called the *square root* of 16; that is,



The square root of a number is one of its two equal factors.

The square root of 16 is indicated by the symbol $\sqrt{16}$, which is read "the square root of 16."

Approximate Square Roots. A number like 16 is called a *perfect square*, but 10 is not a perfect square. Just as we say, however, that $\frac{2}{3} = 0.667$, to the nearest 0.001, so we may say that the square root of 10 is 3.16, to the nearest 0.01. If we find the value of 3.16^2, we shall see that it is nearly 10.

Square Root of a Perfect Square. Square roots of perfect squares may often be found by factoring.

For example, in finding the square root of 441, we separate 441 into its factors, as here shown, and then separate these factors into the two equal groups, 3 × 7 and 3 × 7. Hence we see that

$$\sqrt{441} = \sqrt{21 \times 21} = 21.$$

$$\begin{array}{r} 3\overline{)441} \\ 3\overline{)147} \\ 7\overline{)49} \\ 7 \end{array}$$

FINDING SQUARES AND SQUARE ROOTS

Numbers 1 to 24, oral

State the squares of the following numbers:

1. 2.
2. 4.
3. 6.
4. 8.
5. 10.
6. 11.
7. 3.
8. 7.
9. $\frac{1}{2}$.
10. $\frac{1}{3}$.
11. 0.2.
12. 1.1.

State the square roots of the following numbers:

13. 9.
14. 64.
15. 16.
16. 81.
17. 121.
18. 144.
19. 169.
20. 225.
21. 1.44.
22. 0.64.
23. 4900.
24. 2500.

Find the squares of the following numbers:

25. 2.3.
26. 4.7.
27. $2\frac{1}{2}$.
28. $3\frac{1}{3}$.
29. 2.4.
30. 3.6.
31. 0.7.
32. 0.81.
33. 1200.
34. 3602.

By factoring, find the square roots of the following numbers:

35. 196.
36. 484.
37. 625.
38. 324.
39. 729.
40. 576.
41. 1296.
42. 1089.
43. 12.25.
44. 40.96.

Find the sides of squares whose areas are as follows:

45. 4.84 sq. in.
46. 1.96 sq. ft.
47. 4.41 sq. in.
48. 4356 sq. in.
49. 5929 sq. yd.
50. 10.89 sq. yd.

Find the perimeters of squares whose areas are as follows:

51. 6561 sq. ft.
52. 12,100 sq. in.
53. 19,600 sq. ft.
54. 146.41 sq. in.
55. 16,384 sq. in.
56. 16,900 sq. ft.

57. At the corner of a square piece of land that contains 57,600 sq. ft., a square lot containing 6400 sq. ft. is laid out. To a convenient scale, draw a plan of the land and find the perimeter of each piece.

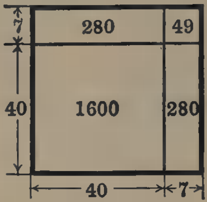
58. If a square lot contains 18,225 sq. ft., what is the perimeter? What is the perimeter of a square lot of four times this area? The perimeter of the second lot is how many times the perimeter of the first?

59. The combined area of two square building lots which front on the same street is 8900 sq. ft., and the area of the larger lot is 6400 sq. ft. What is the frontage of each lot?

60. If a square lot has an area of 28,900 sq. ft., what is the perimeter? What is the perimeter of a square lot that contains nine times this area? The perimeter of the second lot is how many times the perimeter of the first? The perimeter of the first lot is what part as long as that of the second?

Square of the Sum of Two Numbers. Since $47 = 40 + 7$, the square of 47 may be obtained in the following manner:

$$\begin{array}{rl}
 & 40 + 7 \\
 & 40 + 7 \\
\hline
 & (40 \times 7) + 7^2 \\
40^2 + & (40 \times 7) \\
\hline
40^2 + 2 \times & (40 \times 7) + 7^2 \\
 & = 1600 + 560 + 49 \\
 & = 2209.
\end{array}$$



Every number consisting of two or more figures may be regarded as composed of tens and units. That is, as shown by both the multiplication and the figure,

The square of a number contains the square of the tens, plus twice the tens times the units, plus the square of the units.

This principle, which is very important in finding square roots, should be clearly understood before proceeding further.

Locating the Root. Since $1 = 1^2$, $100 = 10^2$, and $10{,}000 = 100^2$, it is evident that the square root of any number between 1 and 100 lies between 1 and 10, and that the square root of any number between 100 and 10,000 lies between 10 and 100. In other words, considering only whole numbers which are perfect squares, the square root of a number of one or two figures is a number of one figure; the square root of a number of three or four figures is a number of two figures; and so on.

Separating into Periods. We thus see that if a whole number is separated into periods of two figures each, beginning at the right, the number of figures in the integral part (the part that is a whole number) of the square root is the same as the number of periods. The left-hand period may, of course, have one figure or two figures. For example, the square root of 47 61 has two figures, and that of 1 61 29 has three.

Finding a Square Root. In finding the square root of 2209, for example, we first separate the number into periods of two figures each, beginning at the right. In so doing, we see that there are two periods in the number, and hence there are two figures in the integral part of the square root.

The first period, 22, contains the square of the tens' figure of the root. Since the greatest square in 22 is 16, then 4, the square root of 16, is the tens' figure.

$$\begin{array}{rl}
 & 22\ 09\,\underline{|\,47} \\
 & 16 \\
80 & \overline{|\,6\ 09} \\
87 & |\,\underline{6\ 09}
\end{array}$$

Subtracting 16, the square of the tens, as shown, 609 must contain twice the tens times the units, plus the square of the units. We then divide 609 by 2 × 4 tens, or 80, to obtain the approximate units' figure, which we find to be 7.

Since in squaring $40 + 7$ we had $2 \times 40 \times 7 + 7^2$, which we may write as $(2 \times 40 + 7) \times 7$, we add 7 to 80 and multiply the result by 7, which gives us 609. Subtracting this result, there is no remainder; that is, $\sqrt{2209} = 47$.

We may check the work by finding that $47^2 = 2209$.

FINDING A SQUARE ROOT

Find the square root of each of the following numbers:

1. 144.
2. 169.
3. 625.
4. 729.
5. 900.
6. 3600.
7. 3249.
8. 3721.
9. 3969.
10. 5041.

Find the side of a square that has an area of

11. 6724 sq. ft.
12. 7569 sq. ft.
13. 9025 sq. ft.
14. 9409 sq. ft.
15. 7921 sq. ft.
16. 6889 sq. yd.

Find the square root of each of the following fractions by taking the square root of each term:

17. $\frac{144}{529}$.
18. $\frac{169}{225}$.
19. $\frac{961}{1024}$.
20. $\frac{1089}{1156}$.
21. $\frac{1936}{5041}$.

Square Root with Decimals. In finding the square root of a decimal, for example, of 151.29, the first step is to separate the number into periods, beginning at the decimal point. We then proceed as shown in the following solution, which should be compared step by step with the summary given below:

Given number		1 51.29	12.3	square root
Square of 1		1		
First trial divisor	20	51		first remainder
First complete divisor	22	44		from 2 × 22
Second trial divisor	24.0	7.29		second remainder
Second complete divisor	24.3	7.29		from 0.3 × 24.3

Check. $1.23 \times 1.23 = 151.29$.

If the number whose root we wish to find is not a perfect square, we may annex pairs of zeros at the right and continue until we have the desired number of decimal places.

Summary of Square Root. To find a square root,

Separate the number into periods of two figures each, beginning at the decimal point.

Find the greatest square in the left-hand period and write its square root as the first figure of the required root.

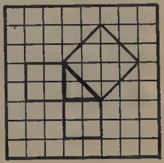
Subtract the square from the left-hand period, and to the remainder annex the next period.

Double the part of the root already found and use it as the trial divisor to find the second figure of the root. Add this result to the trial divisor and multiply by the number represented by the second figure of the root.

Subtract this result, bring down the next period, and proceed as before until all the periods have been used, or until the desired number of decimal places is reached.

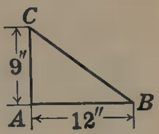
Square of the Hypotenuse. In a right triangle the side opposite the right angle is called the *hypotenuse* of the triangle.

Suppose that on a floor made of square tiles we mark out a right triangle and on each of the sides mark out a square, as here shown. We then see that the square on the hypotenuse contains eight small squares while each square on a side contains four such squares. In geometry it is proved that in any right triangle



The square of the hypotenuse is equal to the sum of the squares of the other two sides.

For example, in finding the length of the hypotenuse BC of the right triangle here shown, we have



$$(BC)^2 = 12^2 + 9^2 = 144 + 81 = 225,$$

and $$BC = \sqrt{225} = 15.$$

That is, the hypotenuse is 15 in. long.

RIGHT TRIANGLES

1. In the triangle ABC, above, find the length of the hypotenuse when $AB = 24$ ft. and $AC = 9$ ft.

2. From the relation $(BC)^2 = (AB)^2 + (AC)^2$ in the triangle ABC, above, we have $(AB)^2 = (BC)^2 - (AC)^2$. Find the length of AB when $BC = 105$ in. and $AC = 63$ in.

3. If the side of a square is 17 in., what is the length of the diagonal to the nearest 0.1 in.?

4. Find, to the nearest 0.1 in., the side of a square whose area is 3 sq. in.; whose area is 6 sq. in.

5. What, to the nearest 0.01 in., is the length of the diagonal of a square whose area is 81 sq. in.? of one that has an area four times as large?

6. Find the length of the diagonal of the floor of a hall that is 51 ft. by 68 ft.

7. Find, to the nearest 0.01 ft., the length of the diagonal of a square that contains 4 sq. ft.

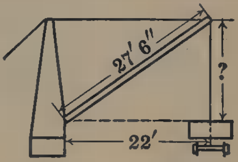
8. The two sides of a right triangle are 20 in. and 30 in. Find the length of the hypotenuse to the nearest 0.01 in.

9. The two sides of a right triangle are 57 in. and 76 in. Find the length of the hypotenuse.

10. What is the length of a wire drawn taut from the top of a 100-foot flagpole to a spot 75 ft. from the base?

11. In order to hold a telegraph pole vertical, a wire is stretched from a point on the pole 18 ft. above the ground to a stake 13 ft. 6 in. from the foot. How long is the wire?

12. The length of the arm of a derrick for unloading coal is 27 ft. 6 in. When the arm swings over a car 22 ft. from its foot, as here shown, how far is the top of the arm above the car?



Find, to the nearest 0.01, the length of the hypotenuse of a right triangle in which the other two sides are

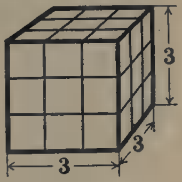
13. 32 ft., 28 ft.
14. 40 ft., 75 ft.
15. 321 ft., 428 ft.
16. 36 in., $67\frac{1}{2}$ in.
17. 72 ft., 135 ft.
18. $56\frac{1}{2}$ in., $72\frac{1}{2}$ in.

19. Find the diagonal of a square that contains 9409 sq. ft.

20. The foot of a 45-foot ladder is placed $13\frac{1}{2}$ ft. away from a building against which the top of the ladder rests. How high does the ladder reach on the building?

21. How far from the wall of a house must the foot of a 25-foot ladder be placed in order that the top of the ladder may just touch a window sill 20 ft. from the ground?

Cube Root. If a cube has an edge of 3 units, it has a volume of $3 \times 3 \times 3$, or 27 cubic units. Since 27 is the cube of 3, we can easily see why 3 is called the *cube root* of 27. That is, we see that



The cube root of a number is one of its three equal factors.

The cube root of 27 is indicated by the symbol $\sqrt[3]{27}$, which is read "the cube root of 27."

A number like 27 is a *perfect cube*, but 28 is not. We use approximate cube roots, however, just as we use approximate square roots; that is, $\sqrt[3]{28} = 3.04$, to the nearest 0.01.

Cube Root of a Perfect Cube. The cube root of a perfect cube may often be found by factoring.

For example, in finding the cube root of 216, we factor as here shown, and see that the factors form the three equal groups 3×2; that is,

$$\sqrt[3]{216} = \sqrt[3]{6 \times 6 \times 6} = 6.$$

In the same way, we can show that

$$\sqrt[3]{\tfrac{1}{27}} = \sqrt[3]{\tfrac{1}{3} \times \tfrac{1}{3} \times \tfrac{1}{3}} = \tfrac{1}{3}.$$

2)216
2)108
2)54
3)27
3)9
3

FINDING CUBE ROOTS

Find each of the following cube roots by factoring:

1. $\sqrt[3]{125}$. 3. $\sqrt[3]{729}$. 5. $\sqrt[3]{512}$. 7. $\sqrt[3]{1728}$. 9. $\sqrt[3]{4096}$.

2. $\sqrt[3]{\tfrac{1}{125}}$. 4. $\sqrt[3]{\tfrac{1}{729}}$. 6. $\sqrt[3]{\tfrac{1}{512}}$. 8. $\sqrt[3]{\tfrac{1}{1728}}$. 10. $\sqrt[3]{5832}$.

11. Find the edge of a cube whose volume is 13.824 cu. in.

Find the edge of a cube whose volume is

12. 343 cu. in. 14. 2744 cu. in. 16. 15,625 cu. ft.

13. 1000 cu. in. 15. 3375 cu. ft. 17. 21,952 cu. ft.

Using a Table of Roots. There are methods of finding the cube root of any number, just as we have seen that there is a method of finding the square root. These methods, however, are seldom used in ordinary work. When we wish to find square roots or cube roots, we generally use a table like the one given on page 273. The following examples show how this table can be used:

1. Find the square root of 39.

Looking for 39 in the column headed "No." (number), we follow along this line to the column marked "Square Roots." We there find 6.245; that is, $\sqrt{39} = 6.245$, to the nearest 0.001.

2. Find the cube root of 88.

On line 88 in the column marked "Cube Roots," we find 4.448; that is, $\sqrt[3]{88} = 4.448$, to the nearest 0.001.

3. Find the square root of 841.

The column of numbers ("No.") goes only to 100. We therefore look to see if 841 is in the column marked "Squares." We thus find that $841 = 29^2$, and so $\sqrt{841} = 29$.

4. Find the cube root of 884,736.

In the column marked "Cubes" we find that $884{,}736 = 96^3$, and so $\sqrt[3]{884{,}736} = 96$.

5. Find the square root of 2000.

Although 2000 is not given in the column marked "Squares," we may find $\sqrt{2000}$ by observing that $\sqrt{2000} = \sqrt{100 \times 20}$. Then, from the table, $\sqrt{100} = 10$ and $\sqrt{20} = 4.472$, and so $\sqrt{2000} = 10 \times 4.472 = 44.72$.

6. Find the square root of 0.45.

First writing $\sqrt{0.45} = \sqrt{0.01 \times 45}$, we see that $0.01 = 0.1 \times 0.1$, and, from the table, we find that $\sqrt{45} = 6.708$. Hence we have $\sqrt{0.45} = 0.1 \times 6.708 = 0.6708$.

POWERS AND ROOTS

No.	Squares	Cubes	Square Roots	Cube Roots	No.	Squares	Cubes	Square Roots	Cube Roots
1	1	1	1.000	1.000	**51**	2 601	132 651	7.141	3.708
2	4	8	1.414	1.260	52	2 704	140 608	7.211	3.733
3	9	27	1.732	1.442	53	2 809	148 877	7.280	3.756
4	16	64	2.000	1.587	54	2 916	157 464	7.348	3.780
5	25	125	2.236	1.710	55	3 025	166 375	7.416	3.803
6	36	216	2.449	1.817	56	3 136	175 616	7.483	3.826
7	49	343	2.646	1.913	57	3 249	185 193	7.550	3.849
8	64	512	2.828	2.000	58	3 364	195 112	7.616	3.871
9	81	729	3.000	2.080	59	3 481	205 379	7.681	3.893
10	100	1 000	3.162	2.154	**60**	3 600	216 000	7.746	3.915
11	121	1 331	3.317	2.224	61	3 721	226 981	7.810	3.936
12	144	1 728	3.464	2.289	62	3 844	238 328	7.874	3.958
13	169	2 197	3.606	2.351	63	3 969	250 047	7.937	3.979
14	196	2 744	3.742	2.410	64	4 096	262 144	8.000	4.000
15	225	3 375	3.873	2.466	65	4 225	274 625	8.062	4.021
16	256	4 096	4.000	2.520	66	4 356	287 496	8.124	4.041
17	289	4 913	4.123	2.571	67	4 489	300 763	8.185	4.062
18	324	5 832	4.243	2.621	68	4 624	314 432	8.246	4.082
19	361	6 859	4.359	2.668	69	4 761	328 509	8.307	4.102
20	400	8 000	4.472	2.714	**70**	4 900	343 000	8.367	4.121
21	441	9 261	4.583	2.759	71	5 041	357 911	8.426	4.141
22	484	10 648	4.690	2.802	72	5 184	373 248	8.485	4.160
23	529	12 167	4.796	2.844	73	5 329	389 017	8.544	4.179
24	576	13 824	4.899	2.884	74	5 476	405 224	8.602	4.198
25	625	15 625	5.000	2.924	75	5 625	421 875	8.660	4.217
26	676	17 576	5.099	2.962	76	5 776	438 976	8.718	4.236
27	729	19 683	5.196	3.000	77	5 929	456 533	8.775	4.254
28	784	21 952	5.292	3.037	78	6 084	474 552	8.832	4.273
29	841	24 389	5.385	3.072	79	6 241	493 039	8.888	4.291
30	900	27 000	5.477	3.107	**80**	6 400	512 000	8.944	4.309
31	961	29 791	5.568	3.141	81	6 561	531 441	9.000	4.327
32	1 024	32 768	5.657	3.175	82	6 724	551 368	9.055	4.344
33	1 089	35 937	5.745	3.208	83	6 889	571 787	9.110	4.362
34	1 156	39 304	5.831	3.240	84	7 056	592 704	9.165	4.380
35	1 225	42 875	5.916	3.271	85	7 225	614 125	9.220	4.397
36	1 296	46 656	6.000	3.302	86	7 396	636 056	9.274	4.414
37	1 369	50 653	6.083	3.332	87	7 569	658 503	9.327	4.431
38	1 444	54 872	6.164	3.362	88	7 744	681 472	9.381	4.448
39	1 521	59 319	6.245	3.391	89	7 921	704 969	9.434	4.465
40	1 600	64 000	6.325	3.420	**90**	8 100	729 000	9.487	4.481
41	1 681	68 921	6.403	3.448	91	8 281	753 571	9.539	4.498
42	1 764	74 088	6.481	3.476	92	8 464	778 688	9.592	4.514
43	1 849	79 507	6.557	3.503	93	8 649	804 357	9.644	4.531
44	1 936	85 184	6.633	3.530	94	8 836	830 584	9.695	4.547
45	2 025	91 125	6.708	3.557	95	9 025	857 375	9.747	4.563
46	2 116	97 336	6.782	3.583	96	9 216	884 736	9.798	4.579
47	2 209	103 823	6.856	3.609	97	9 409	912 673	9.849	4.595
48	2 304	110 592	6.928	3.634	98	9 604	941 192	9.899	4.610
49	2 401	117 649	7.000	3.659	99	9 801	970 299	9.950	4.626
50	2 500	125 000	7.071	3.684	**100**	10 000	1 000 000	10.000	4.642

USING THE TABLE OF ROOTS ON PAGE 273

Using the table, find the square root of

1. 25.
2. 49.
3. 64.
4. 81.
5. 7.
6. 13.
7. 29.
8. 47.
9. 67.
10. 91.

Using the table, find the cube root of

11. 8.
12. 27.
13. 64.
14. 13.
15. 17.
16. 19.
17. 28.
18. 46.
19. 51.
20. 99.

Using the table, find the square root of

21. 0.97.
22. 0.56.
23. 961.
24. 2116.
25. 5184.
26. 6889.
27. 7744.
28. 8649.
29. 9025.
30. 9604.

Using the table, find the cube root of

31. 5832.
32. 9261.
33. 19,683.
34. 42,875.
35. 85,184.
36. 97,336.
37. 531,441.
38. 804,357.

39. The volume of a cube is 2197 cu. in. Using the table of roots, find the length of an edge.

40. Find the length of an edge of a cube which has a volume of 39 cu. ft.

41. The area of one face of a cube is 78 sq. in. Find the volume of the cube.

42. The total area of all six faces of a cube is 36 sq. in. Find the volume of the cube.

43. From the table on page 273, what is the square root of 1764? of 1849? The square root of 1775, which lies between these numbers, is therefore 42 plus some fraction. Find the figures in the integral part of the square root of 230.

44. Find the integral part of the square root of 509; of 850; of 6730; of 8675.

V. GENERAL REVIEW TO PERCENTAGE

REVIEW DRILL CHART ON THE OPERATIONS

Add rapidly, recording the time taken:

1. 1276	**2.** 9089	**3.** $12.75	**4.** $23.07	**5.** $371.42
2309	1627	26.42	41.68	126.09
5462	2314	3.97	33.99	83.54
1526	5403	40.21	8.27	102.07
7384	8675	16.95	65.45	29.68

Subtract rapidly, recording the time taken:

6. 7023	**8.** 6021	**10.** $29.43	**12.** $80.12	**14.** $928.30
4681	5236	18.75	9.47	429.72
7. 8274	**9.** 9231	**11.** $36.71	**13.** $92.20	**15.** $875.00
5328	4763	19.82	46.60	39.57

Multiply the first of these numbers by the second, and then divide the first by the second, giving the results in inexact divisions only to the nearest 0.01:

16. 75 by 840.
17. 960 by 25.
18. 125 by 960.
19. 880 by 125.
20. 625 by 440.
21. 625 by 640.
22. 640 by 375.
23. 480 by 375.
24. 800 by 625.
25. 144 by 12.
26. 240 by 144.
27. 288 by 24.
28. 288 by 48.
29. 1728 by 12.
30. 1728 by 48.
31. 1500 by 25.
32. 25 by 1280.
33. 1240 by 75.
34. 3714 by 39.
35. 2806 by 73.
36. 4026 by 123.
37. 2708 by 236.
38. 7432 by 247.
39. 6296 by 793.
40. 1225 by 326.
41. 3265 by 472.
42. 2766 by 395.

REVIEW DRILL CHART IN FRACTIONS

Add rapidly, recording the time taken:

1.	**2.**	**3.**	**4.**	**5.**	**6.**
$\frac{1}{2}$	$1\frac{1}{2}$	$3\frac{1}{2}$	$8\frac{7}{8}$	$9\frac{3}{8}$	$7\frac{3}{16}$
$\frac{3}{4}$	$3\frac{3}{4}$	$5\frac{1}{4}$	$2\frac{3}{4}$	$7\frac{5}{16}$	$2\frac{5}{8}$
$\frac{5}{8}$	$2\frac{5}{8}$	$6\frac{7}{8}$	$5\frac{1}{2}$	$9\frac{3}{4}$	$9\frac{1}{2}$

Subtract rapidly, recording the time taken:

7.	**8.**	**9.**	**10.**	**11.**	**12.**
$\frac{7}{8}$	$\frac{7}{8}$	$9\frac{1}{2}$	$8\frac{1}{16}$	9	$8\frac{1}{3}$
$\frac{3}{4}$	$\frac{1}{2}$	$3\frac{3}{4}$	$3\frac{7}{8}$	$3\frac{2}{3}$	$3\frac{3}{5}$

Multiply rapidly, recording the time taken:

13.	**14.**	**15.**	**16.**	**17.**	**18.**
$1\frac{1}{2}$	$3\frac{3}{4}$	$5\frac{7}{8}$	$6\frac{3}{4}$	$7\frac{7}{8}$	$4\frac{2}{3}$
$\frac{3}{4}$	$\frac{1}{2}$	$2\frac{1}{2}$	$2\frac{1}{2}$	$3\frac{1}{3}$	$4\frac{2}{3}$

Divide rapidly, recording the time taken:

19. $2 \div \frac{3}{4}$. **21.** $\frac{1}{2} \div \frac{7}{8}$. **23.** $\frac{2}{3} \div \frac{3}{5}$. **25.** $2\frac{7}{8} \div 3\frac{3}{4}$.

20. $\frac{3}{4} \div 2$. **22.** $\frac{7}{8} \div \frac{1}{2}$. **24.** $\frac{3}{5} \div \frac{2}{3}$. **26.** $3\frac{3}{16} \div 3\frac{3}{4}$.

27. Add $\frac{2}{3}$ and $\frac{3}{4}$ and divide the result by $2\frac{1}{2}$.

28. Multiply $3\frac{1}{2} - 1\frac{3}{4}$ by $5\frac{2}{3}$.

29. From $5\frac{3}{4} + 2\frac{7}{8}$ subtract $2\frac{3}{8} - \frac{5}{16}$.

30. Find the sum of $1\frac{1}{2}$, $2\frac{7}{8}$, $1\frac{3}{4}$, $5\frac{1}{2}$, and $6\frac{5}{8}$.

31. Find the difference between $3\frac{1}{2} + 2\frac{7}{8}$ and $1\frac{1}{4} + 2\frac{5}{8}$.

32. Find the product of $1\frac{1}{2} + 9\frac{7}{8}$ and $3\frac{1}{4} + 2\frac{7}{8}$.

33. Divide $8\frac{1}{2} + 3\frac{5}{8} + 2\frac{3}{4}$ by $4\frac{1}{4} + 1\frac{13}{16} + 1\frac{3}{8}$.

34. Add $2\frac{3}{4}$ and $5\frac{7}{8}$ and subtract $1\frac{3}{8}$ from the sum.

35. What number multiplied by $2\frac{5}{8}$ becomes $5\frac{1}{4}$? $7\frac{7}{8}$?

36. What number subtracted from $2\frac{1}{2}$ becomes $\frac{7}{8}$? $\frac{21}{32}$?

REVIEW DRILL CHART IN FRACTIONAL PARTS

Read, supplying the missing numbers:

1. $12\frac{1}{2}$¢ = $\frac{1}{8}$ of \$1.
2. 25¢ = —— of \$1.
3. $66\frac{2}{3}$¢ = —— of \$1.
4. $6\frac{1}{4}$¢ = —— of \$1.
5. $87\frac{1}{2}$¢ = —— of \$1.
6. 40¢ = —— of \$1.
7. $37\frac{1}{2}$¢ = —— of \$1.
8. $16\frac{2}{3}$¢ = —— of \$1.
9. $62\frac{1}{2}$¢ = —— of \$1.
10. 60¢ = —— of \$1.
11. 75¢ = —— of \$1.
12. $33\frac{1}{3}$¢ = —— of \$1.
13. $33\frac{1}{3}$¢ = —— of \$2.
14. 75¢ = —— of \$2.
15. 60¢ = —— of \$2.
16. $62\frac{1}{2}$¢ = —— of \$2.
17. $6\frac{1}{4}$¢ = —— of \$2.
18. $66\frac{2}{3}$¢ = —— of \$2.
19. 25¢ = —— of \$2.
20. $12\frac{1}{2}$¢ = —— of \$2.
21. $37\frac{1}{2}$¢ = —— of \$3.
22. $62\frac{1}{2}$¢ = —— of \$3.
23. $87\frac{1}{2}$¢ = —— of \$3.
24. $\frac{1}{6}$ of \$1 = $16\frac{2}{3}$¢.
25. $\frac{7}{8}$ of \$1 = ——.
26. $\frac{2}{3}$ of \$1 = ——.
27. $\frac{5}{8}$ of \$1 = ——.
28. $\frac{3}{8}$ of \$1 = ——.
29. $\frac{1}{3}$ of \$1 = ——.
30. $\frac{1}{20}$ of \$1 = ——.
31. $\frac{1}{25}$ of \$1 = ——.
32. $\frac{7}{8}$ of \$1 = ——.
33. $\frac{5}{6}$ of \$1 = ——.
34. $\frac{1}{2}$ of \$2 = ——.
35. $\frac{1}{3}$ of \$2 = ——.
36. $\frac{3}{4}$ of \$2 = ——.
37. $\frac{2}{3}$ of \$3 = ——.
38. $\frac{1}{2}$ of \$3 = ——.
39. $\frac{3}{4}$ of \$3 = ——.
40. $\frac{1}{8}$ of \$3 = ——.
41. $\frac{3}{8}$ of \$3 = ——.
42. $\frac{1}{8}$ of \$4 = ——.
43. $\frac{7}{8}$ of \$4 = ——.
44. $\frac{1}{6}$ of \$4 = ——.
45. $\frac{1}{6}$ of \$15 = ——.
46. $\frac{1}{6}$ of \$25 = ——.

REVIEW DRILL CHART IN DECIMALS

Add rapidly, recording the time taken:

1. 12.07	**2.** 16.08	**3.** 231.4	**4.** 4.008
3.00	12.72	126.9	2.079
16.21	24.00	39.8	.630
4.09	76.75	107.0	8.000

Subtract rapidly, recording the time taken:

5. 32.01	**7.** 87.00	**9.** 501.21	**11.** 9.000
4.96	9.63	96.42	.078
6. 48.72	**8.** 75.05	**10.** 612.30	**12.** 8.123
19.39	62.78	149.79	4.690

Multiply rapidly, recording the time taken:

13. 24.09	**14.** 87.62	**15.** 722.31	**16.** 4.126
2.72	14.68	48.79	3.528

Divide, giving each result to the nearest 0.01:

17. 9273 ÷ 8.7.	**19.** 40.89 ÷ 28.	**21.** 8.289 ÷ 6.7.
18. 896.8 ÷ 9.2.	**20.** 49.72 ÷ 0.39.	**22.** 9.763 ÷ 0.52.

Find the square of

23. 1.7.	**25.** 2.43.	**27.** 16.8.	**29.** 0.23.
24. 3.9.	**26.** 4.71.	**28.** 29.5.	**30.** 0.09.

Find the square root of

31. 0.01. **32.** 0.09. **33.** 0.25. **34.** 6.25.

Find the cube root of

35. 1. **36.** 1000. **37.** 0.027. **38.** 64,000.

VI. REVIEW OF PERCENTAGE

REVIEW DRILL CHART IN PERCENTAGE

State the results as rapidly as you can:

1. 1 % of $10 ; of $15.
2. 4 % of $10 ; of $15.
3. 40 % of $10 ; of $15.
4. 25 % of $4 ; of $8.
5. 25 % of $5 ; of $10.
6. 20 % of $50 ; of $80.
7. 20 % of $60 ; of $70.
8. 20 % of $8 ; of $20.
9. 5 % of $10 ; of $60.
10. 5 % of $3 ; of $30.
11. 6 % of $40 ; of $48.
12. 6 % of $4 ; of $12.
13. 10 % of $10 ; of $18.
14. 10 % of $25 ; of $50.
15. 30 % of $10 ; of $30.
16. 30 % of $40 ; of $45.
17. $33\frac{1}{3}$ % of $60 ; of $90.
18. $33\frac{1}{3}$ % of $9 ; of $12.
19. $66\frac{2}{3}$ % of $9 ; of $27.
20. $66\frac{2}{3}$ % of $60 ; of $90.
21. 60 % of $2500.
22. 60 % of $2000.
23. 2 % of $1200.
24. 2 % of $2700.
25. 1 % of $35.40.
26. 2 % of $33.50.
27. $\frac{1}{2}$ of $4000.
28. $\frac{1}{2}$ % of $4000.
29. $\frac{1}{4}$ % of $8000.
30. $\frac{3}{4}$ % of $8000.
31. 3 × $6000.
32. 3 % of $6000.
33. 0.3 % of $6000.
34. 100 % of 1000.
35. 100 % of 0.01.
36. 200 % of 1.
37. 4 % of $2000.
38. 0.4 % of $2000.
39. $\frac{4}{5}$ % of $2000.
40. 400 % of $2000.

REVIEW DRILL CHART IN PERCENTAGE

Read, supplying the missing numbers:

1. 32 is ——% of 64.
2. 64 is ——% of 32.
3. 64 is ——% of 64.
4. 8 is ——% of 100.
5. 8 is ——% of 200.
6. 80 is ——% of 100.
7. 80 is ——% of 200.
8. 72 is ——% of 144.
9. 72 is ——% of 720.
10. 15 is ——% of 90.
11. 1 is ——% of 2.
12. 2 is ——% of 2.
13. 100 is ——% of 200.
14. 200 is ——% of 100.
15. 125 is ——% of 100.
16. 125 is ——% of 200.
17. 3 is ——% of 8.
18. 33 is ——% of 88.
19. 330 is ——% of 990.
20. 990 is ——% of 330.

Copy, supplying the missing numbers:

21. 25 is 20% of ——.
22. 60 is 10% of ——.
23. 48 is $37\frac{1}{2}$% of ——.
24. $12\frac{1}{2}$ is 25% of ——.
25. 27 is $37\frac{1}{2}$% of ——.
26. 70 is $87\frac{1}{2}$% of ——.
27. 80 is $16\frac{2}{3}$% of ——.
28. 20 is $8\frac{1}{3}$% of ——.
29. 25 is $12\frac{1}{2}$% of ——.
30. 80 is 40% of ——.
31. 25 is $\frac{1}{5}$ of ——.
32. 25 is $\frac{1}{5}$% of ——.
33. 60 is $\frac{1}{10}$% of ——.
34. 48 is $\frac{3}{8}$% of ——.
35. $12\frac{1}{2}$ is $\frac{1}{4}$% of ——.
36. 27 is $\frac{3}{8}$% of ——.
37. 70 is $\frac{7}{8}$% of ——.
38. 80 is $\frac{1}{6}$% of ——.
39. 20 is $\frac{1}{12}$% of ——.
40. 150 is $\frac{1}{3}$% of ——.

REVIEW OF DISCOUNT

A department store advertises a January mark-down sale. Find the sale prices of overcoats which are advertised at

	Regular Price	Discount
1.	$67.50	20%
2.	$65.00	20%
3.	$55.00	25%

	Regular Price	Discount
4.	$75.00	30%
5.	$50.00	20%
6.	$62.50	25%

Find the sale prices of suits which are advertised at

	Regular Price	Discount
7.	$47.00	25%
8.	$45.00	25%
9.	$42.00	25%
10.	$40.00	25%

	Regular Price	Discount
11.	$37.50	15%
12.	$35.00	15%
13.	$32.50	15%
14.	$30.00	15%

Find the per cent of discount offered on the regular price of fur-trimmed coats for women which are advertised at

	Regular Price	Sale Price
15.	$125.00	$75.00
16.	$115.00	$70.00
17.	$110.00	$68.00

	Regular Price	Sale Price
18.	$100.00	$60.00
19.	$90.00	$59.50
20.	$85.00	$49.90

Find the per cent of discount on the regular price of

21. Men's suits, regular price, $45.00; reduced to $39.50.

22. Boys' suits, regular price, $15.00; reduced to $12.75.

REVIEW OF PROFIT OR LOSS

Disregarding the expenses of doing business, which must, of course, be subtracted from the gain or added to the loss, state rapidly, in each of the following cases, the per cent of gain or loss on the cost, stating also whether it is a gain or a loss:

	COST	SELLING PRICE		COST	SELLING PRICE
1.	1¢	2¢	6.	$10	$12
2.	5¢	10¢	7.	$10	$25
3.	4¢	3¢	8.	$25	$20
4.	$1\frac{1}{2}$¢	3¢	9.	$50	$30
5.	$10	$15	10.	$75	$100

11. Cost, $1000; selling price, $500; selling price, $1500.

12. Cost, $800; selling price, $600; selling price, $1000.

As above, state the per cent of gain or loss, but base it on the selling price in the following cases:

13. Cost, 1¢; selling price, $1\frac{1}{2}$¢; selling price, 2¢.

14. Cost, 5¢; selling price, 6¢; selling price, 8¢.

15. Cost, 90¢; selling price, 80¢; selling price, $1.

16. Cost, $800; selling price, $750; selling price, $1000.

Find the amount of gain or loss and also the selling price when the cost and the per cent of gain or loss based on the cost are

17. Cost $500, gain 20%.
18. Cost $250, loss 20%.
19. Cost $600, loss 25%.
20. Cost $800, gain 10%.
21. Cost $75, loss $33\frac{1}{3}$%.
22. Cost $1260, loss 10%.
23. Cost $100, gain 30%.
24. Cost $1.48, loss 10%.

Copy the following table, which shows amounts and per cents of gain figured on the selling price, and fill the blank columns, checking the selling price in each case by taking the given per cent of it and comparing the result with the given amount of gain:

	COST	GAIN	PER CENT	SELLING PRICE
25.		\$25	50 %	
26.		\$50	$33\frac{1}{3}$ %	
27.		\$25	25 %	
28.		\$20	$16\frac{2}{3}$ %	
29.		\$20	20 %	
30.		\$20	25 %	
31.		\$5	$6\frac{1}{4}$ %	
32.		\$75	$33\frac{1}{3}$ %	
33.		\$125	$62\frac{1}{2}$ %	
34.		\$13.94	17 %	
35.		\$213.44	28 %	
36.		\$668.25	45 %	

37. If you buy articles at 5 for 3¢ and sell them at 5 for 10¢, what is your per cent of profit figured on the selling price? figured on the cost?

38. If $\frac{1}{2}$¢ is a gain of $33\frac{1}{3}$%, figured on the selling price, what is the selling price? What is the cost?

39. If $3\frac{1}{3}$¢ is a gain of $33\frac{1}{3}$%, figured on the selling price, what is the selling price? What is the cost? What is the per cent of gain on the cost?

40. Solve Ex. 39 for $3\frac{3}{4}$¢ and $37\frac{1}{2}$%; for $7\frac{1}{2}$¢ and 75%; for $16\frac{2}{3}$¢ and $33\frac{1}{3}$%.

REVIEW OF INSURANCE

1. The owner of a row of eight dwellings insures each one of them against loss by fire for $5800. If the annual rate for fire insurance on this class of property is 34¢ per $100, how much does he pay in premiums if the policies are written for 1 yr.?

2. If, in Ex. 1, the rate for a 3-year policy is $2\frac{1}{2}$ times the annual rate, what will the owner pay in premiums if he takes the insurance for 3 yr.?

3. If, in Ex. 1, the rate for a 5-year policy is four times the annual rate, what will the owner pay in premiums if he insures his property for a 5-year period?

4. A mill owner insures his building against loss by fire for $134,500, his machinery for $75,000, and his stock of materials for $65,500. The annual rate on the building is $1.25 per $100, and on the machinery and materials it is $1.45 per $100. What premium does he pay for a 3-year policy (see Ex. 2)?

5. A department-store building is valued at $125,000, and the contents are valued at $225,000. The owner takes out fire insurance for 80% of the value at an annual rate of 89¢ on the building and $1.19 on the contents. What premium does he pay for a 5-year policy (see Ex. 3)?

6. The owner of a dwelling insured it for $5000 for 3 yr. at an annual rate of 36¢ (see Ex. 2). At the end of 1 yr. 4 mo. he sold the property to a man who agreed to pay for the unexpired insurance. How much did the purchaser pay to the owner on this account?

7. A doctor insured his dwelling for $6500, his furniture for $3250, and his office fixtures and medical supplies for $1200. The annual rate on the dwelling was 35¢, on the furniture 39¢, and the rate on the fixtures was the furniture rate plus 10%. What premium did he pay for a 3-year policy (see Ex. 2)?

8. The fire-insurance premiums in a town which had no public fire protection amounted, during a certain year, to \$3675.75. When a fire department and a water system were installed, the insurance companies reduced the rates 15%. For the same amount of insurance, what were the premiums the year after the reduction?

9. The total premiums received by a fire-insurance company in a certain year were \$1,341,225. During the year the fire losses paid were $52\frac{1}{2}$% of the premiums, and the expenses were $45\frac{3}{4}$%. How much did the company pay that year to meet the losses and expenses?

10. A merchant paid a premium of \$929.50 annually on a fire-insurance policy for \$65,000. What was the rate?

11. On his twenty-seventh birthday, when the rate was \$23.14 per \$1000, a man insured his life for \$3500. If he lived to be 47 yr. old, how much did he pay the company in premiums?

12. A man takes out a 20-payment life-insurance policy for \$8000 at an annual rate of \$35.26 per \$1000. During the 20 yr. the dividends paid to the policy holders averaged \$6.80 per \$1000 per year. What was the actual cost to him of the policy by the time it was paid up?

13. As in Ex. 12, find the actual cost of a 30-payment life-insurance policy for \$15,000, when the annual rate is \$25.34, and the dividends average \$4.20 per \$1000 of insurance.

14. A man had an accident-insurance policy that provided weekly payments of \$50 for total disability and \$18.75 for partial disability. An accident disabled him totally for 6 wk. and partially for 12 wk. How much did he receive?

15. A vessel was insured for \$750,000, and the cargo for \$356,750. On account of damage by a storm the insurance company paid 6% of the face of the policy on the vessel and 32% on the cargo. How much was paid?

REVIEW OF COMMISSION

In each of the following cases, which show the amounts of sales and the rates of commission paid, find the amount of the commission and the proceeds:

1. \$650, 2%.
2. \$1300, 3%.
3. \$1650, 4%.
4. \$1895, 5%.
5. \$3200, $3\frac{1}{2}\%$.
6. \$3520, $4\frac{1}{2}\%$.
7. \$1675, 8%.
8. \$2250, $8\frac{1}{2}\%$.
9. \$8500, $3\frac{1}{2}\%$.
10. \$12,000, $4\frac{1}{4}\%$.
11. \$12,500, $3\frac{1}{4}\%$.
12. \$16,550, $3\frac{3}{4}\%$.
13. \$18,500, $5\frac{1}{2}\%$.
14. \$22,500, $6\frac{7}{8}\%$.
15. \$18,625, $5\frac{1}{4}\%$.
16. \$20,750, $4\frac{3}{4}\%$.
17. \$80.50, 8%.
18. \$69.35, 7%.
19. \$51.27, 10%.
20. \$98.25, $4\frac{1}{2}\%$.
21. \$67.20, $12\frac{1}{2}\%$.
22. \$86.40, 15%.
23. \$112.25, 12%.
24. \$150.60, 17%.

In each of the following cases, which show the amounts of sales and the net amounts received from the sales, find the amount and rate of commission paid:

25. \$500, \$450.
26. \$600, \$576.
27. \$700, \$665.
28. \$650, \$624.
29. \$320, \$302.40.
30. \$480, \$441.60.
31. \$640, \$598.40.
32. \$730, \$697.15.
33. \$10,000, \$9650.
34. \$12,500, \$11,875.
35. \$14,225, \$13,656.
36. \$16,250, \$14,950.

37. If possible, find out the customary rates of commission charged by local real-estate agents for selling various classes of property. Make up and bring to class two or three problems about selling pieces of property near the school.

38. Find out, if possible, the rates of commission charged by commission merchants in your city or in a neighboring city for handling farm produce. Make up and bring to class several problems about selling farm produce on commission.

39. Find out about architects' fees and the customary rates, and make up problems about them.

REVIEW OF TAXES

At a tax rate of $1.36 per $100 find the taxes paid on property which is assessed at

1. $600.
2. $250.
3. $375.
4. $840.
5. $1250.
6. $8000.
7. $9200.
8. $5850.
9. $6250.
10. $8375.
11. $10,000.
12. $12,000.
13. $13,500.
14. $16,250.
15. $20,500.
16. $55,000.
17. $75,500.
18. $125,000.
19. $225,000.
20. $675,000.

At a tax rate of $14.70 per $1000 find the taxes paid on property which is assessed at

21. $350.
22. $780.
23. $1220.
24. $9300.
25. $7560.
26. $8525.
27. $12,500.
28. $16,250.
29. $15,500.
30. $60,000.
31. $72,500.
32. $128,500.

33. A city with a total assessed valuation of $75,000,000 has to raise $1,845,000 to meet the year's budget of expenses. What will be the tax rate on each $1000 of assessed valuation?

34. Solve Ex. 33 for a town which has to raise $56,250 and has a total valuation of $3,500,000.

35. After deducting the exemptions to which he is entitled, a man must pay an income tax on $3750.68 at a rate of 2%. Find the amount of his income tax.

36. In a year in which the receipts of the national government totaled $4,012,000,000 the amount raised by taxes on incomes was $1,842,000,000. Income taxes yielded what per cent of the total receipts?

37. Of the total income-tax receipts mentioned in Ex. 36, $506,600,000 were collected in New York State. What per cent of the total was paid by New York?

A SILENT READING LESSON

Besides the uses of per cents which we have been considering, we shall find that there are many others. In fact, in everyday life we shall find that per cents, because of their particular value in helping us to see the relation between numbers, are used in almost every situation in which we wish to compare numbers.

For example, the attendance in our schools is increasing at a more rapid rate than our population; that is, it is increasing more rapidly than the number of those who pay the taxes to support the schools. This, of course, increases the amount that each taxpayer must contribute for the education of our boys and girls.

The following table shows, for a period of several years, the estimated population of New York State and the average daily attendance in our elementary schools and high schools:

YEAR	ESTIMATED POPULATION	PER CENT OF INCREASE	SCHOOL ATTENDANCE	PER CENT OF INCREASE
1915	9,687,744		1,288,047	
1916	9,827,240		1,318,605	
1917	9,966,736		1,318,287	
1918	10,106,232		1,297,271	
1919	10,245,728		1,310,826	
1920	10,385,227		1,361,600	
1921	10,524,723		1,443,657	
1922	10,664,219		1,518,781	
1923	10,803,715		1,582,364	

As we look at the table, we cannot tell whether the population or the school attendance is increasing more rapidly until we find the per cents of increase from year to year, a problem which we shall consider on page 289.

SCHOOL EXPENDITURES

1. Copy the table given on page 288 and fill the blank columns, finding the per cent of increase from one year to the next to the nearest 0.1%. Write a statement of the conclusions that you can then make about the rates of increase.

2. The high-school enrollment in the United States in 1890 was 297,894; in 1900, 630,048; in 1910, 1,032,461; and in 1920, 2,041,308. Find the per cent of increase in this enrollment from 1890 to 1900; from 1900 to 1910; from 1910 to 1920; from 1890 to 1920.

3. The population of the United States was 62,947,714 in 1890 and 105,710,620 in 1920. Find the per cent of increase. This is how much less than the last result found in Ex. 2?

4. Copy the following table, which shows the cost of maintaining the different parts of the Rochester school system in a recent year, and fill the blank column at the right, finding the cost of maintaining each of the units to the nearest cent:

SCHOOL UNITS	PER CENT OF TOTAL	COSTS
1. Kindergarten	6.3%	
2. Elementary Grades . . .	46.6%	
3. Junior High Schools . . .	19.3%	
4. Senior High Schools . . .	16.5%	
5. City Normal	.8%	
6. Other Teacher Training .	.7%	
7. Special Education	5.7%	
8. Continuation Schools . .	2.1%	
9. Adult Education	1.7%	
10. Summer Schools	.3%	
Totals	100.0%	$6,549,838.20

CAREFUL BUYING

1. In Boston, consumers pay higher prices for brown eggs, while in New York City they pay higher prices for white eggs, although chemical analysis has shown that there is no difference in the quality of white and brown eggs. If white eggs are selling for 72¢ per dozen and brown eggs are selling for 66¢ in the same city, what per cent of the lower price can be saved by buying brown eggs?

2. Bright red apples often bring higher prices than other apples, although they may be much poorer for eating. If red apples sell for $5 per bushel, and equally good yellow apples sell for $3.50 per bushel, what per cent of the lower price can be saved by buying the cheaper apples?

3. In the North yellow corn meal is in demand, while in the South white corn meal is more popular, although the nutritive value of each is the same. When yellow meal is selling at $6\frac{1}{2}$¢ a pound and white meal at 12¢, what is the per cent of saving by buying the cheaper meal?

4. Southern markets prefer yellow to red onions, although, when their skins are removed, the onions are just the same in taste and food value. A saving of 18¢ on a purchase amounting to 98¢ is what per cent of saving?

5. Cheaper cuts of beef, such as the neck, have the same nutritive value as the more expensive cuts, and with proper cooking can be made just as palatable. If a housekeeper serves a chuck roast costing 22¢ a pound in place of a sirloin roast costing 40¢ a pound, what per cent of the lower cost does she save by so doing?

6. If possible, obtain at local stores prices of different cuts of meat. Make up and bring to class five problems about saving money in purchasing meats.

Food in Packages. There are many foods that can be purchased either in packages or in bulk. The package increases the cost, because the boxes, or containers, the printing, the paper, and the labor must all be paid for by the consumer. The following table shows the prices per pound of several foods which can be bought either in bulk or in packages:

	Bulk Price per Pound	Package Price per Pound	Extra Cost in Packages	Per Cent of Extra Cost
Bacon	42.0¢	58.2¢	16.2¢	38.6%
Cocoa	25.0¢	45.0¢		
Coffee	42.0¢	48.0¢		
Corn meal . .	6.5¢	10.0¢		
Graham flour .	9.5¢	10.0¢		
Macaroni . . .	6.25¢	16.0¢		
Oatmeal . . .	10.0¢	10.4¢		
Oyster crackers	20.0¢	30.0¢		
Rye flour . . .	7.5¢	9.0¢		
Soda crackers .	22.0¢	30.0¢		
Tapioca . . .	15.0¢	25.0¢		

PAYING FOR THE PACKING

1. Copy the above table and fill in the blank columns.

2. Find the average cost per pound in bulk and in packages of the articles in the table. Find also the average extra cost per pound and the average per cent of extra cost.

3. Get similar prices from local stores and find the per cent of saving by buying in bulk. State whether or not you think that the cleanliness of the package offsets the extra cost.

4. Make up five problems to show how much can be saved by a family in making purchases in bulk of various amounts of some of the foods in the table.

Meat and Health. Our school nurse says that the American people eat too much meat and not enough vegetables. The following table shows the average amount of meat consumed by each person in seven different countries in a recent year:

Country	Pounds per Person per Year
United States	187
Germany	115
England	105
Belgium and Holland	75
France	74
Russia	59
Italy	23

PROBLEMS ABOUT MEAT-EATING

1. Find the average consumption for all the countries.

2. What per cent more meat did the average person in the United States eat than the average person in Italy? than the average for all the countries?

3. What per cent of the consumption of meat in the United States was the consumption in Italy? in Russia?

4. What per cent more meat did we use than England? than Germany? than France?

5. In a year when the average consumption of meat in this country was 194 lb. per person per year, the consumption was what per cent greater than in the year for which the figures are given in the table above?

6. If the same food value can be had from 14¢ worth of milk as from 38¢ worth of sirloin steak, what per cent of saving could be made by using milk instead of steak?

Using Vegetables. Reliable authorities say that in the United States we eat only about one third as many green vegetables as we need to keep in good health. In food value, the leading green vegetable is spinach, which is not sufficiently used in this country. The following table shows the number of units of food value in a standard amount of several common vegetables:

VEGETABLES	VALUES
Spinach	810
Green peas . . .	475
Green beans . .	472
Potatoes	414
Sweet potatoes .	374
Asparagus . . .	368

VEGETABLES	VALUES
Cabbage	367
Celery	350
Carrots	338
Lettuce	299
Onions	295
Tomatoes . . .	192

FOOD VALUE OF VEGETABLES

1. Spinach has what per cent greater food value than asparagus? than onions?

2. Green peas have what per cent greater food value than carrots? than celery?

3. As in Exs. 1 and 2, compare the food value of potatoes with that of cabbage; with that of lettuce; with that of asparagus.

4. Compare the food value of green beans with that of sweet potatoes; with that of carrots; with that of onions.

5. The food value of asparagus is what per cent greater than that of onions? The food value of onions is what per cent less than that of asparagus?

6. The food value of lettuce is what per cent less than that of green peas? The food value of green peas is what per cent greater than that of lettuce?

How We Use Milk. In a recent year the milk production in the United States was 89,842,000,000 lb. This table, in which the numbers represent millions, shows how it was used:

How Used	Million Pounds	Per Cent of Total
Household purposes . . .	38,619	
Creamery butter	18,375	
Dairy butter	14,385	
Condensed milk	4,813	
Cheese	4,200	
Feeding calves	3,500	
Ice cream	3,450	
Wasted	2,500	
Totals	89,842	100 %

USES OF MILK

1. Copy the above table and then complete it by finding the per cent of the total amount used for each purpose.

2. What per cent more milk was used for household purposes than for creamery and dairy butter combined?

3. Compare the amount wasted with that used for making cheese by finding what per cent the former is of the latter.

4. What per cent of the total amount used for making butter was used for dairy butter?

5. What per cent of the ice cream that we use could be made with the wasted milk?

6. What per cent of the calves could be fed with the amount of milk that is wasted?

7. What per cent less milk is used for making dairy butter than is used for creamery butter?

Butter Fat in Milk. In 1890 Dr. Babcock invented the Babcock milk tester, which makes it possible to determine easily the per cent of butter fat (the chief constituent in butter) in milk. He donated his invention to the world, although he could have sold it for millions of dollars.

The composition of standard butter, as determined by the United States Department of Agriculture, is as follows:

Butter fat . .	82.41%	Salt	2.51%
Water	13.90%	Curd	1.18%

PROBLEMS ABOUT BUTTER FAT

1. Using the above table, find the weight of each of the constituents in 720 lb. of standard butter.

2. The dairymen who sell milk to a large factory in central New York are paid in proportion to the per cent of butter fat in the milk which they sell. This table shows the price recently paid per quart for milk containing the per cent of butter fat stated. Find the amount received for 50 qt. of each of the grades of milk given in the table.

Per cent	Price
5.6%	10.6¢
4.7%	9.7¢
4.6%	9.6¢
4.1%	9.1¢
4.0%	9.0¢
3.9%	8.9¢
3.8%	8.8¢

3. From this table find what per cent more the milk that tests 5.6% butter fat is worth than that which tests 3.8%.

4. Farmers who sell cream are paid for the butter fat the cream contains and not for the weight of the cream. How many pounds of cream testing 25% butter fat will it take to equal in value 2500 lb. testing 35% butter fat? How many pounds testing 35% will equal in value 2500 lb. testing 25%?

MISCELLANEOUS PROBLEMS

1. A farmer was offered \$1400 for his potato crop in October, but by waiting until spring he received \$1620 for it. Deducting \$42 to offset the interest lost on the \$1400, find the per cent gained by holding the crop.

2. Last year the total registration in our schools was 1830, and the average daily attendance was 1710. The attendance was what per cent of the total registration?

3. Mr. Park bought his winter supply of potatoes last fall at \$1.40 a bushel. His neighbors, who bought their potatoes during the winter in small quantities, had to pay an average price of 4¢ a pound. Allowing 60 lb. to the bushel, what per cent of the higher price did he save by this method?

4. Mr. Smith and his son bought a motor truck for \$5000. Mr. Smith paid \$3700 and his son paid the balance. What per cent did each pay?

5. Which gives the better per cent of profit, an investment of \$12,000 which yields a profit of \$900 or an investment of \$10,000 which yields a profit of \$800?

6. The estimated population of New York State in 1925 was 11,105,625, and of this number 6,015,496 lived in New York City. What per cent lived in New York City?

7. In 1925 the estimated population of the United States was 113,493,720, and of this number 11,105,625 lived in New York State. What per cent lived in New York State?

8. There were 720 pupils in School No. 5 in one of our smaller cities, and of this number 288 were in the first three grades. What per cent were in these three grades?

9. Ben's scores on eight tests were 84%, 88%, 90%, 90%, 92%, 85%, 87%, and 96%. Find his average score.

VII. REVIEW AND DRILL

MINIMUM ESSENTIALS

Add and also subtract as follows:

$$\begin{array}{rrrr} \textbf{1. } \$427.32 & \qquad \textbf{2. } \$932.01 & \qquad \textbf{3. } \$870.23 & \qquad \textbf{4. } \$900.01 \\ \underline{198.56} & \underline{687.56} & \underline{592.68} & \underline{48.75} \end{array}$$

Multiply as follows:

$$\begin{array}{rrrr} \textbf{5. } \$423.78 & \qquad \textbf{6. } \$573.81 & \qquad \textbf{7. } \$609.98 & \qquad \textbf{8. } 23 \text{ ft. } 8 \text{ in.} \\ \underline{0.65} & \underline{0.74} & \underline{127} & \underline{15} \end{array}$$

Divide, in turn, as follows:

9. 864 by 16, 32, and $\frac{4}{5}$.

10. 9.72 by 0.36 and 36.

11. 58 ft. 8 in. by 8 and 4.

12. 64 lb. 2 oz. by 9 and 12.

Express the following as decimals and also as per cents:

13. $\frac{1}{2}$. **14.** $\frac{3}{4}$. **15.** $\frac{5}{8}$. **16.** $\frac{2}{3}$. **17.** $\frac{5}{6}$. **18.** $\frac{5}{12}$. **19.** $\frac{11}{12}$.

20. At 35¢ per square foot, find the value of a rectangular building lot with a frontage of 83.2 ft. and a depth of 102.5 ft.

21. Find the volume of a rectangular stone block which is 28 in. long, 14 in. wide, and 18 in. high.

22. At 190 lb. per cubic foot, find the weight in Ex. 21.

23. Find the area of a circle with a radius of 4.9 in.

24. Find the area of the curve surface and the volume of a cylinder 18.9 in. high and 1 ft. in diameter.

Find the number represented by x in each of these proportions:

25. $\frac{x}{24} = \frac{13}{26}$.

26. $\frac{x}{57} = \frac{14}{42}$.

27. $\frac{x}{1.4} = \frac{3.5}{17.5}$.

28. $\frac{x}{3.2} = \frac{6.7}{53.6}$.

29. $\frac{14.5}{4.35} = \frac{x}{0.9}$.

30. $\frac{6.75}{1.35} = \frac{x}{25}$.

PROBLEMS WITHOUT NUMBERS

All work oral

1. If you know the number of cups of flour needed for a certain number of loaves of bread, how do you find the number of cups of flour needed if you wish to make a certain other number of loaves?

2. If you know how much water flows through a pipe in a minute, how do you find the amount of water that will flow through the pipe in a whole day, at the same rate?

3. If you know the capacity of a water tank in cubic feet, how do you find its capacity in gallons?

4. If you know what per cent of its original length an iron rod expands when heated from an ordinary temperature to red heat, how do you find the expansion in heating to red heat an iron rod which is a given number of inches long?

5. If you know the number of hours that a man works each day of the week, and the rate at which he is paid per hour, how do you find the amount of money which is due him at the end of the week?

6. At a given price per square yard, how do you find the cost of laying a concrete walk in front of the schoolhouse?

7. At a given price per cubic yard, how do you find the cost of making a rectangular excavation for the cellar of a house?

8. If you know how much a cubic foot of granite weighs, how do you find the weight of a granite cylinder when you know the radius and the height?

9. How can you find the height of your school building by means of shadows and the height of a post?

10. How can you find the height of a tree by sighting over the top of a stick of known length?

PROBLEMS FOR COMPLETION

1. A man bought 5 loads of coal which weighed 2475 lb., 2830 lb., 1970 lb., 2150 lb., and 1950 lb. The coal cost him $11.45 a ton. Complete the problem in any reasonable way and then solve it.

Complete and solve the following problems:

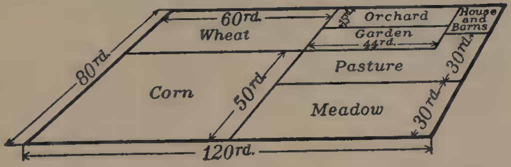
2. A grocer sold Mrs. Towne $4\frac{1}{4}$ lb. of butter @ 52¢, 3 bunches of celery @ 18¢, 12 lb. of beans @ 16¢, 8 cans of corn @ 18¢, and 2 bottles of olives @ 60¢.

3. An electric meter indicated 198 K. W. H. on May 1 and 251 K. W. H. on June 1. The company which supplies the electricity charges $11\frac{1}{2}$¢ per K. W. H.

4. On a bill of goods amounting to $170, a dealer is allowed a discount of 10%, 8%.

5. A cow gave 675 lb. of milk in a month, and the milk tested 4.2% butter fat.

6. A rectangular farm 120 rd. long and 80 rd. wide is laid out as shown in this plan. Some of the dimensions are not given, but are easily found from the plan.



7. A cylindric water tank is 10 ft. in diameter and 8 ft. high. A cubic foot is equivalent to $7\frac{1}{2}$ gal.

8. A grain bin is 18 ft. long, 14 ft. wide, and 6 ft. deep. A bushel is approximately equivalent to $1\frac{1}{4}$ cu. ft., and the exact equivalent of the bushel is 2150.42 cu. in.

9. A city lot in the form of a rectangle is 22.5 ft. along the front, and the area of the lot is 1080 sq. ft.

FARM–CLUB PROBLEMS

1. In a calf-feeding contest held by a farm club, a boy fed a calf during a period of 120 da. The calf gained, on an average, $1\frac{3}{4}$ lb. per day. If the calf weighed 70 lb. when the contest began, how much did it weigh when the contest ended?

2. A boy bought a calf for $35 and entered a calf-feeding contest. His expense account showed the following items:

600 lb. whole milk @ $2.50 per hundredweight;
1200 lb. skim milk @ 50¢ per hundredweight;
2 bu. whole oats @ 65¢ per bushel;
50 lb. oil meal @ $3 per hundredweight;
120 lb. clover hay @ $1 per hundredweight.

If he sold the calf for $72, how much did he receive, above these expenses, for his labor?

3. If a boy feeds some calves 150 lb. of hay at $20 per ton, how much should he charge to his expense account?

4. A boy fed a calf for a certain period, during which the calf gained 325 lb. in weight. The expenses were as follows:

Whole milk	$11.00	Hay	$2.00
Skim milk	5.00	Oil meal	.50
Oats	1.65	Labor	6.50
Bran	1.20	Medicine	.75

Find the total cost per hundredweight of gain.

5. How long will $2\frac{1}{2}$ bu. of oats, weighing 32 lb. per bushel, last if a boy feeds 1 lb. twice each day to each of two calves?

6. In a certain year the corn clubs in this country reported that they raised 523,000 bu. of corn at an expense of $143,000. If the corn was sold at an average price of 61¢ a bushel, how much, above expenses, was received?

7. Each of two boys in a farm club planted an acre of potatoes. One boy treated his seed potatoes chemically (so that they would grow better) at an expense of $1.10, but the other did not. The first boy raised 200 bu. of potatoes, which he sold at an average price of $1.48 per bushel. The other boy raised only 140 bu., which he had to sell at 98¢ per bushel, because they were of poorer quality. How much did the first boy gain by treating the seed potatoes?

8. A bushel of potatoes weighs 60 lb. If a boy carries 30 lb. and his sister carries 20 lb. on a trip from the field, how many bushels will they both carry together in 24 trips?

9. A boy raised 240 bu. of potatoes. He sold 35 bu. for seed potatoes at $2.25 a bushel, and of the remainder of the crop he sold 120 bu. at $1.35 a bushel and the rest at 90¢ a bushel. How much money did he receive?

10. A girl raised 10,800 lb. of potatoes, but the yield would have been increased one fifth by spraying. At 60 lb. to the bushel, how many bushels would she have gained by spraying?

11. A member of a farm club turned in the following account of his receipts and expenses in a recent year:

Receipts		Expenses	
Value of seed on hand	$4.00	Cost of seed	$1.80
Value of table stock	14.40	Chemical treatment	.20
Sales, selected	6.80	Rental of land	1.50
Sales, common	3.20	Spraying	.50
Prize won	2.00	Labor	12.80

Find the amount of the member's profit.

12. Allowing 60 lb. to the bushel, find the amount reported to a farm club as the receipts from the sale of 16,800 lb. of potatoes at $1.40 a bushel.

CANNING CLUBS

1. In a recent year the canning clubs reported that 7900 members canned, on an average, $25\frac{1}{2}$ qt. of food per member. Find the total number of quarts canned.

2. The clubs in Ex. 1 reported that the total cost of canning averaged 14¢ a quart. If the clubs sold the food at an average of 21¢ a quart, how much did they receive above expenses?

3. The canning club in a school near Dunkirk put up 3304 cans in one season. Sarah Kelley put up 104 of these cans. This was what per cent of the total number?

4. There are eleven girls in a canning club. Six of them put up 35 qt. each and the others put up 29 qt. each. How many quarts did the club put up?

5. A girl put up 4 qt. of fruit in pint cans each week day during the month of July, in a year in which that month had four Sundays. How many pint cans of fruit did she put up during the month?

6. A girl raised tomatoes, spending 60¢ for plants, 85¢ for fertilizer and spraying, $1.20 for lumber for trellises, and 5¢ apiece for 25 cans. She put up 25 quart cans, which she sold at 30¢ each. How much did she receive for her labor?

7. In canning peaches, a woman estimates that for each can the peaches cost 8¢, the sugar 3¢, the fuel 1¢, and the can itself 5¢. If she could buy peaches already put up at 25¢ a can, how much does she save on 50 cans by putting up the peaches herself?

8. Margaret raised some beans and sold 63 lb. at 5¢ a pound. The woman who bought the beans put them up in 30 one-quart cans. How much more would Margaret have received if she had canned the beans herself, at an expense of $8\frac{1}{2}$¢ per can, and sold them at 30¢ per can?

ECONOMY IN THE HOME

1. If each of the 114,000,000 persons in this country wastes on an average $1.10 worth of food per month, how much is wasted by all the population in a year?

2. If a housekeeper throws away, on an average, a slice of bread every day, and if there are 16 slices to a 10-cent loaf, how much money does she waste in a year (365 da.)?

3. If 10,000,000 housekeepers are as wasteful in this respect as the one referred to in Ex. 2, how much money do they all waste in this way in a year?

4. If the average waste of wheat flour in this country is 1 oz. per week for each of the 24,000,000 families, how many pounds does the country waste per year (52 wk.) in this way?

5. There are 196 lb. of flour in a barrel, and $4\frac{1}{2}$ bu. of wheat are required to make a barrel of flour. Find to the nearest hundred bushels the amount wasted annually in Ex. 4.

6. Allowing 16 bu. of wheat to the acre, how many acres of land would be required to supply the waste found in Ex. 5?

7. The average consumption of meat per person in this country was at one time 194 lb. per year. Some experts on diet recommend 4 oz. per day as sufficient for the average person. The average consumption for each person is how much more per year than is recommended?

8. If meat costs, on the average, 28¢ a pound, how much might be saved in a year by our population of 114,000,000 people if we brought the average consumption of meat down to 4 oz. per day for each person, as mentioned in Ex. 7?

9. If each of the 24,000,000 families in this country wastes, on an average, 1 oz. of meat per day, how many pounds are thus wasted in a year? How much is this worth at an average price of 28¢ per pound?

VIII. LITTLE EXAMINATIONS

I. 1. $\$1850.75 + \$260.50$.
2. $\$1127.38 - \$999.90$.
3. $125 \times \$72.16$.
4. $\$867.75 \div 0.35$.
5. $\frac{1}{2} + \frac{7}{8} - \frac{5}{16}$.
6. $1\frac{3}{4} + 2\frac{7}{16} - 3\frac{3}{8}$.
7. $\frac{3}{8} \times \frac{7}{16} \times 5\frac{1}{3}$.
8. $6\frac{3}{4} \div 3\frac{7}{8}$.

II. 1. Find the area of a rectangle 16.5 in. by 28.4 in.
2. Find the volume of a box 18 in. by 22 in. by 12 in.
3. Find the capacity in bushels of a bin holding 250 cu. ft.
4. Find the area of a circle with a radius of $5\frac{1}{4}$ in.
5. How do you find the volume of a cylinder?

III. 1. Separate 625 into two parts having the ratio 2 to 3.
2. Separate 1728 into two parts having the ratio 7 to 5.

Find the number represented by x in these proportions:

3. $\frac{x}{18} = \frac{7}{9}$.
4. $\frac{x}{15} = \frac{7}{3}$.
5. $\frac{x}{2\frac{1}{2}} = \frac{\frac{3}{4}}{\frac{2}{3}}$.

IV. 1. Find the square of 15; of 15.3; of 625.
2. Find the cube of 8; of 0.09; of 1.5.
3. Find the square root of 4489; of 6724; of 98.01.
4. Find, to the nearest 0.01: $\sqrt{1625}$; $\sqrt{88.69}$; $\sqrt{92.75}$.
5. Find the length of the diagonal of a floor 21 ft. by 28 ft.

V. 1. $0.37\frac{1}{2}\%$ of \$560.
2. 125% of \$200.
3. $66\frac{2}{3}\%$ of \$978.
4. $4\frac{3}{4}\%$ of \$1250.
5. $66\frac{2}{3}\% \div 8\frac{1}{3}\%$.
6. \$35 is 6% of \$ ——.
7. \$60 is $2\frac{1}{2}\%$ of \$ ——.
8. \$45 is —— % of \$135.
9. \$6.20 is —— % of \$49.60.
10. \$42.60 is —— % of \$284.

CHAPTER IV

I. BANKING PRACTICE

A SILENT READING LESSON

Last year we learned that banks exist chiefly to keep money safely and to lend the money needed to carry on business and industry. We learned about *savings banks* and *commercial banks,* the latter including *trust companies* which also manage estates and trust funds, and we learned that all these banks are closely supervised by *bank examiners* appointed by the state or national government.

We also learned about *bank books, deposits,* and *checks,* and about the *monthly statements* that the bank sends to a depositor so that he can compare his own record of his account with that kept by the bank.

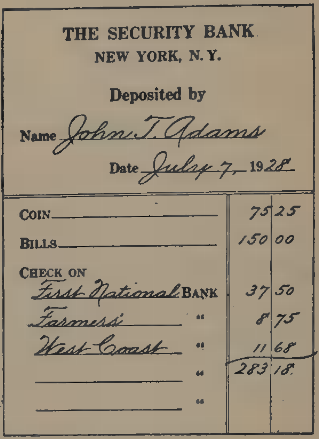
If we should visit a bank it would give us a better understanding of modern banking practice. We should then learn the difference between a *savings account* and a *checking account,* and why the former pays a higher rate of interest. We should also see why it is unlawful for anyone to write a check on a bank where he has no account, or why a bank regards as a poor business man one who writes a check for a larger amount than he has on deposit.

While at the bank we can also learn about *certificates of deposit,* which are used when money is to remain on deposit for some time, and about the *safe deposit boxes* that people rent for storing their valuables. All these things would make an interesting subject for class discussion.

Bank Forms. The most important things about banks that the average citizen needs to know are how to make deposits and how to draw checks.

The first form here given shows the *deposit slip* that is filled out when making a deposit in order to show the bank to whose account the amount deposited is to be credited.

The second form shows a *check* such as Mr. M. R. St. John will give to Mr. A. P. Jones if he wishes to pay Mr. Jones $4.68. In this case Mr. Jones is the *payee* and Mr. St. John is the *maker* of the check.



THE SECURITY BANK
NEW YORK, N. Y.

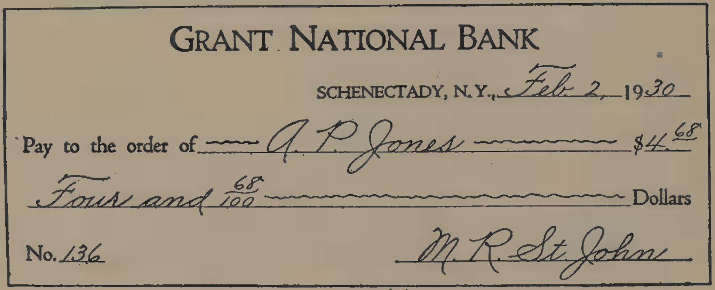
Deposited by

Name John T. Adams

Date July 7, 1928

COIN	75	25
BILLS	150	00
CHECK ON First National BANK	37	50
Farmers' "	8	75
West Coast "	11	68
"	283	18
"		

In cashing or depositing the check Mr. Jones will have to *indorse* it by signing his name across the back of the check.



GRANT NATIONAL BANK

SCHENECTADY, N.Y., Feb. 2, 1930

Pay to the order of A. P. Jones $4 68

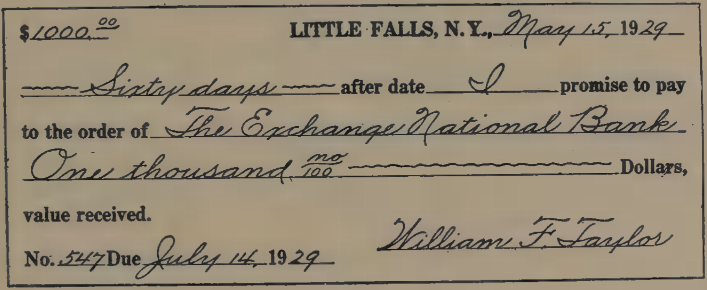
Four and $\frac{68}{100}$ Dollars

No. 136

M. R. St. John

As we learned last year, in order to remove any doubt as to whether or not a man has sufficient funds on deposit to pay a check, the maker may have it *certified* by taking it to his bank and having it stamped across the face to indicate that the bank guarantees its payment.

Promissory Note. When a bank lends money it requires the borrower to sign a *promissory note* for the amount of the loan in some such form as is shown in the following model:



$1000.00 LITTLE FALLS, N.Y., May 15, 1929
Sixty days after date I promise to pay
to the order of The Exchange National Bank
One thousand no/100 Dollars,
value received.
No. 547 Due July 14, 1929 William F. Taylor

In order to be sure that the note will be paid, the bank may require the borrower to "put up *collateral*"; that is, to leave with the bank some property of value, such as stocks or bonds, that can be sold by the bank in case the loan is not repaid. This collateral is, of course, returned to the borrower if the note is paid when due.

Bank Discount. As we learned last year, banks frequently lend money on notes like the one above, and deduct the interest, called *discount*, in advance. The balance, called the *proceeds*, is then paid to the borrower.

In case Mr. Taylor wishes to pay off part of the note (that is, to make a *partial payment*) before it is due, the bank will *rebate* (pay back to him) a proportionate part of the discount that he paid, and will indorse on the back of the note the amount paid off. Since some banks, in such cases, cancel the old note and issue a new note for the balance of the loan, the customs of local banks should be ascertained.

On interest-bearing notes, the general rule is first to apply the payment to the interest due to date and then to apply the balance to reducing the principal.

REVIEW DRILL CHART IN BANK DISCOUNT

Find as rapidly as you can the discount at 6% when the face and the period of discount are as follows:

1. $1500, 30 da.
2. $3600, 36 da.
3. $1960, 28 da.
4. $3750, 72 da.
5. $1690, 18 da.
6. $6300, 24 da.
7. $7100, 48 da.
8. $2950, 54 da.
9. $4200, 12 da.
10. $3700, 30 da.
11. $6200, 42 da.
12. $3200, 24 da.
13. $2750, 18 da.
14. $1590, 42 da.
15. $1850, 72 da.
16. $5100, 24 da.
17. $2300, 30 da.
18. $3000, 18 da.
19. $1890, 13 da.
20. $7550, 30 da.
21. $3650, 12 da.
22. $5700, 48 da.
23. $8375, 72 da.
24. $6450, 24 da.
25. $4500, 54 da.
26. $7200, 18 da.
27. $3680, 42 da.
28. $1570, 38 da.
29. $3500, 12 da.
30. $2500, 30 da.
31. $5800, 15 da.
32. $9400, 36 da.
33. $2000, 60 da.
34. $6000, 15 da.
35. $470, 48 da.
36. $450, 30 da.
37. $870, 20 da.
38. $900, 40 da.
39. $820, 90 da.
40. $550, 15 da.
41. $620, 60 da.
42. $150, 72 da.
43. $300, 90 da.
44. $650, 60 da.
45. $190, 30 da.
46. $840, 10 da.
47. $240, 15 da.
48. $850, 60 da.
49. $700, 80 da.
50. $350, 120 da.
51. $730, 90 da.
52. $320, 45 da.
53. $480, 45 da.
54. $530, 60 da.
55. $530, 75 da.
56. $560, 15 da.
57. $625, 30 da.
58. $750, 90 da.
59. $825, 60 da.
60. $850, 30 da.
61. $800, 15 da.
62. $900, 75 da.
63. $650, 90 da.
64. $140, 90 da.
65. $740, 48 da.
66. $480, 36 da.

REVIEW OF BANK FORMS

1. Suppose that you have an account at the People's Bank in your city or town. Write the deposit slip which you would fill out when making a deposit consisting of \$48.50 in cash and checks for \$17.25, \$6.50, and \$10.63.

2. As in Ex. 1, make out a deposit slip for a deposit consisting of \$68.73 in cash, a check on the Reed National Bank in payment of a bill for \$56.78 from which 2% was deducted for prompt payment, a check on an Oswego bank in payment of a bill for \$425.80 less 30%, and a check on the State Trust Co. in payment of a bill for \$274.65 less 3%.

3. If Mr. Clark's bank balance is \$273.50, and he draws checks for \$17.68 and \$29.75, how much money does his check book then show that he has in the bank?

4. Write a check, dated today, for \$2.50 on the Corner Trust Company, Buffalo, N. Y., payable to the order of John Doe, and sign it with your name. Then indorse the check as John Doe should indorse it in order to cash it.

5. Suppose that Mr. Rogers has \$247.50 in the bank on the first day of the month and that during the month he deposits \$48.75, \$187.60, \$268.90, \$232.75, \$14.68, \$142.30, \$72.70, and \$68.65. If during the month he draws checks for \$192.68, \$95.43, \$110.70, \$182.50, \$73.20, \$15.40, \$7.26, \$6.93, \$3.27, and \$28.75, what is his balance on the last day of the month?

6. Suppose that you pay by check the following bills for goods purchased and are allowed the discounts specified: \$427.60, less 2%; \$106.80, less 20%, 2%; \$649.50, less 10%, 5%, 2%; and \$327.75, less 15%, 5%, 5%. If your bank balance was \$1878.93 before you wrote the checks for the bills, how much money does your check book show that you now have in the bank?

7. If Richard Roe borrows $175 at his bank for 30 da. at 6%, how much discount (interest) does he pay?

8. In Ex. 7, if Mr. Roe borrows twice as much for twice as long, how much discount does he pay?

9. Write a promissory note for the amount borrowed in Ex. 7, making it payable to the order of the bank (inserting the name of some bank that you know). The note should not specify any interest, since the discount was paid at the time the loan was made.

10. Mr. Sinclair borrows $750 from his brother and gives a note for 6 mo., bearing interest at 6%. If he pays the principal and interest by check on the date the note is due, what is the amount of the check which he draws?

11. On an invoice amounting to $1782.50, for goods for the holiday trade, a merchant is allowed a discount of 3% for prompt payment. In order to take advantage of the reduction, he will have to discount a 60-day note for $1750 at his bank. If the discount rate is 6%, how much does he save by borrowing the money and paying the invoice promptly?

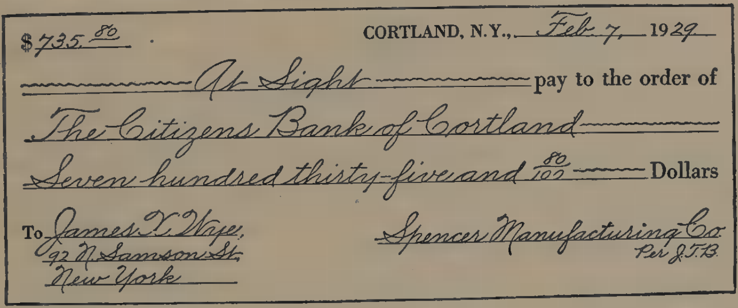
12. A man can buy an automobile for $2275 in cash, or by paying $1000 down and dividing the balance into three notes for equal amounts, due in 30 da., 60 da., and 90 da. respectively, with interest at 6%. If he takes the second plan, how much does he pay for the car? How much could he have saved by paying cash?

13. An automobile dealer borrowed $20,000 on a 90-day note, which his bank discounted at 6%. If at the end of 30 da. he paid off $5000 on the note, how much discount was rebated? If at the end of 60 da. he pays off $8000 more, how much discount is rebated? What is the balance due on the original note at the end of the 90 da.?

A SILENT READING LESSON

Besides rendering the services about which we have already learned, modern banks act as agents for their depositors in collecting accounts, in transmitting funds from one place to another, and in many other ways.

For example, suppose that Mr. James X. Wye, a hardware dealer in New York City, owes money to the Spencer Manufacturing Co. of Cortland, N. Y., and that the company is unable to collect the amount due. The company may "draw" on Mr. Wye by means of the following *sight draft*, which is so called because payment is demanded upon its presentation:



$735.80 CORTLAND, N.Y., Feb. 7, 1929

At Sight pay to the order of

The Citizens Bank of Cortland

Seven hundred thirty-five and $\frac{80}{100}$ Dollars

To James X. Wye,
92 N. Samson St.
New York

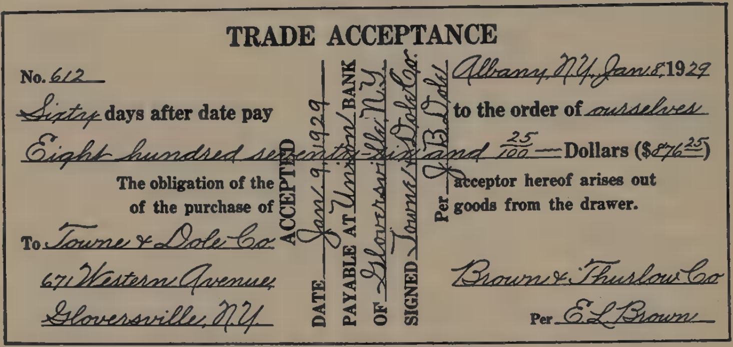
Spencer Manufacturing Co.
Per J.T.B.

The Spencer Manufacturing Co. (the *drawer*) deposits this draft with its bank, the Citizens Bank (the *payee*) in Cortland. The bank then forwards it to some bank in New York, which presents the draft to Mr. Wye (the *drawee*) and asks him if he desires to "honor" it. If so, Mr. Wye pays it, and the New York bank then notifies the Cortland bank that the draft is paid. The Citizens Bank then places the amount to the credit of the company's account at the bank and notifies the Spencer Manufacturing Co. that it has done so.

Banks usually charge a small commission for collecting drafts, which is deducted from the amount collected.

A SILENT READING LESSON (CONTINUED)

In some kinds of business another sort of draft, called a *trade acceptance,* is sometimes used in making payments for goods. The following form shows a common type of such a draft:



TRADE ACCEPTANCE

No. 612

Albany, N.Y. Jan. 8, 1929

Sixty days after date pay to the order of ourselves

Eight hundred seventy-six and $\frac{25}{100}$ Dollars ($876.25)

The obligation of the acceptor hereof arises out of the purchase of goods from the drawer.

To Towne & Dole Co.

671 Western Avenue

Gloversville, N.Y.

Brown & Thurlow Co.

Per E. L. Brown

ACCEPTED

DATE Jan. 9, 1929

PAYABLE AT Union BANK

OF Gloversville, N.Y.

SIGNED Towne & Dole Co.

Per J. B. Dole

In the above case we may suppose that the Brown & Thurlow Co. (the *drawer*) sends this draft with the invoice covering a shipment of goods. Upon receipt, it is at once accepted by the Towne & Dole Co. (the *drawee*); that is, as shown in the model form, the drawee signs the acceptance across the face. The acceptance, which is now returned to the drawer, has practically the standing of a promissory note that does not bear interest, and may be discounted at a bank. The drawer, if he wishes, can thus get his money immediately, less the discount, even though the goods were sold on credit for a period of 30 da. or 60 da.

Trade acceptances have an advantage over the method of letting accounts run for a long time. By using acceptances a manufacturer, for example, does not have to carry on his books a large number of accounts with individual customers, but may transfer all these details to his bank, which collects the acceptances for him when they are due.

SIGHT DRAFTS AND ACCEPTANCES

1. The Westinghouse Linen Co. of Syracuse, N.Y., draws a sight draft on R. T. Dow of Troy, N. Y., for $150. If the bank charges 0.2% for collection, what are the proceeds? Write the draft, dating it today and making it payable to some bank, as in the model form on page 311.

2. Robertson Bros. draw on J. P. Shipley at sight for $37.50. The bank charges 10¢ for collection. The charge for collection is what per cent of the face of the draft?

3. A. F. Reed owes M. S. Coe for 2 doz. suits @ $271.50, 1½ doz. suits @ $234, and 7½ doz. pairs trousers @ $40. Coe draws on Reed for the money, and the bank charges him $1.25 for collecting. What rate is this?

4. Suppose that John Bentley, of Yonkers, N. Y., owes you $250, and that you draw on him today for this amount. Write the draft, payable to the order of some bank near your home. If the bank charges you $\frac{1}{10}$% for collecting the money, what are the proceeds?

5. The terms on which a miller in Buffalo sells an order of flour amounting to $500 are "30 da. net, covered by trade acceptance." The miller sends the acceptance (which the purchaser has signed and returned) to his bank 20 da. before it is due and has it discounted at ½% per month for this period. What are the proceeds of the draft; that is, how much does the miller's bank place to his credit?

6. A 30-day trade acceptance for $975, which was signed by the drawee on Aug. 10, is discounted by the drawer on Aug. 16 at 6%. What is the period of discount; that is, what is the result of subtracting the number of days from Aug. 10 to Aug. 16 from the number of days that the acceptance runs? What are the proceeds?

REVIEW OF COMMERCIAL FORMS

1. Write a bill for four items of groceries purchased from some grocer in your vicinity, giving his name and address. Then add the amounts and receipt the bill.

2. As in Ex. 1, write a bill for four items of dry goods purchased at some store in your vicinity.

3. Write two pages of a cash account such as you might keep for personal receipts and expenses. The left-hand page should show the receipts, and the opposite page the expenses. Give six items on a page and balance the account.

4. Write a deposit slip for four items of money or checks to be deposited at some bank in your vicinity.

5. Write a check for $2.75 to pay for some purchase. If you have $68.93 in the bank before making out the check, what is your bank balance after the check is paid? Show how the check should be indorsed.

6. Write a promissory note for $25, payable to John Doe one year after today, with interest at 6%, and sign it with the name "Richard Roe."

7. How much interest is due on the note in Ex. 6 on the date on which payment is promised?

8. In Ex. 7 suppose that Richard Roe paid the note and interest by giving a check. What is the date of the check? To whom would he make the check payable? Write the check and show how it should be indorsed.

9. Write a check payable to the order of John Doe for $1.50. Date it properly, and sign it with the name "Richard Roe." Show how Mr. Doe should indorse the check.

10. Write a sight draft for $200 such as might be made by a wholesaler on a retail dealer.

II. STOCKS AND BONDS

A SILENT READING LESSON

A group of boys decided to organize a baseball club and found that the cost of uniforms and equipment would be $112. If 14 boys had been willing to join, each would have had to pay $\frac{1}{14}$ of $112, or $8. Some of the boys could not afford to contribute as much as others, and so they divided the $112 into 224 shares (or parts) of 50¢ each, selling to each boy as many shares as he cared to buy. Fred was one of the chief promoters and took 40 shares, thus making his payment $20. James took 20 shares, and the others took various amounts.

At the first three games the club made large profits. If the boys had all contributed equally, each would have received $\frac{1}{14}$ of the profits. Since this would not be fair to those who put in the most money, they divided the profits according to the number of shares that each boy owned.

James now tried to buy some of Fred's shares, so as to get more of the profits; but since the profits were so large, Fred would not agree to sell for less than 60¢ a share.

The boys had formed a *corporation* with a *capital* of $112. There were 224 *shares of stock*, which had a *par value* of 50¢ each. The profit each boy received was a *dividend*, and these dividends were so large that a share was worth more than 50¢; that is, the stock was *above par*.

In practical life men form corporations in much the same way, only they play the game of business instead of playing the game of baseball.

The laws of our state specify the provisions under which corporations may be formed. There are many advantages, both from a legal and a business point of view, in organizing as a corporation, and a large part of modern business is now done by firms that are incorporated.

FORMING CORPORATIONS

All work oral

1. In organizing a tennis club some boys issue shares at 50¢ each. If they need $150 to start the club, how many shares should they issue?

2. If William buys 30 shares, how much does he pay?

3. If Frank pays in $10, how many shares does he take?

4. If the profits from renting the courts and from the sale of tickets to the match games are so large that each share is worth 55¢, what is the per cent of increase in value?

5. If the stock rises to 20% above the par value of 50¢, how much is each share then worth?

6. A heavy storm injures the courts, and the boys have to raise more money. Frank offers to sell his stock for 45¢ a share, thinking that someone may wish to buy at this price despite the poor prospects of the club. This price is what per cent below par?

7. The boys elect six of their number to direct, or manage, the club. These six members are called "directors," and at a meeting they order an "assessment" of 10% on the par value of each share. How much has to be paid on each share?

8. If you own $5 worth of stock in a baseball club, and the stock rises to 20% above par, how much is it then worth?

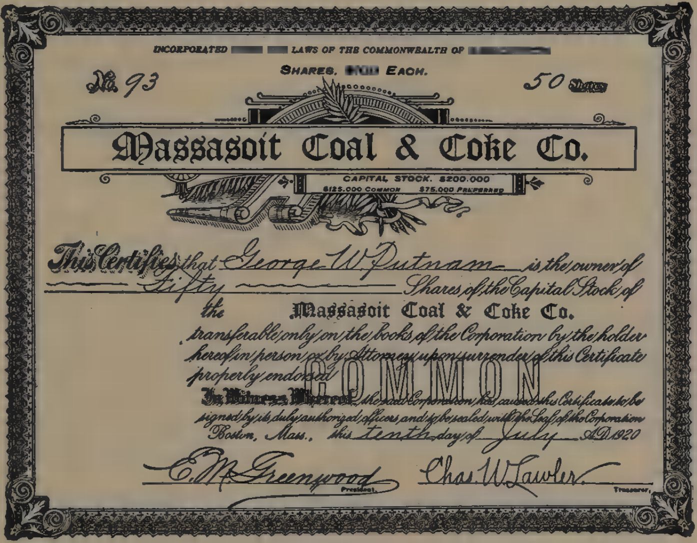
9. If you own $5 worth of stock in a tennis club, and the stock drops to 10% below par, how much is it then worth?

10. Why may stock be above or below par?

11. Why is it necessary in business to have stock companies, or corporations, instead of having one man furnish the capital?

12. Why is it necessary for the stockholders to select a few men to direct the business instead of managing it themselves?

Stockholders. The *capital*, or *capital stock*, which is the amount invested for the purpose of carrying on the business of a corporation, is divided into *shares*. A person who buys shares is called a *stockholder*, and receives a *stock certificate* showing how many shares he owns. A stockholder usually has one vote for each share at the stockholders' meetings.



INCORPORATED [redacted] LAWS OF THE COMMONWEALTH OF [redacted]

SHARES, [redacted] EACH.

No. 93

50 [redacted]

Massasoit Coal & Coke Co.

CAPITAL STOCK. $200.000
$125.000 COMMON $75.000 PREFERRED

This Certifies that George W. Putnam is the owner of Fifty Shares of the Capital Stock of the Massasoit Coal & Coke Co. transferable only on the books of the Corporation by the holder hereof in person or by Attorney upon surrender of this Certificate properly endorsed

COMMON

In Witness Whereof the said Corporation has caused this Certificate to be signed by its duly authorized officers and to be sealed with the Seal of the Corporation
Boston, Mass., this tenth day of July A.D. 1920

E. M. Greenwood
President.

Chas. W. Lawler
Treasurer.

A Stock Certificate

In our work the par value of a share of stock is to be taken as $100 unless otherwise stated, although in many cases the par value may be $50, $25, $5, or even $1. Sometimes stock is issued without a stated par value.

Directors and Officers. The stockholders elect a few of their number, called *directors*, to supervise the business. The directors elect the *officers* of the company, who are generally a president, a vice president, a secretary, and a treasurer.

Dividends. The net profits (the profits after all expenses are paid) are divided among the stockholders in the form of *dividends*. There are frequently two kinds of stock: *preferred stock*, which often pays a fixed rate of dividend, and *common stock*, which pays a dividend depending upon the profits left after the preferred dividend is paid.

For example, if the capital of a corporation is \$100,000, and the par value is \$100, there are 1000 shares outstanding. If in a year the company earns \$6000 above all expenses, the directors may declare a dividend of \$6000 ÷ \$100,000, or 6%. Then, for each share of stock that a man owns, he will receive 6% of \$100, or \$6.

$$\textit{Dividend} = \textit{rate} \times \textit{par value}.$$

CORPORATION PROBLEMS

1. A company capitalized at \$3,000,000 declares a dividend of 5%. How much does the holder of 100 shares receive?

2. A company with a capital of \$500,000 divides \$45,000 in dividends. How much does the holder of 30 shares receive?

3. A company divides \$50,000 in dividends, the stockholders receiving \$4 per share. What is the capital?

4. A company with a capital of \$50,000,000 declares a dividend of $2\frac{3}{4}\%$ every 6 mo. How much money does it distribute annually among the stockholders?

5. If a company with a capital of \$500,000 earned \$11,725.40 above expenses in the last quarter (3 mo.), what rate of dividend can it declare for this quarter and still carry about \$1700 to the next quarter?

6. A company with a capital of \$250,000 has earned \$15,000 this year above all expenses, and decides to divide all but \$2500 of this amount in dividends. What is the dividend rate?

Buying and Selling Stock. Stocks are usually bought and sold through a *broker*, who buys and sells stock in the *stock exchange*, a kind of auction room for such business.

Stock Quotations. A *quotation*, or *quoted price*, of $118\frac{3}{4}$ means that the *market price* per share is $118.75. If a stock is paying a high rate of dividend, people will pay more than $100 for a share whose par value is $100, and the stock is said to be *above par*, or *at a premium*. If a stock with a par value of $100 is quoted at 100, it is said to be *at par*; and if quoted below 100, it is said to be *below par*, or *at a discount*.

Brokerage. The broker charges *brokerage*, or commission, for his services. A charge of $15 per 100 shares on stocks selling between $10 and $125 per share, and $20 per 100 shares on stocks selling at $125 per share or over, is added to the cost of the stock at the market price when the broker buys for a customer. When the broker sells stock, he deducts the brokerage from the amount received and pays the balance to the customer. There is a minimum brokerage charge of $1 in all purchases or sales of stock.

For example, how much does a man, buying through a broker, pay for 300 shares of stock quoted at $117\frac{1}{8}$?

The cost of the stock at the market price is 300 × $\$117.12\frac{1}{2}$, or $35,137.50. The brokerage is 3 × $15, or $45, so that the man pays $35,137.50 + $45, or $35,182.50, for the stock.

Cost = number of shares × quotation + brokerage.

The man who sells the above stock receives from his broker $35,137.50 – $45, or $35,092.50.

Proceeds = number of shares × quotation – brokerage.

The brokerage on 6 shares or less of the above stock would be the minimum charge of $1; on 10 shares the brokerage would be $\frac{1}{10}$ of $15, or $1.50.

BUYING AND SELLING STOCK

Numbers 1 to 6, oral

State the brokerage on the following sales of stock:

1. 100 shares @ $97\frac{7}{8}$.
2. 200 shares @ 137.
3. 500 shares @ $69\frac{3}{4}$.
4. 50 shares @ $87\frac{3}{8}$.
5. 75 shares @ $172\frac{1}{2}$.
6. 3 shares @ 149.

Find the amounts paid in making the following purchases of stock through a broker:

7. 100 shares @ $87\frac{1}{8}$.
8. 300 shares @ $136\frac{1}{2}$.
9. 400 shares @ $54\frac{3}{8}$.
10. 1000 shares @ $47\frac{7}{8}$.
11. 50 shares @ $142\frac{5}{8}$.
12. 75 shares @ $172\frac{5}{8}$.
13. 125 shares @ $97\frac{3}{4}$.
14. 10 shares @ $119\frac{7}{8}$.
15. 5 shares @ $63\frac{3}{8}$.
16. 60 shares @ 147.

Find the amounts received from selling the following stocks through a broker:

17. 75 shares @ 67.
18. 500 shares @ $37\frac{1}{8}$.
19. 600 shares @ $96\frac{3}{8}$.
20. 50 shares @ $99\frac{7}{8}$.
21. 200 shares @ $127\frac{1}{4}$.
22. 4 shares @ $158\frac{3}{8}$.
23. 15 shares @ $56\frac{1}{4}$.
24. 2400 shares @ $80\frac{1}{2}$.
25. 40 shares @ $129\frac{5}{8}$.
26. 3 shares @ $88\frac{3}{4}$.

27. A man bought 30 shares of American Locomotive @ 124, 20 shares of Bethlehem Steel @ $40\frac{3}{4}$, and 5 shares of Union Pacific @ 139. How much did he pay, including brokerage?

28. A man sold 40 shares of General Motors @ $71\frac{1}{2}$, 75 shares of American Sugar @ $66\frac{7}{8}$, and 5 shares of Baldwin Locomotive @ $112\frac{7}{8}$. How much did he receive from his broker?

Bond. A *bond*, like a note, is a written promise to pay money. A corporation that wanted to borrow \$5000 for 2 mo. would do so on its note; but if it wanted to borrow \$500,000 and not pay it back for 10 yr., it would do so by issuing bonds, probably in denominations of \$1000 or \$500 each.

All the conditions under which the bonds are issued are stated in the bond, including the amount for which issued (the *face*), the rate of interest, the date of maturity, and so on. The interest on bonds is usually paid semiannually.

Bonds are often secured by a *mortgage*, an agreement by which the bondholders may sell the property pledged if the bonds are not paid when due.

The stockholders of a corporation are the owners of the property, while the bondholders are owed money by the corporation and so are creditors. A stock certificate is like a deed to a house; a bond is like a mortgage on it. Bonds bear a fixed rate of interest, but the dividends on stock depend upon the net profits of the company.

Income from Stocks and Bonds. Bond interest and stock dividends are always computed on the *face value*; that is, on the par value.

For example, in finding the *amount* of income received on 4% bonds which have a par value of \$5000, we have

$$0.04 \times \$5000 = \$200, \text{ the income.}$$

Similarly, in finding the *rate* of income received from a stock which pays 5% and is bought at $112\frac{1}{4}$, we have

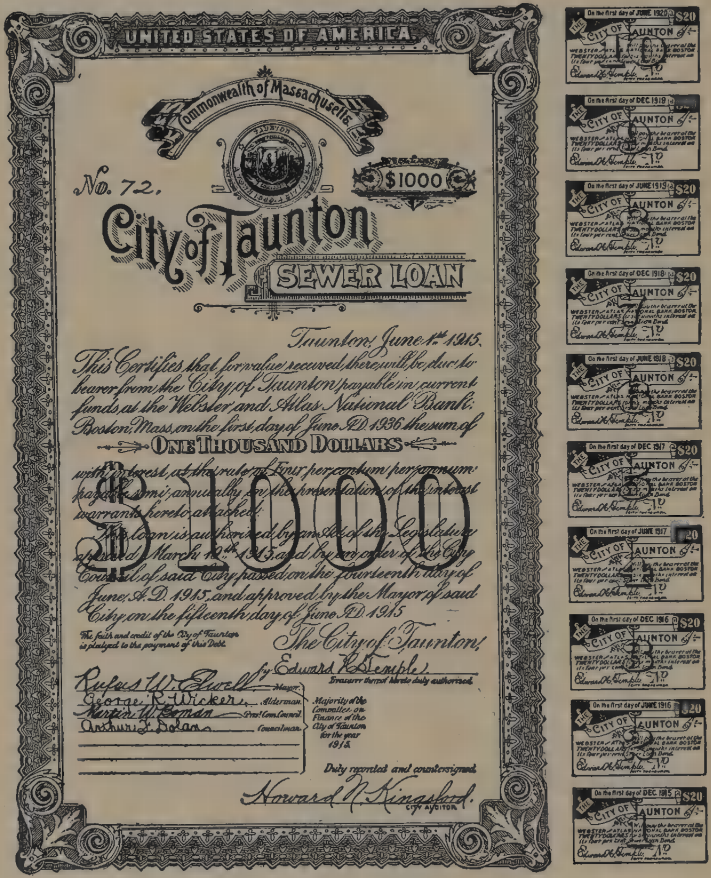
$$\$112.25 + \$0.15 \text{ (brokerage)} = \$112.40, \text{ the cost per share,}$$

$$0.05 \times \$100 = \$5.00, \text{ the income per share,}$$

and
$$\$5.00 \div \$112.40 = 0.044 \cdots.$$

That is, the rate of income, although 5% on the par value, is approximately 4.4% *on the investment.*

Bond Coupons. Bonds are often issued with small attached notes, which are called *coupons*, as in the specimen here shown:



UNITED STATES OF AMERICA

Commonwealth of Massachusetts

No. 72.

$1000

City of Taunton

SEWER LOAN

Taunton, June 1st 1915.

This Certifies that for value received there will be due to bearer from the City of Taunton payable in current funds at the Webster and Atlas National Bank, Boston, Mass. on the first day of June A.D. 1936 the sum of

ONE THOUSAND DOLLARS

with interest at the rate of Four per centum per annum payable semi-annually on the presentation of the interest warrants hereto attached.

This loan is authorized by an Act of the Legislature approved March 19th 1915, and by an order of the City Council of said City passed on the fourteenth day of June, A.D. 1915, and approved by the Mayor of said City on the fifteenth day of June A.D. 1915

$1000

The faith and credit of the City of Taunton is pledged to the payment of this Debt.

The City of Taunton,
by Edward H. Temple
Treasurer thereof hereto duly authorized.

Rufus W. Elwell Mayor.
George R. Wicker Alderman.
Martin W. Boman Pres't Com. Council.
Arthur F. Dolan Councilman.

Majority of the Committee on Finance of the City of Taunton for the year 1915.

Duly recorded and countersigned.
Howard N. Kingsford.
CITY AUDITOR.

On the first day of JUNE 1920 $20 THE CITY OF TAUNTON 10

On the first day of DEC 1919 THE CITY OF TAUNTON 9

On the first day of JUNE 1919 $20 THE CITY OF TAUNTON 8

On the first day of DEC 1918 $20 THE CITY OF TAUNTON 7

On the first day of JUNE 1918 $20 THE CITY OF TAUNTON 6

On the first day of DEC 1917 $20 THE CITY OF TAUNTON 5

On the first day of JUNE 1917 $20 THE CITY OF TAUNTON 4

On the first day of DEC 1916 $20 THE CITY OF TAUNTON 3

On the first day of JUNE 1916 $20 THE CITY OF TAUNTON 2

On the first day of DEC. 1915 $20 THE CITY OF TAUNTON 1

A Coupon Bond

When a payment of interest is due on the bond, the bond holder cuts off the proper coupon and presents it for payment. Banks accept and collect bond coupons for their depositors.

Newspaper Quotations. The newspapers give the market prices of the leading stocks and bonds. The problems on the next page may be solved by using the current stock quotations found in the newspapers or by using the following list:

At., Top., & S.F.	$120\frac{7}{8}$	United Drug	$121\frac{1}{2}$
N.Y. Central	$114\frac{1}{2}$	U.S. Rubber	$38\frac{1}{4}$
Union Pacific	$139\frac{1}{2}$	General Electric	269

A par value of $100 per share and the brokerage rates on page 319 are to be used in all problems about stocks.

Profit or Loss on Stock. In figuring the profit or loss on a stock transaction,

The difference between the total cost (cost + brokerage) and the net selling price (selling price − brokerage) is the gain or loss.

It is often easier, however, to figure the brokerage separately, as in the following examples:

1. If a man buys 100 shares of Western Union Telegraph at $118\frac{1}{4}$ and sells the stock at $117\frac{1}{2}$, before receiving any dividends, how much does he lose?

The man loses $118.25 – $117.50, or $0.75, on each share.

On 100 shares he loses 100 × $0.75, or $75.

He must, however, pay two brokerage charges, one when he buys the stock and one when he sells it; and hence he pays 2 × $15, or $30, for brokerage.

His total loss on the transaction is $75 + $30, or $105.

2. If a man buys 1000 shares of American Locomotive at 116 and sells the stock at $119\frac{1}{2}$, before receiving any dividends, how much does he gain or lose?

The man gains on each share $119.50 – $116, or $3.50.

On 1000 shares he gains 1000 × $3.50, or $3500.

The brokerage is 2 × 10 × $15, or $300.

His total gain is therefore $3500 – $300, or $3200.

STOCKS AND BONDS

1. What is the annual income from 50 shares of stock which pays a dividend of $1\frac{3}{4}\%$ quarterly?

2. If a bond which has a par value of $100 and pays 6% is bought for $104, including brokerage, what is the rate of income from the investment?

Disregarding dividends, find the gain or loss on the following sales of stocks, bought at the quotations given on page 323:

3. 100 shares of At., Top., & S. F. @ $129\frac{1}{4}$.

4. 50 shares of N. Y. Central @ $105\frac{7}{8}$.

5. 15 shares of Union Pacific @ $137\frac{3}{4}$.

6. 3 shares of United Drug @ $126\frac{7}{8}$.

7. 400 shares of U. S. Rubber @ $45\frac{5}{8}$.

8. 150 shares of General Electric @ $255\frac{5}{8}$.

9. If a Fourth Liberty Loan bond, which has a par value of $100 and pays $4\frac{1}{4}\%$, is bought for $108.25, including brokerage, what is the rate of income from the investment?

10. If the bond in Ex. 9 is bought for $101.80, including brokerage, what is the rate of income from the investment?

11. A man buys 100 shares of stock at $129\frac{3}{4}$. If the stock pays dividends at the rate of $6\frac{1}{2}\%$ annually, find the rate of income from the investment.

12. At the time the man in Ex. 11 bought his stock, he could have invested $13,000 in a mortgage paying $5\frac{1}{2}\%$, but he preferred the stock. What is the difference in income?

13. If you can consult a daily newspaper which gives the prices of stocks and bonds, keep track of the *fluctuations* (changes in price) of some bond or stock for a week, and report to the class.

Government Bonds. The governments of states, counties, cities, and towns, and our national government as well, often issue bonds when they are financing some public enterprise that is too large to be paid for at one time by the money raised by taxes.

Bonds issued by a national government are called *government bonds,* while those issued by state, city, and other local governments are called *municipal bonds.*

The United States could borrow money before the World War at from 2% to 3%, and states could borrow at about 4%, while cities, counties, and villages paid a little more. While at present the rates are somewhat higher, they vary so much from time to time that no standard rate can be given.

Cities also raise money by special *assessments.* For example, if a street is to be paved, the city will require owners of property fronting on the street to pay a special assessment in addition to their ordinary taxes, because of the increase in the value of their property which is caused by improving the street.

GOVERNMENT AND MUNICIPAL BONDS

1. If a city borrows \$5,000,000 at $4\frac{1}{2}$%, what is the annual interest charge?

2. If a state borrows \$40,000,000 at $4\frac{1}{4}$%, what is the annual interest charge?

3. If a nation wishes to borrow \$6,000,000,000 and must pay $4\frac{1}{4}$%, what will be the annual interest charge?

4. A certain suburb of New York, with an assessed valuation of \$3,500,000, decided to borrow \$125,000 in order to erect a new school building. At $5\frac{1}{4}$% by how much will the annual interest charge on the loan increase the tax rate on each \$1000 of assessed valuation?

5. During the World War the first issue of government bonds amounted to \$2,000,000,000 and bore interest at $3\frac{1}{2}\%$. This was called the First Liberty Loan, and was oversubscribed by a little more than 50%. By how many dollars was the loan oversubscribed? What is the annual interest charge for borrowing this \$2,000,000,000?

6. The second issue of bonds amounted to \$3,808,766,150. This was called the Second Liberty Loan, and bore interest at 4%. How much is the annual interest on this loan?

7. A little later the government borrowed \$4,176,516,850 (the Third Liberty Loan) at $4\frac{1}{4}\%$. Find the amount of interest paid annually by the government on this loan.

8. The Fourth Liberty Loan, amounting to \$6,989,047,000, was issued still later, and bore interest at $4\frac{1}{4}\%$. Find the amount of interest paid annually on this loan.

9. The government borrowed also \$4,500,000,000 at $4\frac{1}{4}\%$. How much was the annual interest charge on this loan?

10. If a man owns \$5000 worth of government bonds which pay $4\frac{1}{4}\%$, how much interest does he receive semiannually from the bonds?

11. In Ex. 10, how much more interest does the man receive semiannually than he would receive if he owned \$5000 worth of bonds paying $3\frac{1}{2}\%$?

12. If a man owns \$8500 worth of bonds of the Fourth Liberty Loan, which pays $4\frac{1}{4}\%$, how much interest does he receive semiannually?

13. If a man owns \$12,000 worth of bonds of the city of Rochester, and the bonds pay interest at the rate of 4%, how much interest does he receive in a year?

14. Solve Ex. 13, supposing that the man owns \$15,500 worth of the bonds.

III. THRIFT AND INVESTMENTS

A SILENT READING LESSON

Formerly people hoarded their savings in hidden places about their homes. Now they usually invest money that is not needed for immediate use, so that it will not only be safer than when kept at home but will also bring in an income. Unfortunately, some fraudulent investments are offered to the public, while others are so risky as to be undesirable. There are, however, many types of investments that are perfectly safe. It is well to remember that, in general, if the rate of income offered by any form of investment is larger than the current rates on sound investments, there is usually a greater risk of losing the money put into it.

We should learn how to distinguish between good investments and bad investments, so that we may invest wisely whatever money we may be able to save. Hundreds of millions of dollars are lost in this country every year through unscrupulous promoters of worthless stock, despite such laws as have been passed by New York and other states to prevent the sale of fraudulent stocks.

Buying stocks *on a margin,* which means that the purchaser pays only a small amount down and hopes that the stock will rise so that he can then sell it at a profit, is a very dangerous way to invest money. The stock may go down instead of up, and, if it goes down as many dollars as he paid, he will lose all that he put in. He also, of course, must pay the brokerage charges, whether he wins or loses, and so buying stocks in this way is a gamble. Never gamble in stocks; the chances are against you.

We should never make any kind of investment without asking the opinion of some reliable banker who has no financial interest in the property or stock that we plan to buy.

THRIFT AND INVESTMENTS

1. If you have $85 to invest and deposit it in a savings bank that pays 4 % a year, compounded semiannually, how much interest is due you at the end of 6 mo.?

2. A woman has saved $2750. She can deposit it in a savings bank which pays 4 %, compounded semiannually, or can invest it in an oil company which advertises that it pays 10 % on its stock. Her banker, however, advises her that the oil stock is a risky speculation. What is the difference in income for the first year? Which investment should she make?

3. A man has saved $4500. He can invest it in a well-managed manufacturing business, about which he is thoroughly informed, so as to receive 7 % on his investment. Is this a good investment? How much income will he receive a year?

4. A man comes to town and offers investments in a fruit plantation in Mexico. Nobody has heard of the man or of his plantation. He says the business will pay 15 %, and he asks Mr. Doe to invest $5000. How much income does he promise to Mr. Doe? Mr. Doe can invest the money near home at 6 %. How much will this bring him per year? Which investment should you advise him to make? Write your reasons.

5. Mrs. St. John has inherited $4500, and has a chance to invest it at 5 % in a first-class security or at 12 % in a banana plantation that her banker tells her is a very risky speculation. How much would be the income from each? Which investment should she make?

Find the income from each of the following investments:

6. $600 at 5 %.
7. $575 at 5 %.
8. $875 at 4 %.
9. $625 at $5\frac{1}{2}$ %.
10. $777 at $4\frac{1}{2}$ %.
11. $1450 at 4 %.
12. $9750 at 6 %.
13. $6225 at 6 %.
14. $3500 at 3 %.

INVESTING IN REAL ESTATE

1. Besides depositing money in banks or purchasing stocks and bonds, people who have saved money often buy real estate, which means land and buildings. A man paid $6000 for a house and rents it for $50 a month. He pays $186 a year for taxes, insurance, and repairs. What per cent on his investment does he receive?

2. Mr. Lowe rents to Mr. Ryan a house in Elmira for $75 a month. If one year Mr. Lowe pays $165 for taxes, $30 for insurance and repairs, and $130 for a street-paving assessment, how much was his net income from the property that year?

3. A man owns a house which he rents for $80 a month, but in 3 yr. the house is vacant for 2 mo. The house cost him $9000, and his annual expenses for taxes, repairs, and insurance are $187.50. What per cent of income does the man receive from his investment?

4. Mr. Phillips bought a house for $6000 and rented it for $45 a month. He paid annually $150 for taxes, $4.50 for insurance, and $45 for repairs. Three years later he sold the property for $6800. How much more did he gain than he would have gained by investing the $6000 for 3 yr. at 6%?

5. A man bought a house for $4800 and placed on it a mortgage of $2000 on which he has to pay 6%. Allowing $150 a year for taxes, insurance, and repairs, and interest at 5% on his investment of $2800, how much does the house cost him each year? This is how much less than paying $50 a month rent for a place in which to live?

6. Mr. Young paid $8000 for a house. He rents it for $60 a month. In a certain year he pays $166 for taxes, $6 for insurance, and $48 for repairs. What per cent does he receive on his investment that year?

CHOOSING INVESTMENTS

1. If on Jan. 1 you have $1000 to invest, and put it in a savings bank which pays 4%, compounding the interest every 6 mo., how much money do you have at the end of 6 mo.? of 2 yr.? of 3 yr.? of 4 yr.?

2. Instead of depositing in the savings bank the $1000 mentioned in Ex. 1, suppose that with it you had been able to buy 10 shares of stock which paid 3% every 6 mo. How much would you receive in dividends in 3 yr.?

3. After finding the incomes in Exs. 1 and 2, write a statement of the reasons why, under certain circumstances, you might prefer to put your money in the savings bank and why, under other circumstances, you might prefer to buy the stock.

4. If you should receive through the mails a circular letter offering you stock in a moving-picture concern and promising dividends of at least 20%, should you invest your $1000 in this way? If so, what would lead you to do it? If not, why should you hesitate? If in doubt, whom should you consult?

5. If you were advised to do so by a man who possessed good judgment and who was not financially interested in the property, would you invest the $1000 in a building lot in a growing section of a city? What are the arguments in your mind for and against such an investment?

6. One of the best ways to save money is to set aside a fixed sum, even if small, each week. Many banks conduct Thrift Clubs, in which the members deposit each week a fixed amount, such as 50¢, $1, $2, or $5, for 50 wk. The banks pay interest that is equivalent to $1\frac{1}{2}$% on the total of the deposits during the year, and at the end of that time send to each member a check for his deposits and interest. How large is the check which a member receives if each week he deposits 50¢? $1? $2? $5?

IV. DRAWING TO SCALE

A SILENT READING LESSON

Most of us have probably seen the workmen on buildings looking at *blue prints*, which are photographic copies, on blue paper, of the architects' drawings. These drawings are made to *scale*. In our work in measurement we have already used drawings that were made to scale; that is, drawings in which 1 in. represented some greater length, such as 1 ft., 1 yd., or 1 mi. If we draw to the scale $\frac{1}{10}$, each line in the drawing is $\frac{1}{10}$ as long as it is in the object represented.

Maps are always drawn to scale. If a map of New York State could be drawn full size, it would be exactly as large as the state itself. In the same way, a plan of a floor, drawn full size, would be as large as the floor. For practical purposes, therefore, scale drawings are necessary.

We must be very careful in drawing to scale, because the computations made from the drawing will be incorrect if the drawing is not made precisely. We should handle our *rulers* carefully so as not to get nicks in them, we should keep our pencils sharp, and we should have a pair of *compasses* and, if possible, a *protractor* for measuring angles.

The common kinds of lines that we shall use are the *straight line* and the *curve*. The *broken line*, made up of straight lines though not itself straight, is sometimes used. The four sides of a square, taken as one line, illustrate a broken line.

Whenever we speak simply of a line, we always mean a straight line and not any one of the other kinds of lines that are mentioned above.

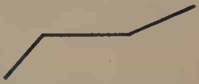
We should also know how to divide a line or an angle into two equal parts, how to draw triangles of different kinds, and how to make other constructions of this kind. We shall soon learn how to do these things.

LINES AND ANGLES

1. Using a ruler, draw on the board a straight line 1 ft. long. Beneath this line draw another line to represent the first one, using the scale $\frac{1}{12}$; that is, make it $\frac{1}{12}$ as long.

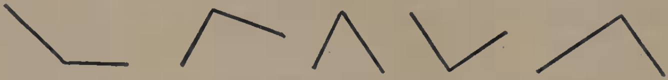
2. Measure the length of the teacher's desk and draw a line on the board to represent this length, using the scale $\frac{1}{4}$.

3. Copy this broken line on the board, drawing the angles freehand (that is, making them as nearly like those here shown as you can without measuring them), and making each of the three parts of the broken line ten times the length here shown.



4. Draw three curves of different shapes on the board, letting them represent some curves that you see in the room.

5. Here are five angles. Copy them freehand on the board and under each figure that you draw write the name *acute, right*, or *obtuse*, depending upon which kind of angle it is:



6. Tell the class about three places in some building, street, or pattern where you have seen a right angle.

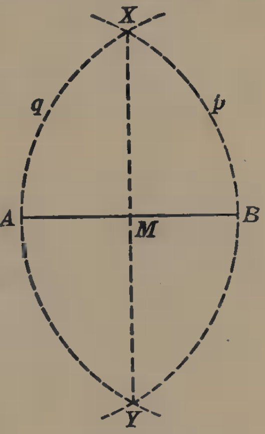
7. As in Ex. 6, tell about three acute angles and three obtuse angles that you have seen.

8. Tell the class about three places in some building or pattern where you have seen curves used.

9. Tell the class what scale you would use if you were making on the board or on your drawing paper a plan of the floor of your schoolroom, and tell why that would be a good scale.

10. As in Ex. 9, tell the class what scale you would use if you were drawing a plan of the top of your desk.

Bisecting a Line. To *bisect* a straight line means to find the mid-point between the ends; that is, to cut the line into two equal parts. We might bisect the line AB here shown by measuring off half of it, but it would not be very accurate.



A much better way is to place the steel point of our compasses at A and the pencil point at B, and then draw part of a circle, as shown by the curve p. Then place the steel point at B and the pencil point at A and draw the curve q. These two curves cross at X and Y, and if we join X and Y, the point M will bisect AB.

In making figures like this on the board, a string tied to a crayon may be used in drawing the circles.

Drawing a Perpendicular. We can draw a perpendicular to the line AB at M by using a method like the one above.

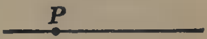
To do so we first place the steel point of the compasses at M, and mark off two points A and B (they do not have to be at the ends of the line) so that MA is equal to MB. Then using A and B as centers, as before, we draw parts of two circles like p and q, which cross at X. Drawing a line between X and M, this line will be perpendicular to AB and the angles at M will be right angles, as must always be the case with a perpendicular.

Perpendiculars can also be drawn by using a small square like a carpenter's square or a right triangle made of some stiff material like cardboard or celluloid. This is perhaps the best way when we wish to draw a perpendicular at the end of a line that we do not care to extend (a thing which happens too rarely to be considered further).

BISECTORS AND PERPENDICULARS

1. Draw a line on the board and then bisect it by the method given on page 333. Check your work by measuring the length of each part with a ruler.

2. Draw a line on the board, mark a point *P* on the line, as here shown, and then draw a perpendicular to the line at the point *P*.



3. Draw on the board a vertical line like the one here shown, but about 1 ft. long. Bisect this 1-foot line by the method given on page 333, and check your work by measuring the parts with a ruler.

4. Draw a vertical line on your paper, mark on it any point *Q*, and then at *Q* draw a line that shall be perpendicular to the vertical line.

5. Draw on the board a slanting line like the one here shown and then bisect it.

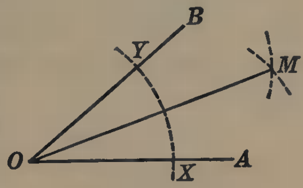
6. Draw on your paper a slanting line, mark on it any point *T*, and then at *T* draw a line perpendicular to the slanting line.

7. On the board draw a vertical line, a horizontal line, and a slanting line, each 2 ft. long. Bisect each line and check your results by measuring to see if one part of each line is equal to the other part of the line.

8. Draw on paper a line 3 in. long. At a point 1 in. from one end draw a perpendicular. If you have a protractor, measure the angles to see if they are each 90°.

9. Draw on paper a line 2 in. long and bisect it with ruler and compasses. To check your work, fold the paper along the bisector (the line which bisects the 2-inch line), stick a small pin through one end of the 2-inch line, and see if the pin goes through the other end of the line.

Bisecting an Angle. To *bisect* an angle means to draw a line through the *vertex* (the point where the lines meet to form the angle) so as to cut the angle into two equal angles.

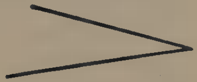


In speaking of the angle formed by the lines *OA* and *OB* in this figure, we call it "the angle *AOB*." The vertex is always designated by the middle letter.

To bisect the angle *AOB* here shown, place the steel point of the compasses at *O* and draw part of a circle which crosses both *OA* and *OB*, thus fixing the points *X* and *Y*. Now place the steel point at *X* and the pencil point at *Y* and draw part of a circle, and then, placing the steel point at *Y* and the pencil point at *X*, draw another part of a circle crossing the first one at *M*. Now draw *OM*, which will bisect the angle.

BISECTING AN ANGLE

1. Draw on the board an angle like the one here shown and then bisect it.



2. Draw on the board any angle that you wish and then bisect it.

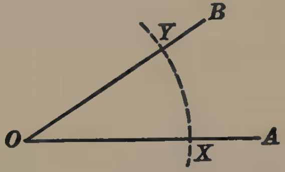
3. Draw on the board any triangle and bisect each of the three angles. If your work is accurate, the three bisectors will all meet at one point.

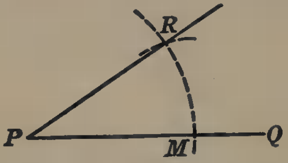
4. Draw on paper three triangles of different shapes and draw the bisectors of the angles, as in Ex. 3. If in any one of them the bisectors do not all meet at one point, your work has not been carefully done.

5. Draw on paper three triangles of different shapes. At the mid-point of each side draw a perpendicular. As in Ex. 4, the three perpendiculars should all meet in one point.

Making One Angle Equal to Another. When we copy a figure, we often have to make one angle equal to another.

For example, suppose that *AOB* is an angle of a figure that we wish to copy, and that *PQ* is a line on which we wish to make an angle that shall be equal to the angle *AOB*.





First, with the center *O*, draw part of a circle which cuts off the equal lines *OX* and *OY*. Then, keeping the compasses opened just as they were, place the steel point at *P* and with the pencil point mark *M* by drawing part of a circle. Then *PM* will be the same length as *OX*.

Now place one point of the compasses at *X* and the other at *Y*, and then, without changing the opening of the compasses, place the steel point at *M* and draw part of a circle crossing the first one at *R*.

Then draw *PR*, and the angle *QPR* thus formed will be exactly equal to the angle *AOB*.

MAKING EQUAL ANGLES

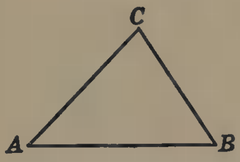
1. Draw any angle that you wish on the board and draw an angle equal to it by the method described above.

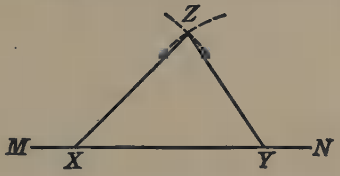
2. Repeat Ex. 1 on paper, checking your work by using a protractor, if one is available, to measure the angles.

3. Draw one line perpendicular to another, and then bisect one of the right angles thus formed.

4. Draw two lines which cross each other at any convenient angle, and bisect each of the four angles thus formed.

Making One Triangle Equal to Another. Suppose that we wish to draw a triangle that shall be exactly equal in all respects to the triangle *ABC*, which is shown below at the left.





First draw any line *MN*. Then place the two points of the compasses at *A* and *B* and, without changing the opening of the compasses, transfer the length *AB* to the line *MN*, as shown by the points *X* and *Y*.

Now place the compasses so that the points lie on *A* and *C*, and, without changing the opening, place the steel point at *X* and with the pencil point draw part of a circle. Then doing the same for *B* and *C*, place the steel point at *Y* and draw another part of a circle crossing the first one at *Z*.

Then, drawing the lines *XZ* and *YZ*, the triangle *XYZ* will be exactly equal to the triangle *ABC*.

In the same way we can make a triangle with its sides of any given lengths, provided only that the given lengths are such as will make a triangle possible.

EQUAL TRIANGLES

1. On paper draw a triangle of any shape and then make a triangle exactly equal to it.

2. Draw a triangle with sides $1\frac{1}{2}$ in., 1 in., and $1\frac{3}{4}$ in. long.

Draw triangles with sides of the following lengths:

3. 1 in., 1 in., 1 in.

4. $1\frac{1}{2}$ in., $1\frac{1}{2}$ in., 2 in.

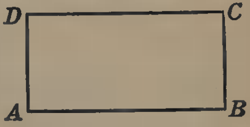
5. 2 in., 3 in., 3 in.

6. $1\frac{1}{4}$ in., $1\frac{1}{4}$ in., $1\frac{3}{4}$ in.

OTHER EASY FIGURES

1. Draw a rectangle with its base 2 in. long and a height of 1 in., being careful to draw accurate right angles by the method given for drawing perpendiculars.

2. If your work in Ex. 1 was accurate, the two diagonals (the lines from one corner to the opposite one, as from *A* to *C* and from *B* to *D* in this figure) are equal. If this is not the case, do your work over again until, when you apply this test, you find that the diagonals are equal.

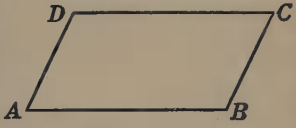


3. Enlarge the figure of Ex. 2 to three times its size; that is, so that each side of the new rectangle is three times the length of the one shown above.

4. The area of the new rectangle in Ex. 3 is how many times the area of the original one?

5. Reduce the figure that you drew in Ex. 3, making each side two thirds as long. The area of this rectangle is how many times the area of the original one in Ex. 2?

6. Enlarge this parallelogram so that each side is four times as long. Be careful to make the angles equal to those that are here shown, using the method given on page 336.



7. Check your work in Ex. 6 by seeing if each diagonal in your figure is four times as long as the corresponding diagonal in the figure shown above.

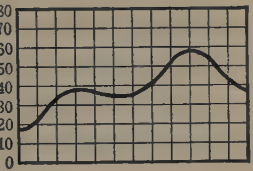
8. Reduce the figure that you drew in Ex. 6, making each side three fourths as long.

9. Draw a plan of your desk top to the scale $\frac{1}{8}$, and then enlarge the plan, making each side twice as long.

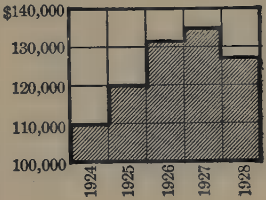
V. GRAPHS

A SILENT READING LESSON

In the newspapers, we have probably seen diagrams like the one here shown. Perhaps they were used in advertisements, or in weather reports, or in setting forth certain important facts about railroads or other business corporations. As we have already seen, pupils can keep a record of their progress in arithmetic and other studies in this way. The ups and downs in their work are shown by the ups and downs of the irregular curve. A diagram like the one here shown is called a *curve-line graph.*



If a large business house wishes to compare the amounts of its sales for each of the five years from 1924 to 1928, it might do so by means of the *bar graph* here shown. In this graph the length of each bar shows the sales above $100,000, each vertical space represents $10,000 and each horizontal space a year. As we can easily see, the graph tells the story about the sales much more clearly than columns of figures would tell it.



The bars of this graph might have started at $0 or at any other convenient point, and the spaces might have each represented $1000 or some other amount. If this had been done, the graph would have been larger, but it would not have told the story any more effectively.

Graphs must be accurate, clear, neat, and interesting if they are to tell the story better than it could be told by a mass of figures.

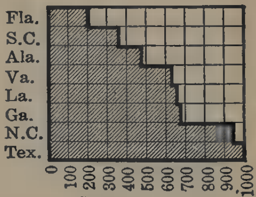
Horizontal Bars. Instead of placing the bars in a graph vertically we may, if it is more convenient, place them horizontally as in the illustration given below.

For example, in some of our states agriculture is the chief source of wealth, in others mining, and in others manufacturing. Taking the value of manufactured goods, to the nearest $1,000,000, as a basis of comparison, the government reports show the following statistics for eight states in a recent year:

Florida . . .	$213,000,000	Louisiana . .	$676,000,000
South Carolina .	381,000,000	Georgia . . .	693,000,000
Alabama . . .	492,000,000	North Carolina .	944,000,000
Virginia . . .	644,000,000	Texas . . .	1,000,000,000

Let us now make a graph and see how much clearer these facts will stand out. The most convenient way is to use a piece of *squared paper*, or *coordinate paper*, which can be purchased already ruled in squares. If squared paper is not available, it is easy to rule some.

Since the numbers run from $200,000,000 to $1,000,000,000, and we wish to make only a general comparison of the amounts, we may let each horizontal space represent $100,000,000.

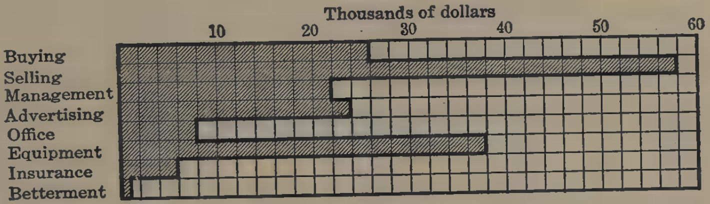


We see at a glance that Texas produced manufactured goods to the value of about $1,000,000,000 in that year, that North Carolina stood next, and that Virginia, Louisiana, and Georgia were very close together, while Florida, whose chief source of wealth is in fruit growing and market gardening, produced the least.

The term "graph" is generally applied to all pictorial representations of this kind, although a bar graph is sometimes called a *bar pictogram*.

BAR GRAPHS

1. The following graph shows the distribution of the various expenses of running a large department store in a recent year:



Write a statement as to the purpose for which the largest amount was spent; the smallest amount. How does the amount spent for advertising compare with that for buying? for selling? for insurance?

Represent the following statistics by either kind of bar graph:

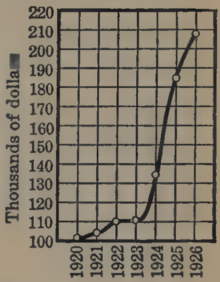
2. The approximate areas of several counties in New York State, to the nearest 100 sq. mi., are as follows: Allegany, 1000; Cattaraugus, 1300; Chautauqua, 1000; Hamilton, 1800; Herkimer, 1500; Onondaga, 800; St. Lawrence, 2700; Cayuga, 700; Chemung, 400; Delaware, 1400; Essex, 1800; Jefferson, 1300; Cortland, 500. Let one space in your graph represent 100 sq. mi.

3. The total yearly sales of a company were as follows: 1923, $58,000; 1924, $82,000; 1925, $98,000; 1926, $135,000; 1927, $140,000; 1928, $145,000; 1929, $160,000. After drawing the graph, interpret it; that is, state your conclusions about the business of the company.

4. The total yearly income of a company for the last ten years was as follows: $275,000, $285,000, $295,000, $325,000, $310,000, $330,000, $360,000, $340,000, $375,000, $400,000. Interpret the graph as in Ex. 3.

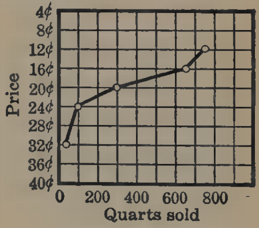
Curve-Line Graph. In representing certain kinds of statistics the most convenient graph is the curve-line graph. This kind of graph is particularly useful in telling the story about changes from day to day or from year to year.

For example, suppose that a company whose sales were $103,000 in 1920, $105,000 in 1921, $110,000 in 1922, and $111,000 in 1923 found that its sales increased to $135,000 in 1924, $185,000 in 1925, and $208,000 in 1926. While these figures show a decided increase in sales since 1923, we can see the sudden change much more easily by glancing at the curve in the graph shown above.



Demand Curve. In some graphs we use a series of short straight lines connected so as to give the effect of a curve, as in the graph, called a *demand curve*, which is here shown.

For example, it was found in a certain market that when strawberries sold at 32¢ a quart, 25 qt. were sold daily; when the price dropped to 24¢, 100 qt. were sold daily; at 20¢, 300 qt. were sold; and so on. It is evident that the average number of quarts sold per day is related to the price, and this relation may be indicated graphically as here shown. The story is thus told much more vividly than by a mere recital of figures. We can also see that it is not always more profitable to sell at the highest prices, because the sales may be so small as to give but a small total profit.



Business houses often find that a study of demand curves, like the one above, helps them in making plans for purchasing stock to meet future requirements.

CURVE-LINE GRAPHS

1. In six consecutive years the average price per pound of lake copper was as follows: 12.7¢, 16.7¢, 16.7¢, 13.5¢, 28¢, 32¢. Represent the changes by a curve-line graph.

2. The total yearly sales of a grocery for a period of ten years were as follows: $16,000, $18,000, $20,000, $25,000, $24,000, $26,000, $28,000, $36,000, $43,000, $52,000. Draw a graph to show the trend of business.

3. From the graph in Ex. 2 estimate the probable amount of business for the next year. Basing your conclusion upon the general trend of business shown by the graph, do you think it will be advisable to plan to enlarge the store and its stock in this neighborhood?

4. Construct a demand curve from the following facts which were ascertained with respect to a certain article:

Price	12¢	10¢	8¢	6¢	4¢	2¢
Number sold . .	400	500	1800	3000	8000	8200

5. A grocer's records of sales show that he has sold 10 baskets of peaches at $1.75 a basket, 30 baskets at $1.50, 70 baskets at $1.20, 80 baskets at $1.10, and 90 baskets at $1. Draw the demand curve and estimate the probable sale of peaches if the price drops to 90¢ a basket.

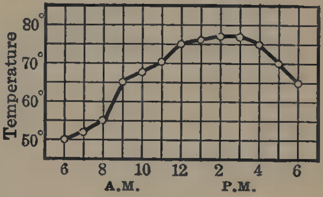
6. If the business of the grocer mentioned in Ex. 5 has increased 20% since last year, when the above facts were recorded, draw a demand curve for this year, assuming that he will sell 20% more at each price mentioned above than he did last year. Using your curve as a guide, estimate how many baskets of peaches he will probably be able to sell this year when the price is $1.45 a basket.

TEMPERATURE GRAPHS

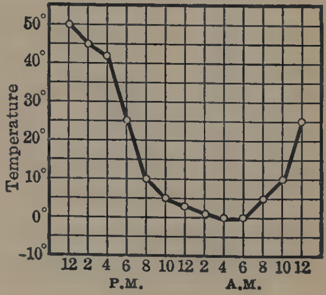
1. The temperature in a certain place in the summer was taken every noon for a week. It was found to be 64° on Sunday, 68° on Monday, 74° on Tuesday, 60° on Wednesday, 62° on Thursday, 70° on Friday, and 75° on Saturday. Represent these statistics by means of a bar graph.

2. Would the night temperatures in Ex. 1 be lower or would they be higher than the noon temperatures? With what we have given in the problem, explain why it would not be proper to use a curve line that might be taken to represent the general range of temperature for the week.

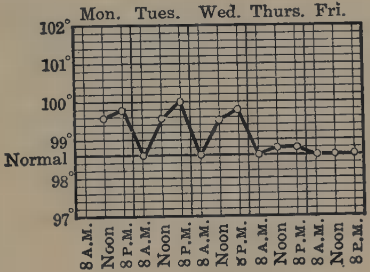
3. On May 1 the temperature between 6 A.M. and 6 P.M. at a certain place was as shown by this graph. Because the hours at which the temperature was taken are so near together, there is probably no decided change between them, and so we use a line graph. Write the approximate temperature for each hour and half hour of the day.



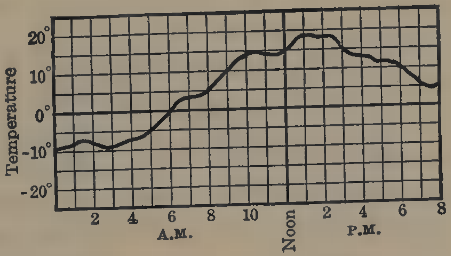
4. When the temperature is below zero, as is often the case in the North, this fact is indicated by a minus sign. For example −5° means 5° below zero. This graph shows the changes in temperature from noon on a day in January to noon on the following day. Write the approximate temperatures for every hour of this period, and a statement telling when the temperature fell most rapidly and when it rose most rapidly.



5. John was out of school one week because he was sick with the mumps. On Monday noon the doctor called, and took John's temperature by placing a small thermometer under his tongue. Instead of being 98.6°, which is the temperature of a healthy person, John's temperature was 99.6°; and so the doctor said it was 1° above normal. He asked John's mother to take the temperature at 8 A.M., noon, and 8 P.M. each day. She did so, and kept the record on a temperature chart, which is usually made on specially ruled paper like that here shown. On Thursday evening, when the doctor called, he looked at the chart, and said that the temperature was so nearly normal that John would be all right the next day. From the chart find John's temperature at 8 A.M. and at 8 P.M. on Tuesday and on Wednesday; at noon and at 8 P.M. on Thursday; and at 8 A.M., noon, and 8 P.M. on Friday.

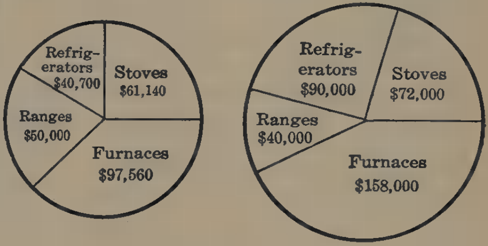


6. The Weather Bureau uses self-recording thermometers that trace graphs on moving strips of paper. In one of our Northern cities on a winter day the graph from midnight to 8 P.M. was as here shown. State the temperature at midnight; at noon; at about 2 30 A.M.; at 6 P.M. How many degrees did the temperature rise between midnight and 1 P.M.?



Circular Graph. In showing certain kinds of statistics, we sometimes use *circular graphs,* like the ones on this page.

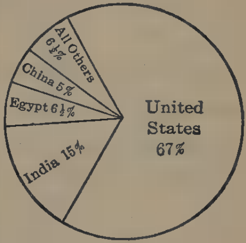
For example, the two circles in this graph make a much more effective comparison than does the table below of the sales, in two consecutive years, of a factory which makes furnaces, ranges, refrigerators, and stoves.



Articles	First Year	Second Year
Furnaces	$97,560	$158,000
Ranges	50,000	40,000
Refrigerators	40,700	90,000
Stoves	61,140	72,000

Since the second circle is considerably larger than the first, the graph shows that the total business has increased. It also shows that although the sale of stoves in the second year was greater, there was a relative shrinkage, and that the sale of ranges fell off both actually and relatively the second year.

A circular graph is particularly convenient when we are dealing with per cents. For example, this graph shows the per cents of the world's mill supply of cotton contributed by each country in a recent year. It tells us at a glance that the United States contributed considerably more than half the supply of the whole world. Although the construction of circular graphs is too difficult for us at this time, we can easily learn how to read the stories that such graphs tell.



TELLING STORIES BY GRAPHS

1. The distances of a train from its starting point in a run of 10 min. were measured and were found to be as follows:

Minutes	1	2	3	4	5	6	7	8	9	10
Miles	0.2	0.5	0.9	1.5	2.2	3	3.8	4.6	5.1	5.5

Draw a curve-line graph and tell what it shows about the speed of the train during this time.

2. The height of a growing plant, which was measured on eight consecutive days in June, was found to be as follows:

Day of the month	1	2	3	4	5	6	7	8
Height in inches	3.1	4.2	5.2	6.1	6.8	7.5	8.2	8.7

Draw a curve-line graph showing the growth of the plant, and from it determine in what part of the period the plant grew most rapidly.

3. Between 6 A.M. and noon on a rainy day in April the amount of rain which had fallen was found to be as follows:

Time	6 A.M.	7 A.M.	8 A.M.	9 A.M.	10 A.M.	11 A.M.	Noon
Inches	0.1	0.4	0.6	0.6	0.8	1.1	1.5

Draw a curve-line graph showing the variation in rainfall.

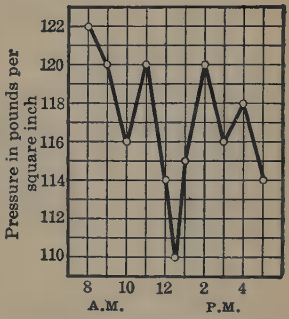
4. The population of the earth in millions, by continents, is approximately as follows: Africa, 143; North America, 140; South America, 56; Asia, 873; Australasia, 16; Europe, 465. Draw a graph, using that kind which you think tells the story most effectively.

5. A book on health gives the following average heights, in inches, for a growing boy between the ages of 5 yr. and 15 yr.:

Age . . .	5	6	7	8	9	10	11	12	13	14	15
Height .	42	44	46	49	52	54	56	58	61	63	68

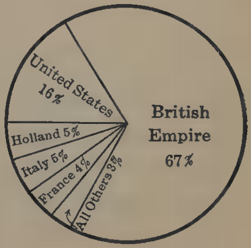
Draw a curve-line graph and from it determine in what part of this period the average boy grows most rapidly.

6. At 8 A.M. in a certain mill the steam gauge on the boiler showed a pressure of 122 lb. to the square inch. The pressure varied through the day as shown by this graph. Write the pressure for every hour from 8 A.M. to 5 P.M. and also for 12 30 P.M. Why was the pressure so high at 8 A.M.? Give a probable reason why the pressure was so low at 12 30 P.M.



7. Tom took a test in arithmetic every week for 10 wk. His scores for the number of "rights" in these tests, beginning with the first week, were as follows: 24, 26, 34, 30, 36, 36, 38, 44, 42, 46. Represent Tom's progress in arithmetic by means of a broken-line graph.

8. In a recent year the relative standing of the countries of the world having ships of over 15,000 T. was as shown in this circular graph. Write a statement of what the graph tells you about the standing of our country as compared with the British Empire; with Holland and Italy; with France; about the standing of all countries as compared with the British Empire.

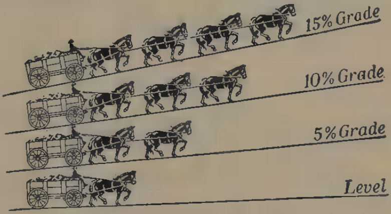


PICTORIAL GRAPHS

1. The following table shows the power required to haul 1 T. on various types and grades of roads, a 5% grade being one that rises 5 ft. in every 100 ft. measured horizontally:

GRADE	MACADAM ROAD	GRAVEL ROAD	SAND OR EARTH ROAD
Level	50 lb.	140 lb.	240 lb.
1%	70 lb.	160 lb.	260 lb.
2%	90 lb.	180 lb.	280 lb.
5%	150 lb.	240 lb.	340 lb.
10%	250 lb.	340 lb.	440 lb.
15%	350 lb.	440 lb.	540 lb.

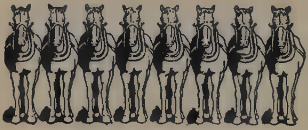
This illustration, which shows another kind of graph, the *pictorial graph*, represents the increase of power that is necessary to haul the same load on various grades of a gravel road. From the above table find out whether this graph tells a true story, considering, of course, any fraction of a horse as a whole horse in determining the number needed to haul a given load on each of the different grades.



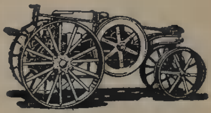
2. The power required to haul 1 T. on level macadam is what part of the power required on a 10% macadam grade? If a horse can haul 5000 lb. on level macadam, how much can it haul with the same effort up a 10% macadam grade?

3. A good horse can haul about 2000 lb. on an improved level gravel road. At 56 lb. to the bushel, how many bushels of corn can two horses haul over this road?

4. An "8–16 horse-power" tractor has about the same power as 8 work horses; but since horses cannot be used continuously, a tractor can do as much in 100 da. as 8 horses can do in 1 yr. The following pictures and table illustrate an effective way of comparing the relative cost of maintenance:



8 Horses for 1 yr.



Tractor for 100 da.

$53.76	Interest on investment	$43.50
80.00	Depreciation	145.00
18.16	Harness depreciation	None
16.32	Shoeing or repairs	36.25
701.76	Feed or fuel	154.00
None	Lubricating oil	35.00
156.56 (Feeding, etc.)	Labor	(Oiling, etc.) 25.00
4.24	Miscellaneous	None

Find the cost of maintaining 8 horses for 1 yr. and of maintaining the tractor for 100 da.

5. If an 8–16 horse-power tractor is used, how much is saved on work requiring the use of 8 horses for 1 yr.?

6. When the tractor is not in use, the only expenses are depreciation and interest on investment. For the horses, however, the expenses are practically the same as given in the table. The cost of one day's idleness for the horses is how much greater than that for the tractor?

7. If the tractor is used only 100 da. in the year, what is the daily cost of fuel, lubricating oil, and oiling (labor)? How does this compare with the cost of feed and feeding (labor) per day for the 8 horses?

VI. LETTERS TO REPRESENT NUMBERS

A SILENT READING LESSON

We live in a more rapid age than our great-grandfathers did. They never saw an automobile, an airplane, or a railroad train that could go a mile a minute. They never dreamed of rapid communication by telegraph, telephone, or radio. They never heard of telling a story by means of a motion picture. They traveled about cities in a horse car instead of an electric car; they wrote letters with a pen instead of a typewriter; and their newspapers were printed by hand, from type that was set by hand.

It is the same way in arithmetic. When our great-grandfathers went to school they learned a rule which was stated like this:

To find the number of square units in the area of any rectangle, multiply the number of linear units in the base by the number of linear units in the height.

A little later this rule was shortened to read:

The area of a rectangle is the base times the height.

This form of the rule was then shortened to read:

$$Area = base \times height.$$

It was then seen that it would be easier to write simply

$$A = b \times h.$$

And so it comes about that today we write this rule as

$$A = b \cdot h,$$

or, more often, in the simpler form

$$A = bh,$$

in which it is understood that when two letters appear together, like bh, their values are to be multiplied.

Arithmetic Shorthand. From the reading lesson on page 351 we see that we can make up a kind of shorthand for arithmetic by using letters to represent numbers.

We can also shorten still more the work of making computations in arithmetic. For example, in finding the area of a rectangle 8 in. long and 3 in. wide, people used to write

$$8 \times 3 \text{ sq. in.} = 24 \text{ sq. in.}$$

When we use arithmetic shorthand we do not need labels like *ft.*, *lb.*, or *sq. in.* We know that the rule for the area is

$$A = bh,$$

and we therefore write the work, without using any labels, simply as

$$A = 8 \times 3 = 24.$$

Our common sense then tells us that this means that the area is 24 sq. in. Similarly, if the rectangle had been 8 ft. long and 3 ft. wide, we should have known that the area was 24 sq. ft.

Letters Represent Numbers. The letters used in this kind of shorthand always represent numbers. Thus,

n means some number,
$3\,n$ means three times this number,
and $\frac{1}{2}\,n$ means half the number.

Similarly, $2.5\,n + 7$ means that 7 is to be added to 2.5 times the number, and $0.3\,n - 5$ means that 5 is to be subtracted from 0.3 of the number.

We may also represent one number by a, another by b, another by c, and so on. Then $a + b$ means that the two numbers represented by these letters are to be added, and $\frac{a}{b+c}$ means that the value represented by a is to be divided by the value of $b + c$.

We speak of numbers represented by letters as *literal numbers* or *literal quantities.*

Adding Literal Numbers. Just as we add 2 ft. and 3 ft., so we can add $2a$ and $3a$; that is, just as

$$2 \text{ ft.} + 3 \text{ ft.} = 5 \text{ ft.},$$

so

$$2a + 3a = 5a.$$

This means that 2 times any number plus 3 times the same number (because it is represented by the same letter) is 5 times that number. We know that this is true because

$$2 \times 7 + 3 \times 7 = 5 \times 7,$$

$$2 \times 1\tfrac{1}{2} + 3 \times 1\tfrac{1}{2} = 5 \times 1\tfrac{1}{2},$$

and so on for whatever number we may let a represent.

Subtracting Literal Numbers. In the same way, just as

$$9 \text{ in.} - 4 \text{ in.} = 5 \text{ in.},$$

so

$$9n - 4n = 5n,$$

and

$$9a - 4a = 5a.$$

If n represents 6 and a represents 2.5, which we shorten to $n = 6$ and $a = 2.5$, these last two statements become

$$9 \times 6 - 4 \times 6 = 5 \times 6,$$

and

$$9 \times 2.5 - 4 \times 2.5 = 5 \times 2.5.$$

We know that this is true, because, as we can easily see by multiplying, the above expressions mean that

$$54 - 24 = 30,$$

and

$$22.5 - 10 = 12.5.$$

Meaning of This Shorthand. When we write $9n - 4n = 5n$, we are simply stating in shorthand the following sentence:

If from nine times any number we take four times the same number, we have five times the number.

Such a long statement might have been good enough for our great-grandfathers, but it is too long for us.

WORKING WITH LITERAL NUMBERS

Perform these additions rapidly:

1. 6 ft.	**2.** $6\,a$	**3.** $6\,n$	**4.** $6\,x$	**5.** $6\,w$
$\underline{4\text{ ft.}}$	$\underline{4\,a}$	$\underline{4\,n}$	$\underline{4\,x}$	$\underline{4\,w}$

6. How much is $7\,a + 5\,a$? What does the addition become when $a = 3$? when $a = 6$? when $a = 9$?

Perform these additions rapidly:

7. 12 in. + 8 in.	**9.** $12\,x + 8\,x$.	**11.** $17\,n + 5\,n$.
8. 12 yd. + 8 yd.	**10.** $12\,y + 8\,y$.	**12.** $16\,a + 10\,a$.

Perform these subtractions rapidly:

13. 7 in.	**14.** $7\,x$	**15.** $7\,a$	**16.** $9\,n$	**17.** $25\,y$
$\underline{2\text{ in.}}$	$\underline{2\,x}$	$\underline{2\,a}$	$\underline{2\,n}$	$\underline{5\,y}$

18. How much is $16\,x - 4\,x$? What does this subtraction become when $x = 1$? when $x = 3$? when $x = \frac{1}{2}$? when $x = \frac{1}{4}$?

Perform these additions rapidly:

19. 3 in.	**21.** $3\,b$	**23.** $3\,x$	**25.** 7 ft.	**27.** $11\,a$
2 in.	$2\,b$	$2\,x$	2 ft.	$15\,a$
$\underline{7\text{ in.}}$	$\underline{7\,b}$	$\underline{7\,x}$	$\underline{8\text{ ft.}}$	$\underline{2\,a}$
20. $7\,a$	**22.** $3\,n$	**24.** $8\,x$	**26.** $7\,x$	**28.** $36\,y$
$3\,a$	$7\,n$	$7\,x$	$2\,x$	$42\,y$
$\underline{2\,a}$	$\underline{2\,n}$	$\underline{4\,x}$	$\underline{8\,x}$	$\underline{81\,y}$

29. How much is $2\,a + 3\,a + 5\,a$? What does this become when $a = 3$? when $a = 5$? when $a = 1$?

30. How much is 7 in. + 2 in. − 3 in.? $7\,a + 2\,a - 3\,a$? $7\,x + 2\,x - 3\,x$? $7\,n + 2\,n - 3\,n$?

A SILENT READING LESSON

There are a few new words of which we need to know the meaning when using literal numbers.

For example, in the expression $3a$ we shall, for our present purposes, speak of the number 3 as the *coefficient* of a. If we study algebra next year, we shall find that coefficients are sometimes letters; that is, in the expression ax we may speak of a as the coefficient of x. If no coefficient is written, it is understood that the coefficient is 1; that is, a is the same as $1a$, and so, in cases like the following, we have

$$a + 2a = 3a,$$

and

$$5n - n = 4n.$$

In the expression $a + b$, the letters, which represent numbers as usual, are known as the *terms* of the expression. In the expression $a - b$, the terms are also a and b. Terms are always separated from one another by the + or − sign. Thus, in the expression $2a + 3b - c$ the terms are $2a$, $3b$, and c.

In the expression bh, which was given on page 351, b and h are *factors* of the expression; they are not terms, because they are not separated by the + or − sign. Thus, the factors of $2a$ are 2 and a, and the factors of $6ax$ are 6, a, and x. In such cases, we do not, for ordinary purposes, need to give the factors of a coefficient like 6.

If we know the value of b in the expression bh, but do not know the value of h, we speak of b as a *known quantity* and of h as an *unknown quantity*; that is, it is customary in this work to say known or unknown quantity instead of known or unknown number.

In work with literal expressions we may thus have known and unknown terms, known and unknown factors, known and unknown numbers, and known and unknown quantities.

WRITING LITERAL EXPRESSIONS

Write an expression that shall mean

1. The sum of any two numbers; for example, a and b.

2. The difference between any two numbers; for example, x and y with x the larger.

3. The difference, as in Ex. 2, but taking x as the smaller.

4. The product of any two numbers.

5. The quotient of any number divided by any other number, letting m represent the dividend and x represent the divisor and writing the result in the fractional form customarily used with literal numbers.

6. The quotient, as in Ex. 5, but taking m as the divisor and x as the dividend.

7. The sum of any three numbers.

8. The sum of twice one number and three times another number.

9. Five times one number less three times another number.

10. In Ex. 9, what is the value of your result if the first number is 8 and the second is 2? if the first is 3 and the second is 5? if the first is $\frac{1}{5}$ and the second is $\frac{1}{3}$?

11. What is the value of $2a$ when $a = 17\frac{1}{2}$? when $a = 1$? when $a = 0$? when $a = 32.7$?

12. What is the value of $\frac{1}{2}x$ when $x = 8$? when $x = 17$? when $x = 0$? when $x = 1.37$? when $x = \frac{1}{2}$?

13. What is the value of the expression $\frac{a+b}{c}$ when $a = 9$, $b = 6$, and $c = 3$? when $a = 7$, $b = 15$, and $c = 2$?

14. What is the value of $\frac{2a+3b-c}{3a-b+5c}$ when $a = 5$, $b = 2$, and $c = 1$? when $a = \frac{1}{2}$, $b = \frac{1}{3}$, and $c = 2$?

A SILENT READING LESSON

When we learned about the powers and roots of numbers, we found that the square of 3 means 3×3, or 9, and that we write 3^2 for "the square of 3." Just as in 3^2 the 2 is called the *exponent* of 3, so in a^2 the 2 is the exponent of a, and shows that a is to be raised to the *second power* or is to be *squared*.

Similarly, a^3 means that a is to be raised to the *third power*, or *cubed*. In this case, the exponent of a is 3.

Just as the expression x means the same as $1\,x$, so it also means x^1; that is, we never write the exponent 1.

In our work with numbers we also learned that, because $4^2 = 4 \times 4 = 16$, we call 4 the *square root* of 16, and because $2^3 = 2 \times 2 \times 2 = 8$, we call 2 the *cube root* of 8. Similarly, n^2 means $n \times n$ and x^3 means $x \times x \times x$, and so

n is the *square root* of n^2,

x is the *cube root* of x^3,

y is the *fourth root* of y^4,

and so on. We shall not in our work, however, need to use exponents larger than 3, or any roots except the square root and the cube root.

Thus, if we wish to represent the square of any number, we may write simply n^2; if we wish to represent twice the square of any number, we may write $2\,n^2$; and if we wish to represent half the square root of any number, we may write $\frac{1}{2}\sqrt{n}$; and so on.

Similarly, when studying the measurement of the circle, we found that the area of a circle is approximately $\frac{22}{7}$ times the square of the radius. We can now express this important relation in the more convenient shorthand, as

$$A = \frac{22}{7}\,r^2.$$

WRITING LITERAL EXPRESSIONS

1. Write a with the coefficient 3.

2. Write b with the exponent 2. What is the value of the result when $b = 3$? when $b = \frac{1}{2}$? when $b = 1$?

3. Write the sum of twice one number, represented by n, and 7 times the same number. What is the value of the result when $n = 6$? when $n = 1$? when $n = \frac{1}{2}$? when $n = 0$?

4. The letter c comes before the letter e. In $3\,a^2$ the coefficient is written first and the exponent last. Do you see a simple way of remembering that the coefficient comes first and the exponent comes last in an expression like this? If so, what is it?

5. In the expression $5\,m^3$, what is the coefficient? what is the exponent? Write an expression in which 7 is the coefficient and 2 is an exponent.

6. What is the value of $2\,n^2$ when $n = 2$? when $n = \frac{1}{2}$?

7. If the edge of a cube is 2 in., how many cubic inches are there in the volume of the cube? What is the volume when the edge is e?

8. In Ex. 7, what is the result of adding all the edges in the first case? in the second case?

9. Write the expression $3 \times a^2 \times b$ without using the symbol ×. One way is $3 \cdot a^2 \cdot b$, but there is a better way.

10. What is the value of your result in Ex. 9 when $a = 2$ and $b = 7$? when $a = 1$ and $b = 1$?

11. What is the value of x^2y when $x = 2$ and $y = 3$? when $x = 3$ and $y = 2$? when $x = 1$ and $y = 1$?

12. What is the value of $3\,xy$ when $x = 1$ and $y = 2$? when $x = \frac{1}{3}$ and $y = 1$?

13. What is the value of $2\,x + x^2$ when $x = 3$? when $x = 1$?

DRILL CHART IN LITERAL EXPRESSIONS

Perform the following operations, in the order in which the terms appear, writing each result as a single term:

1. $2x + 7x$.
2. $15m - 9m$.
3. $6a + 8a$.
4. $4x + 7x$.
5. $17x - x$.
6. $x + 19x$.
7. $2x + 19x$.
8. $8y - 4y$.
9. $8m + 9m$.
10. $17b - 5b$.
11. $2b + 11b$.
12. $23c - 19c$.
13. $5x + 24x$.
14. $26a - 17a$.
15. $a + 24a$.
16. $\frac{1}{2}a + \frac{1}{2}a$.
17. $\frac{3}{4}a - \frac{1}{4}a$.
18. $\frac{7}{8}x - \frac{1}{2}x$.
19. $\frac{5}{8}x + \frac{7}{8}x$.
20. $\frac{3}{4}y - \frac{1}{8}y$.
21. $2.7x + 4.8x$.
22. $3.1y - 0.7y$.
23. $9x - 3x + 2x$.
24. $9x - 3x - 2x$.
25. $15a + 3a + a$.
26. $15a - 3a + a$.
27. $15a + 3a - a$.
28. $15a - 0.3a - a$.
29. $25n + 7n + 4n$.
30. $2.5n + 7n - 0.4n$.
31. $25n - 7n + 4n$.
32. $25n - 7n - 4n$.
33. $1.5w + 10w + 0.5w$.
34. $15w + 10w - 5w$.
35. $15w - 10w - 5w$.
36. $23b + 24b + 8b$.
37. $32b - 24b + 8b$.
38. $3.2b + 2.4b - 0.8b$.
39. $3.2b - 2.4b - 0.8b$.
40. $4\frac{3}{4}x + 2\frac{1}{2}x - \frac{1}{4}x$.
41. $3.9m + 7.4m - 6.9m$.
42. $5.75a^2 + 2.97a^2 - a^2$.
43. $6.23m^2 + 3.98m^2 - 0.67m^2$.
44. $3.66n^3 + 6.27n^3 - 5.77n^3$.

Order of Operations. If we have an expression containing only multiplications, we may perform the multiplications in any order that we wish.

For example, in finding the value of abc when $a = 2$, $b = 3$, and $c = 5$, the product of 2, 3, and 5 is always 30, whatever order we use in multiplying the factors.

In a case like $\frac{ab}{xy}$, we first multiply a and b, we then multiply x and y, and finally we divide the first result by the second. We no longer make use of literal expressions like $ab \div xy$, because the fractional form given above is much better for practical purposes.

In a case involving only the signs + and −, we perform the operations in the order in which they come. For example, in the case of $4b + 3b - 2b + b$, we have

$$4b + 3b = 7b,$$

then

$$7b - 2b = 5b,$$

and, finally,

$$5b + b = 6b.$$

In a case involving the operations indicated by +, −, ×, and the fraction bar (÷), we first perform the operations of multiplication and division in the order in which they come, and then perform the others in the order in which they come. In our present work, however, we shall not find difficult cases that involve so many operations.

In the case of $3ab^2$, the exponent 2 refers only to the b, and so, if $a = 5$ and $b = 4$, we have

$$3ab^2 = 3 \times 5 \times 4^2 = 3 \times 5 \times 4 \times 4 = 240.$$

If there are parentheses, we do the work within them before doing anything else. For example, we have

$$30a - (12a - a) = 30a - 11a = 19a,$$

and

$$5(2a + 3a - a) = 5(5a - a) = 5 \times 4a = 20a.$$

DRILL CHART IN LITERAL EXPRESSIONS

Write each of the following in its simplest form:

1. aa.
2. bb.
3. xx.
4. $2\,xx$.
5. xxx.
6. aaa.
7. bbb.
8. $3\,aa$.
9. $4\,bb$.
10. abb.
11. aab.
12. acc.
13. aac.
14. ccc.
15. xxy.
16. $3\,aab$.
17. $3\,abb$.
18. $4\,xxy$.
19. $4\,xyy$.
20. $4\,xxy^2$.
21. Divide 2×3 by 3; 2×5 by 5; $2\,x$ by x; $2\,x$ by 2.

Write each of the following in its simplest form:

22. $\frac{4\,x}{4}$.
23. $\frac{4\,x}{x}$.
24. $\frac{5\,a^2}{5}$.
25. $\frac{5\,a^2}{a^2}$.
26. $\frac{7\,ab}{7}$.
27. $\frac{7\,ab}{b}$.
28. $\frac{9\,xy}{y}$.
29. $\frac{9\,xy}{x}$.
30. If $x = 5$, what is the value of $2\,x$? of x^2?

Given that $x = 2$ and $y = 3$, find the value of

31. xy.
32. $\frac{x^2}{x}$.
33. $\frac{2\,x^2}{2}$.
34. y^2.
35. $\frac{3\,y}{y}$.
36. $\frac{3\,y^2}{3}$.
37. x^2y.
38. $\frac{x^3y}{y}$.
39. $\frac{x^3y}{x^3}$.
40. $2\,x^2y$.
41. $\frac{x^2y^2}{x^2}$.
42. $\frac{xy^2}{x}$.

Write each of the following in its simplest form:

43. $2\,xy + xy$.
44. $6\,x^2y - 2\,x^2y$.
45. $5\,ab^2c + 2\,ab^2c$.
46. $6\,a^2b^2c^2 - 3\,a^2b^2c^2$.
47. $7\,a^2bx - 3\,a^2bx$.
48. $5\,a^2b^2x^2 + 5\,a^2b^2x^2$.
49. $5\,a^2b^2x^2 - 5\,a^2b^2x^2$.
50. $64\,a^3b^2x - 28\,a^3b^2x$.

REVIEW DRILL CHART IN LITERAL EXPRESSIONS

Given that $a = 3$, $b = 2$, $c = 5$, *find the value of*

1. $a + b + c$.
2. $a + b - c$.
3. $a - b + c$.
4. $a^2 + 2b$.
5. $a^3 - 3c$.
6. $a^3 - c^2$.
7. $a^2 + c^2$.
8. $\frac{125abc}{125}$.
9. $\frac{125abc}{abc}$.
10. $\frac{125abc}{bc}$.
11. $2a + 3b$.
12. $2a - 3b$.
13. $ab + c$.
14. $ab - c$.
15. $4ab - 3c$.
16. $4c - 3ab$.
17. $a^2 - c$.
18. $\frac{100a^2b}{10}$.
19. $\frac{100a^2b}{b}$.
20. $\frac{100a^2b}{a^2}$.
21. $a^2 + b^2 + c^2$.
22. $a^2 + b^2 - c$.
23. $a^2 + c^2 - b^2$.
24. $2abc - 10$.
25. $2a^2bc - 15$.
26. $3ab^2c - 5$.
27. $5abc^2 - c^2$.
28. $\frac{40(a + b)}{40}$.
29. $\frac{40(a + b)}{a + b}$.
30. $\frac{4a(a + b)}{a}$.

Given that $a = 7$, $b = 6$, $x = 1$, *find the value of*

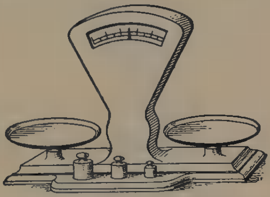
31. abx.
32. abx^2.
33. ab^2x.
34. a^2bx.
35. $15a^2b^2x$.
36. $\frac{20abx}{10}$.
37. $\frac{20abx}{bx}$.
38. $2a^2b$.
39. $20ab^2$.
40. $30ab^2x$.
41. $ab^2 - x$.
42. $a^2b^2 + x$.
43. $\frac{25a^2bx^2}{5}$.
44. $\frac{25a^2b^2x^2}{25a^2}$.
45. $a + b - x$.
46. $a^2 + b^2 - x^2$.
47. $a^2 - b^2 + x^2$.
48. $a^3 + b + 7x$.
49. $a + b^3 - 7x$.
50. $\frac{125a^3b^3x^3}{5b^3}$.
51. $\frac{125a^3b^3x^3}{125a^3b^3}$.

VII. EQUATIONS

SILENT READING LESSON

Any statement of equality, like $3 + 7 = 10$, $x - 3 = 7$, or $A = lw$, is called an *equation*.

If we have a set of scales which balance like those here shown, and if we put equal weights on each side, the scales continue to balance. If we then add more equal weights, the scales still balance; if we take off equal weights, they still balance; if we put twice, or any other number of times, as much weight on each side, they still balance; and if we divide the weights by any number, so that we have only a part as much weight on each side, the scales still balance, as before. That is,



If a pair of scales balance, they continue to balance when we do the same thing (with respect to weight) to each side.

This is such a simple statement that each one of us, by using his common sense, can see that it is true without requiring any further proof.

For example, we can easily see that if we should put a 4-pound weight on each side, the scales would balance. If we should then add a 1-pound weight to each side, they would continue to balance, and if we should then take the 4-pound weight from each side, the scales would still balance. They would also continue to balance if we should multiply the weight on each side by 6 or if we should divide the weight on each side by 5.

What does the equation $12 = 12$ become if we add 6 to each side? if we take 3 from each side? if we multiply each side by 5? if we divide each side by 4?

Axioms. An equation is like a set of scales that balance, and the laws of equations, or the *axioms*, as they are called, are like the common-sense statement on page 363. For example,

1. If you hold 5¢ in each hand, and I add 2¢ more to each amount, how do the amounts then compare? Complete this sentence: *If equals are added to equals, the results* · · · .

2. If you hold 10 coins in each hand, and I take 3 coins from each, how do the coins remaining in each hand compare in number? Complete this sentence: *If equals are subtracted from equals, the* · · · .

3. If in your class the number of boys is equal to the number of girls, how will the numbers compare if each is doubled? Complete this sentence: *If equals are multiplied by the same number, the* · · · .

4. If two boards are of equal length, how do the lengths of one third of each board compare? Complete this sentence: *If equals are divided by the same number, the* · · · .

Stated concisely, the important axioms are as follows:

1. *If equals are added to equals, the results are equal.*
2. *If equals are subtracted from equals, the results are equal.*
3. *If equals are multiplied by the same number, the results are equal.*
4. *If equals are divided by the same number, the results are equal.*

Another axiom that is sometimes used is

5. *Like powers or like roots of equals are equal.*

In an equation we speak of the expressions on opposite sides of the sign of equality as the *sides* or *members* of the equation. We may then say, just as we did for the scales,

Whatever we do to one member of an equation, we must do to the other member.

First Axiom Applied. If we have 50¢ and spend 10¢, and someone then gives us 10¢, we shall still have 50¢; that is,

$$50¢ - 10¢ + 10¢ = 50¢.$$

Similarly, suppose that we are given an equation like

$$x - 10 = 30.$$

If we add 10 to both members, the results will be equal, and we have

$$x - 10 + 10 = 30 + 10,$$

or $$x = 40.$$

$$x - 10 = 30.$$
$$x - 10 + 10 = 30 + 10.$$
$$x = 40.$$
Check. $40 - 10 = 30.$

To *solve* the equation $x - 10 = 30$ means that we are to find the value of the unknown quantity (that is, of x). We do this by finding, as above, that $x = 40$.

We may check our work by *substituting* 40 for x in the original equation $x - 10 = 30$. We then have

$$40 - 10 = 30;$$

that is, we have proved that our value for x is correct.

FIRST AXIOM APPLIED

Solve the following equations:

1. $x - 7 = 36.$
2. $x - 8 = 58.$
3. $x - 5 = 82.$
4. $x - 6 = 41.$
5. $x - 2 = 98.$
6. $x - \frac{1}{4} = \frac{1}{4}.$
7. $x - \frac{1}{2} = \frac{1}{2}.$
8. $x - \frac{1}{2} = \frac{1}{4}.$
9. $x - \frac{1}{8} = \frac{3}{8}.$
10. $x - \frac{7}{8} = \frac{3}{8}.$
11. $x - 0.5 = 2.$
12. $x - 4.5 = 1.$
13. $x - 0.2 = 2.8.$
14. $x - 1.8 = 0.2.$
15. $x - 3.7 = 5.8.$
16. If $x - 4 = 36$, what is the value of x?
17. Solve the equation $x - 8 = 32$ and check the result.

Solve the following equations and check the results:

18. $x - 8 = 39.$
19. $x - 9 = 52.$
20. $x - 7 = 64.$

Second Axiom Applied. Suppose that when 9 is added to a certain number, the result is 41. What is the number?

Letting x represent the number, the shorthand way of telling the story is the equation

$$x + 9 = 41.$$

Solving by subtracting 9 from both members, we have

$$x + 9 - 9 = 41 - 9,$$

or $$x = 32.$$

$$x + 9 = 41.$$
$$x + 9 - 9 = 41 - 9.$$
$$x = 32.$$
Check. $32 + 9 = 41.$

After a little practice in solving equations, we omit the step

$$x + 9 - 9 = 41 - 9.$$

SECOND AXIOM APPLIED

Solve the following equations and check each result:

1. $x + 3 = 43.$
2. $x + 4 = 50.$
3. $x + 5 = 42.$
4. $x + 6 = 56.$
5. $x + 7 = 49.$
6. $y + 7 = 17.$
7. $y + 6 = 27.$
8. $y + 5 = 19.$
9. $n + 4 = 84.$
10. $n + 3 = 51.$
11. $z + 40 = 50.$
12. $z + 40 = 40.$
13. $z + 32 = 33.$
14. $m + 44 = 55.$
15. $m + 86 = 89.$

16. If $2x = x + 3$, what must we subtract from both members so as to have no x in the second member? If you subtract this from both members, what is left in the first member? in the second member? What is the value of x? Prove it.

Solve the following equations and check each result:

17. $2x = x + 9.$
18. $3x = 2x + 7.$
19. $5x = 4x + 11.$
20. $2x = x + 3.5.$
21. $2y = y + 2.8.$
22. $3z = 2z + \frac{1}{2}.$
23. $x + 9 = 19.$
24. $5n = 4n + 1.$
25. $2x - 9 = x.$
26. $2y - 7 = y.$
27. $2x + 9 = 3x.$
28. $3y + 1 = 4y.$

Third Axiom Applied. Suppose someone says that he is thinking of a certain number, and that half of it is 17. What is the number?

Letting x represent the number, the shorthand way of telling the story is the equation

$$\tfrac{1}{2}x = 17.$$

Multiplying these equals by 2, so as to change the $\tfrac{1}{2}$ to 1,

$$2 \times \tfrac{1}{2}x = 2 \times 17,$$

or

$$x = 34.$$

$$\tfrac{1}{2}x = 17.$$
$$2 \times \tfrac{1}{2}x = 2 \times 17.$$
$$x = 34.$$

Check. $\tfrac{1}{2}$ of $34 = 17$.

Instead of $\tfrac{1}{2}x$ we may write $\dfrac{x}{2}$ and proceed in the same way, omitting the second step after a little practice.

THIRD AXIOM APPLIED

Find the value of the letter in each of these equations:

1. $\tfrac{1}{2}x = 7.$
2. $\tfrac{1}{2}x = 5.$
3. $\tfrac{1}{3}x = 4.$
4. $\tfrac{1}{3}x = 9.$
5. $\dfrac{x}{4} = 60.$
6. $\tfrac{1}{4}x = 7.$
7. $\tfrac{1}{4}x = 9.$
8. $\tfrac{1}{5}y = 8.$
9. $\tfrac{1}{6}y = 11.$
10. $\dfrac{y}{8} = 100.$
11. $\tfrac{1}{5}x = 14.$
12. $\tfrac{1}{6}x = 15.$
13. $\tfrac{1}{8}x = 20.$
14. $\tfrac{1}{9}x = 30.$
15. $\dfrac{x}{10} = 140.$
16. $\tfrac{1}{10}y = 2.$
17. $\tfrac{1}{10}y = 2.1.$
18. $\tfrac{1}{10}y = 0.3.$
19. $\tfrac{1}{24}y = 10.$
20. $\dfrac{y}{20} = 200.$

21. If in the equation $\tfrac{1}{2}x + 3 = 16$ you subtract 3 from both members, what do you have? What is the value of x? Check the result.

Solve the following equations and check each result:

22. $\tfrac{1}{3}x + 2 = 12.$
23. $\tfrac{1}{4}x + 5 = 18.$
24. $\tfrac{1}{5}x + 7 = 17.$
25. $\tfrac{1}{8}x + 9 = 20.$
26. $\tfrac{1}{6}x + 7 = 30.$
27. $\tfrac{1}{10}x + 3 = 43.$
28. $\tfrac{1}{2}x + 20 = 21.$
29. $\tfrac{1}{3}x + 11 = 21.$
30. $\tfrac{1}{4}x + 11 = 30.$

Fourth Axiom Applied. Harry is thinking of a number which, when multiplied by 5, gives 75. What is the number?

Letting x represent the number, the shorthand way of telling the story is the equation

$$5x = 75.$$

Dividing both members of this equation by 5, we have

$$x = 15.$$

$$\boxed{\begin{aligned} 5x &= 75. \\ x &= 15. \\ \textit{Check.}\quad 5 \times 15 &= 75. \end{aligned}}$$

FOURTH AXIOM APPLIED

Find the value of the letter in each of these equations:

1. $3x = 6.$
2. $4x = 16.$
3. $5y = 40.$
4. $2x = 34.$
5. $3y = 90.$
6. $4y = 80.$
7. $5x = 20.$
8. $6x = 30.$
9. $7x = 63.$
10. $2.2x = 4.4.$
11. $3.3x = 6.6.$
12. $4.5x = 90.$

Solve the following equations and check each result:

13. $100x = 200.$
14. $117x = 585.$
15. $2.41x = 7.23.$
16. $3\frac{3}{4}x = 15.$
17. $15x = 3\frac{3}{4}.$
18. $2\frac{7}{8}x = 46.$
19. $1.213x = 8.491.$
20. $1.311x = 78.66.$
21. $0.742x = 667.8.$

22. Mary is thinking of a number which, when multiplied by $\frac{2}{3}$, becomes 18. What is the number?

23. In the equation $5x - 7 = 33$ what must be added to both members to leave $5x$ alone in the first member? What is the value of x? Check your result.

Solve the following equations and check each result:

24. $2x - 3 = 7.$
25. $3x - 2 = 13.$
26. $5x - 6 = 34.$
27. $6x - 2 = 16.$
28. $2x - \frac{1}{2} = \frac{3}{4}.$
29. $\frac{3}{4}x - \frac{1}{2} = 2.$
30. $5x - \frac{1}{8} = \frac{1}{2}.$
31. $8x - \frac{1}{5} = \frac{4}{5}.$
32. $\frac{1}{2}x - 1 = \frac{1}{2}.$

Further Work in Solving Equations. In some cases we combine two or more of the methods already learned, as in the following examples:

1. Solve the equation

$$\frac{x}{3}+7=11.$$

First subtracting 7, we have

$$\frac{x}{3}=4,$$

and then, multiplying by 3,

$$x=12.$$

$$\boxed{\begin{aligned}\frac{x}{3}+7&=11.\\ \frac{x}{3}&=4.\\ x&=12.\\ \textit{Check. } \tfrac{12}{3}+7&=4+7\\ &=11.\end{aligned}}$$

Check. Substituting 12 for x in the given equation, we can show, as in the work above, that this result is correct.

2. Solve the equation

$$\frac{x}{2}+\frac{x}{3}-7=6.$$

Adding 7 to both members,

$$\frac{x}{2}+\frac{x}{3}=13.$$

If we now multiply both members by 2, $\frac{x}{2}$ will become x, and if by 3, $\frac{x}{3}$ will become x; that is, if we multiply both members by both 2 and 3, or 6, we have

$$\boxed{\begin{aligned}\frac{x}{2}+\frac{x}{3}-7&=6.\\ \frac{x}{2}+\frac{x}{3}&=13.\\ 3x+2x&=78.\\ 5x&=78.\\ x&=15.6.\end{aligned}}$$

$$6\times\frac{x}{2}+6\times\frac{x}{3}=6\times 13,$$

or $$3x+2x=78;$$

whence $$5x=78,$$

and, dividing by 5, $$x=15.6.$$

Check. $\frac{15.6}{2}+\frac{15.6}{3}-7=7.8+5.2-7=13-7=6.$

DRILL CHART IN EQUATIONS

Solve the following equations and check each result:

1. $x + 3 = 5$.
2. $x - 3 = 6$.
3. $3x = 9$.
4. $\frac{1}{3}x = 7$.
5. $2x = 6 - x$.
6. $2x = 6 + x$.
7. $7x = 2x + 5$.
8. $x - 9 = 12$.
9. $x + 9 = 12$.
10. $x - 12 = 9$.
11. $7x - 12 = 9$.
12. $3x - 12 = 9$.
13. $5x - 9 = 12$.
14. $\frac{1}{2}x + 7 = 16$.
15. $\frac{1}{2}x - 7 = 16$.
16. $\frac{1}{2}x - 16 = 7$.
17. $\frac{2x}{3} + 7 = 9$.
18. $\frac{8x}{3} - 5 = 35$.
19. $\frac{x}{2} + \frac{x}{3} = 12$.
20. $\frac{3y}{4} - \frac{2y}{3} = 17.2$.
21. $2x + 9 = 29$.
22. $2x - 9 = 29$.
23. $2x - 29 = 9$.
24. $3x - 29 = 1$.
25. $3x + 8 = 29$.
26. $5x + 6 = 18$.
27. $5x - 6 = 14$.
28. $5x - 14 = 6$.
29. $14 = 6 + 8x$.
30. $31 = 3x + 22$.
31. $17x - 31 = 20$.
32. $14x - 17 = 11$.
33. $2.6x - 2.2 = 3$.
34. $1.6x - 3.6 = 4.4$.
35. $4.1y - 1.75 = 10.55$.
36. $7.5y - 13 = 18.5$.
37. $\frac{3x}{4} - 7 = 14$.
38. $\frac{2y}{3} - 9 = 27$.
39. $\frac{2n}{5} + \frac{3n}{8} = 62$.
40. $\frac{5n}{8} + \frac{2n}{3} = 124$.

Directions for Solving Equations. 1. *If the equation contains fractions, multiply both members by an expression that will remove the fractions.*

For example, if we are given the equation

$$\frac{8x}{2x+3}=3,$$

and multiply both members (Axiom 3) by $2x+3$, we have

$$8x=6x+9.$$

2. *Subtract from both members such terms as are necessary in order to leave the unknown quantity on one side and the known numbers on the other side.*

For example, in the last equation above we have

$$8x=6x+9.$$

Subtracting $6x$ from both members (Axiom 2), we have

$$8x-6x=9,$$

or

$$2x=9.$$

This operation is called *transposition*, and we may say that we *transpose* the term $6x$.

3. *Divide both sides of the equation by the coefficient of the unknown quantity.*

For example, if in the equation $2x=9$, given above, we divide both members (Axiom 4) by 2, we have

$$x=\tfrac{9}{2}, \text{ or } 4\tfrac{1}{2}.$$

4. *Check the solution by substituting in the original equation the value found for the unknown.*

Thus, substituting $\frac{9}{2}$ for x in the given equation, we have

$$\frac{8\times\frac{9}{2}}{2\times\frac{9}{2}+3}=\frac{36}{9+3}=\frac{36}{12}=3.$$

DRILL CHART IN EQUATIONS

Solve the following equations and check each result:

1. $2x + 7 = x + 19.$
2. $3x - 2 = x + 16.$
3. $5x + 6 = x + 34.$
4. $7x - 9 = 3x + 31.$
5. $3x + 9 = x + 35.$
6. $6x - 8 = 3x + 37.$
7. $9x + 13 = 6x + 85.$
8. $8x - 13 = 6x + 13.$
9. $5 + 8x = 17 + 4x.$
10. $9.1x + 1 = 6.1x + 34.$
11. $3x - 9 = 7 - x.$
12. $8x - 10 - 7x = 4.$
13. $8x - 37 = 2x - 7.$
14. $6x - 44 + 2x = 4x.$
15. $7x + 7 = 31 + x.$
16. $10 + 3x - 21 = 2x.$
17. $\dfrac{9x + 3}{2x + 2} = 3.$
18. $\dfrac{12x + 21}{2x + 5} = 5.$
19. $\dfrac{8x - 7}{3x + 2} = 2.$
20. $\dfrac{7x - 1}{x + 2} = 3.$
21. $5x + 24.2 = 48.7.$
22. $9y - 32.4 = y + 0.4.$
23. $3.8y + 4.3 = 80.3.$
24. $7.4 + 3y = y + 9.8.$
25. $6 - 2x + 8x = 25.$
26. $2 + 21x + x = 25.$
27. $7x + 2 = 3x + 6.$
28. $3x + 3 = x + 73.$
29. $5x + 29 = 2x + 32.$
30. $42x = 39x + 639.$
31. $9x + 3 = 4x + 28.$
32. $4x - 9 = 5 + 2x.$
33. $5 + 22x = 12x + 75.$
34. $5 + 12x = 3x + 50.$
35. $8 + 9x = 9 + x.$
36. $83 + 2.7x = 1.2x + 98.$
37. $\dfrac{35x - 49}{3x + 5} = 10.$
38. $\dfrac{8x - 1}{3x + \frac{1}{2}} = 2.$
39. $\dfrac{42x - 5}{x + 2\frac{1}{2}} + 4 = 30.$
40. $\dfrac{25x - 35}{9 - 2x} = 10.$

Solving Problems. The following examples show how we may use equations in solving problems:

1. Edward is thinking of a number such that half of it plus a third of it is 10. What is the number?

Letting x represent the number, we have the equation

$$\frac{x}{2}+\frac{x}{3}=10.$$

Multiplying both members by 2×3, or 6, we have

$$3x+2x=60,$$

or $$5x=60,$$

and, dividing by 5, $$x=12.$$

Check. $\frac{1}{2}$ of $12+\frac{1}{3}$ of $12=6+4=10$.

2. A carpenter wishes to cut a board 20 ft. long into two pieces such that one piece is four times as long as the other. Into what lengths should he cut the board?

At first reading, it looks as though we had two unknown lengths, but, if we know the length of one piece we can find the other by subtraction. Thus, letting n represent the *number of feet* in the shorter piece, we have

$$n+4n=20,$$

or $$5n=20,$$

and, dividing by 5, $$n=4;$$

that is, he should cut the board into lengths of 4 ft. and 16 ft.

Check. 16 ft. ÷ 4 ft. = 4, and 16 ft. + 4 ft. = 20 ft.

General Directions. In solving a problem we should (1) read it carefully; (2) determine what is to be found and represent this by x, n, or some other letter; (3) state the equation; (4) solve the equation; and (5) check the result by seeing if it meets the conditions given in the problem.

SOLVING PROBLEMS

Find the unknown number described in each of Exs. 1–14 and check each result:

1. If 8 is added to the number, the result is 47.

2. If 6 is subtracted from the number, the result is 8.

3. When the number is multiplied by 7, the result is 63.

4. When the number is divided by 9, the result is 8.

5. The result of adding the number to twice itself is 27.

6. When the number is added to 27, the result is 29.

7. When twice the number is added to 8, the result is 44.

8. If 9 is subtracted from seven times the number, the result is 82.

9. If from three fourths of the number half the number is taken, the result is 14.

10. When the number is increased by three fourths of itself, the result is 28.

11. The number is 42 more than half itself.

12. The number is $3\frac{1}{2}$ more than half itself.

13. The number is 9 more than three fourths of itself.

14. The number is $\frac{7}{8}$ more than three fourths of itself.

15. Ben is thinking of a number whose double is 48. What is the number of which he is thinking?

16. The number 69 is three times the number of which Kate is thinking. Of what number is Kate thinking?

17. If three fourths of the number of marbles a boy has is 18, how many marbles has he?

18. A man spent one fourth of a day's wages for food. If he spent $1.35 for food, what were his wages?

19. A man sold his car for $450, which was five eighths of its cost. What was the cost?

20. When eight times a certain number is diminished by 9, the result is 23. What is the number?

21. In grinding wheat into flour there is a loss of 18%. How much wheat is needed to make 2132 lb. of flour?

22. To a building 60 ft. long and 40 ft. wide, a wing 30 ft. wide and having half the floor space of the original building is to be added. Find the length of the wing.

23. The top of a tree 30 ft. high is broken off by the wind in such a way that the broken part is twice as long as the part of the trunk that is left standing. Find the length of the broken part of the tree.

24. Seven times a certain number is equal to the number increased by 24. Find the number.

25. When seven times a certain number is decreased by 35, the result is twice the number. What is the number?

26. When a certain number is multiplied by 4, and the result is added to 13, the final result is 25. Find the number.

27. Find a number such that the result of adding nine times the number and eleven times the number is 480.

28. The greater of two numbers is seven times the less, and their sum is 96. Letting x represent the lesser number and $96 - x$ the greater, find the numbers.

29. Separate 90 into two parts such that the larger is twice the smaller.

30. Separate 100 into two parts such that the larger is three times the smaller.

31. The sum of three numbers is 21. If the second number is four times the first, and the third is half the second, what are the three numbers?

Further Work in Solving Problems. The following examples illustrate the uses of equations in solving other problems:

1. An agent, who made a profit of \$120 on the sale of some property, said that he made 5%. Find the selling price.

We know that $5\% = \frac{1}{20}$, and so, if we let x represent the *number of dollars* in the selling price, the shorthand way of telling the story is

$$\tfrac{1}{20}x = 120.$$

Multiplying by 20, $x = 2400.$

That is, the selling price of the property was \$2400.

Check. $0.05 \times \$2400 = \$120.$

2. Find two consecutive numbers whose sum is 279.

Let n represent the smaller number. Then, since the numbers are consecutive, the next number must be 1 more than this (that is, $n + 1$ represents the larger number), and so

$$n + n + 1 = 279,$$

or $2n + 1 = 279.$

Subtracting 1, $2n = 278.$

Dividing by 2, $n = 139$, the smaller number,

and $n + 1 = 140$, the larger number.

Check. $139 + 140 = 279.$

3. Find a number such that the tens' digit is 4 more than the units' digit, and the sum of the two digits is 10.

If x is the units' digit, $x + 4$ is the tens' digit; that is,

$$x + x + 4 = 10.$$

Solving, $x = 3$, the units' digit,

and $x + 4 = 7$, the tens' digit.

That is, the required number is 73.

Check. $7 - 3 = 4$, and $7 + 3 = 10.$

SOLVING PROBLEMS

Find the number described in each of Exs. 1–7:

1. The tens' digit is 2 more than the units' digit, and the sum of the digits is 10.

2. The units' digit is 4 more than the tens' digit, and the sum of the digits is 12.

3. The tens' digit is one half the units' digit, and the sum of the digits is 9.

4. The tens' digit is 1 more than the units' digit, and the sum of the digits is 9.

5. The units' digit is 5 less than the tens' digit, and the number is 2 more than ten times the tens' digit. That is, if t is the tens' digit, the units' digit is $t-5$, and the number is $10t+t-5$, because it is made up of t tens, or $10t$, plus $(t-5)$ units. Now solve the equation $10t+t-5=10t+2$, which expresses the conditions of the problem.

6. The tens' digit is four times the units' digit, and the number is 10 more than nine times the tens' digit.

7. The tens' digit is three times the units' digit, and the number is 2 more than ten times the tens' digit.

8. Find two consecutive numbers whose sum is 441.

9. Find two consecutive numbers whose sum is 481.

10. If 8 is subtracted from nine times a certain number, the result is 73. What is the number?

11. If eight times a certain number is decreased by one fourth of itself, the result is 124. Find the number.

12. What number, on being decreased by $33\frac{1}{3}\%$ of itself, becomes 88?

13. What number, on being increased by $12\frac{1}{2}\%$ of itself, becomes 54?

14. A man sold an automobile for \$1500, gaining 20% on what he paid for it. Find the price which he paid for it.

15. If a dealer sold a radio set for \$120, which was 25% above the cost price, what did the set cost the dealer?

16. At a bargain sale a dealer offered a discount of 15% on his entire stock. Find the original marked price of a coat for which the dealer received \$55.25.

17. If 3 is subtracted from three times a certain number, and if 9 is added to twice the number, the two results are equal. What is the number?

18. A rope 90 ft. long is cut into two parts in such a way that the longer part is three times the shorter. Find the length of each part.

19. If $37\frac{1}{2}\%$ of a certain sum of money is \$135, what is the sum of money?

20. A certain sum of money increased by \$14 is equal to eight times itself. Find the sum of money.

21. A boy 5 yr. ago was two thirds as old as he is at present. How old is he now?

22. In 26 yr. from now a boy will be three times as old as he is at present. How old is he now?

23. The length of a rectangle is 8 ft. more than the width, and the perimeter is 64 ft. Find the dimensions.

24. The length of a tennis court is 42 ft. more than the width, and the perimeter is 228 ft. Find the dimensions.

25. If 10 yr. ago a boy was one third as old as he is now, how old is he now?

26. A merchant's sales last year were 10% more than they were the year before. If the sales last year were \$38,500, how much were they the year before?

VIII. FORMULAS

A SILENT READING LESSON

We have learned that representing numbers by letters is helpful in solving certain kinds of problems. We shall now consider another, and much more important, use of this device for shortening work.

For example, we know that the area of a circle is approximately $\frac{22}{7}$ times the square of the radius, and have seen that it is much easier to express this in mathematical shorthand as

$$A = \tfrac{22}{7} r^2.$$

Such an expression is called a *formula*. It is evident that a formula is much more easily written and remembered than a rule expressed in words.

In later work in mathematics we shall find the above formula written with the Greek letter π (pi) in place of $\frac{22}{7}$, because $\frac{22}{7}$ is only an approximation for a number which is really an endless decimal beginning 3.14159; that is, the formula for the area of a circle is usually written as

$$A = \pi r^2.$$

From this formula we can easily find the value of A if we know the value of r. For example, in finding the area of a circle with a radius of 7 in., we see that in this case

$$r = 7,$$

and so, taking $\frac{22}{7}$ as the value of π as usual, we have

$$A = \pi r^2 = \frac{22}{7} \times 7 \times 7 = 154.$$

That is, since the radius is given in inches, the area of the circle is 154 sq. in.

A little later we shall consider the case of finding the value of r from this formula when we know the value of A.

FINDING VALUES FROM FORMULAS

1. The formula for the area (A) of a rectangle with the base b and the height h is $A = bh$. Find the value of A when $b = 8$ and $h = 4$.

Using the formula $A = bh$, find the value of A when

2. $b = 4$, $h = 2$.
3. $b = 7$, $h = 3$.
4. $b = 9$, $h = 5$.
5. $b = 8$, $h = 1\frac{1}{4}$.
6. $b = 6$, $h = 2\frac{1}{2}$.
7. $b = 9$, $h = 3\frac{1}{3}$.
8. $b = 18$, $h = 5$.
9. $b = 16$, $h = 8\frac{1}{4}$.
10. $b = 20$, $h = 9\frac{3}{4}$.

11. The formula for the area (A) of a triangle with the base b and the height h is $A = \frac{1}{2}bh$. Find the value of A when $b = 4$ and $h = 2\frac{1}{2}$.

Using the formula $A = \frac{1}{2}bh$, find the value of A when

12. $b = 3$, $h = 2$.
13. $b = 4$, $h = 7$.
14. $b = 6$, $h = 5$.
15. $b = 8$, $h = 3\frac{1}{2}$.
16. $b = 6$, $h = 2\frac{1}{4}$.
17. $b = 8$, $h = 5\frac{3}{4}$.
18. $b = 12$, $h = 9$.
19. $b = 9$, $h = 12$.
20. $b = 16$, $h = 9\frac{7}{8}$.

21. The formulas for the circumference (C) of a circle with the diameter d and the radius r are $C = \pi d$ and $C = 2\pi r$. Find the value of C when $d = 4$, using $\frac{22}{7}$ for the value of π. Find also the value of C when $r = 2$. Explain why the two results are the same.

Using one of the formulas of Ex. 21, find the value of C when

22. $r = 7$.
23. $d = 14$.
24. $r = 14$.
25. $d = 28$.
26. $r = 3\frac{1}{2}$.
27. $d = 7$.
28. $r = 2.1$.
29. $d = 4.2$.

30. The formula for the area of the entire surface of a cube with the edge e is $A = 6e^2$. Find the total surface area of a cube whose edge is 5 in.

31. The formula for the volume of a cube with the edge e is $V = e^3$. Find the volume of a cube whose edge is 3 in.

A SILENT READING LESSON

The formula is also very convenient as a means of remembering the rules that are needed in business. For example, in finding the interest on \$250 for 6 mo. at 5%, we first find 5% of \$250, which is \$12.50, as the interest for 1 yr. We then take half this result, which is \$6.25, as the interest for 6 mo. If we were to write a rule for this, we should have

To find the interest, multiply the principal by the rate for 1 yr. and then multiply the result by the time expressed in years.

This rule is much too long for a modern business man to bother with. A formula is shorter and is more easily understood and remembered. Letting i stand for the number of dollars of interest, p for the number of dollars of principal, r for the rate of interest expressed as hundredths, and t for the time in years, the above rule may be written

$$i = prt.$$

Similarly, for finding the horse power of a steam engine, there is a long rule which any one of us would find very difficult to remember. But when we write this long rule as a formula, it is simply

$$h = \frac{plan}{33{,}000}.$$

This means that the horse power (h) is equal to the steam pressure (p), times the length (l) of the piston stroke in the cylinder, times the area (a) of the piston, times the number (n) of piston strokes per minute, divided by 33,000.

We are not expected to remember such a formula or even to understand the technical terms used. It is given simply to show how condensed and how clear a formula is. From such an illustration we can readily infer that the formula is an important and useful device in all kinds of work.

Deriving Formulas. Since formulas are equations, we can solve them for any letter we choose. That is, from one formula we can derive other formulas that we may need. The following examples show how we proceed in such cases:

1. From the interest formula, $i = prt$, find a formula for p, and from it find the principal that must be invested in order to yield $12.50 in interest in 1 yr. at 5%.

Dividing $$i = prt$$

by rt, we have $$\frac{i}{rt} = p,$$

or, as usually written, $$p = \frac{i}{rt}.$$

Then, since $i = 12.50$, $r = 0.05$, and $t = 1$, we have

$$p = \frac{12.50}{0.05 \times 1} = 250;$$

that is, the principal that will yield the given interest is $250.

2. From the formula for the area of a circle, $A = \pi r^2$, find a formula for r, and from it find the radius of a circle which has an area of 44 sq. in.

Dividing $$A = \pi r^2$$

by π, we have $$\frac{A}{\pi} = r^2.$$

Taking the square root of both members, we have

$$r = \sqrt{\frac{A}{\pi}}.$$

Then, since $\pi = \frac{22}{7}$ and $A = 44$, we have

$$r = \sqrt{44 \times \tfrac{7}{22}} = \sqrt{14}.$$

From the table on page 273, we have $\sqrt{14} = 3.742$; that is, the radius of the circle is 3.7 in., to the nearest 0.1 in.

DERIVING FORMULAS

1. From the formula $C = 2\pi r$ find a formula for r.

Find to the nearest 0.1 in. the radius of a circle which has a circumference of

2. 22 in. **3.** 4.4 in. **4.** 5 in. **5.** 24.2 in. **6.** 100 in.

7. The formula for the area of the curve surface of a cylinder is $S = 2\pi rh$. Find a formula for r.

Find to the nearest 0.1 in. the radius of a cylinder when the area of the curve surface (S) and the height (h) are as follows:

8. 44 sq. in., 4 in.

9. 88 sq. in., 10 in.

10. 100 sq. in., 8 in.

11. 400 sq. in., 20 in.

12. The formula for the volume (V) of a cylinder of radius r and height h is $V = \pi r^2 h$. Find a formula for h.

Find to the nearest 0.1 in. the height of a cylinder when the volume (V) and radius (r) are as follows:

13. 22 cu. in., 3 in.

14. 44 cu. in., 5 in.

15. 100 cu. in., 7 in.

16. 400 cu. in., 10 in.

17. From the formula of Ex. 12 find a formula for r.

Find to the nearest 0.1 in. the radius of a cylinder when the volume (V) and height (h) are as follows:

18. 66 cu. in., 4 in.

19. 88 cu. in., 11 in.

20. 100 cu. in., 7 in.

21. 150 cu. in., 14 in.

22. From the interest formula, $i = prt$, find a formula for r, and then find one for t.

23. Make up three examples applying the formulas of Ex. 22 and of Ex. 1, page 382, and solve each one.

DEVELOPING FORMULAS

1. If an agent sells $500 worth of goods and receives a commission of 4%, what is the amount of his commission?

2. Letting s represent the amount of a sale and r the rate of commission, develop a formula for c, the amount of commission received.

3. From the formula in Ex. 2, find the amount of commission at 3% on a sale amounting to $1200.

4. A man sells a house that cost him $6500 and makes a profit of 10%. What is the amount of the profit that he makes on the transaction?

5. From Ex. 4 write a formula for the amount (p) of profit when the cost is c dollars and the rate of profit is $r\%$.

6. From Ex. 5 find a formula for r when p and c are given.

7. An insurance policy for i dollars requires the payment of a premium p at the rate r per $100. Write a formula for p.

8. From Ex. 7 find a formula for r.

9. A man's property is assessed at $5000, and the tax rate is $25 per $1000. How large a tax does he pay?

10. From Ex. 9 write a formula for the amount (t) of the tax when the assessed valuation a and the tax rate r per $1000 are given.

11. From Ex. 10 find a formula for r.

12. From Ex. 10 find a formula for a.

13. The formula for the area of a parallelogram with the base b and the height h is $A = bh$. From this derive two other formulas, one for b and one for h.

14. Use all three formulas of Ex. 13 in three problems that you make up about parallelograms.

15. One formula for the area of a trapezoid with the bases b and b' (read b-prime) and the height h, is $A = \frac{1}{2} h(b + b')$. By dividing both members by $b + b'$ and then multiplying both members by 2, find a formula for h.

16. Use the result of Ex. 15 to find the height of a trapezoid which has an area of 50 sq. in. and bases of 6 in. and 4 in.

17. If you drive your car 20 mi. per hour for 3 hr., what distance do you go? If you drive r miles per hour for t hours, what distance (D) do you go?

18. From the formula $D = rt$, find two other formulas, one for r and the other for t.

19. Use the three formulas of Ex. 18 in three problems that you make up about driving a car.

20. If one book costs $2, how much will 3 books cost? If one book costs p dollars, what will be the cost (C) of n books at the same price?

21. From the formula $C = np$, derive two other formulas, and give an example illustrating the use of each one.

22. From the formula $S = 4\pi r^2$, which gives the area of the surface of a sphere, derive a formula for r.

23. The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$. Find the volume of a sphere with a radius of 21 in.

24. A ball with a diameter of 7 in. is placed inside a box having the form of a cube with an edge of 7 in., inside measurement. How many cubic inches are there in the box that are not occupied by the ball?

25. Solve Ex. 24 when the diameter and the edge are each 14 in. instead of 7 in.

26. The formula for the hypotenuse h of a right triangle in which the other two sides are a and b is $h = \sqrt{a^2 + b^2}$. Find the value of h when $a = 6$ and $b = 8$.

REVIEW OF FORMULAS

From the formula $T = 2\,\pi rh + 2\,\pi r^2$ *find the value of* T *when*

1. $r = 7, h = 5.$ **3.** $r = 7, h = 7.$ **5.** $r = 14, h = 28.$

2. $r = 7, h = 6.$ **4.** $r = 7, h = 8.$ **6.** $r = 63, h = 15.$

From the formula $V = \pi r^2 h$ *find the value of* V *when*

7. $r = 7, h = 6.$ **9.** $r = 1, h = 7.$ **11.** $r = 14, h = 1.$

8. $r = 3, h = 8.$ **10.** $r = 7, h = 1.$ **12.** $r = 32, h = 4.$

13. In the formula $i = prt$, find the value of i when $p = 8000$, $r = 0.04$, and $t = 2$. Then write the problem in words and state what the result means.

14. Using the formula of Ex. 13, find the interest on $5000 for 4 mo. at 5%.

15. The amount (A) of a debt at a given time is the principal plus the interest. Write a formula for A in terms of p, r, and t; that is, using only p, r, and t to express the value of A.

16. Using the formula of Ex. 15, compute the amount, at the end of 2 yr., of a debt of $5000 bearing interest at 5%.

Taking π *as* $\frac{22}{7}$, *write the following formulas in words:*

17. $V = \frac{4}{3}\,\pi r^3$, where V stands for the volume of a sphere and r for the radius.

18. $V = \frac{1}{3}\,\pi r^2 h$, where V stands for the volume of a cone, r for the radius of the base, and h for the height.

19. $S = 4\,\pi r^2$, where S stands for the area of the surface of a sphere and r for the radius.

20. From the formula $T = 2\,\pi rh + 2\,\pi r^2$, which gives the area of the entire surface of a cylinder, derive a formula for h.

21. If an automobile averages m miles an hour, what is the distance (d miles) traveled in h hours? Find the value of d when $m = 25$ and $h = 3\frac{1}{2}$.

IX. REVIEW AND DRILL

MINIMUM ESSENTIALS

Add and also subtract as follows:

1. $3\frac{7}{8}$	**2.** $4\frac{7}{8}$	**3.** $7\frac{5}{8}$	**4.** $9\frac{1}{16}$	**5.** $8\frac{1}{32}$	**6.** $6\frac{7}{16}$
$\underline{2\frac{5}{16}}$	$\underline{3\frac{1}{4}}$	$\underline{2\frac{15}{16}}$	$\underline{3\frac{5}{8}}$	$\underline{2\frac{3}{4}}$	$\underline{3\frac{5}{8}}$

Multiply and also divide as follows:

7. $\frac{2}{3}$ by $\frac{4}{5}$. **8.** $\frac{7}{32}$ by $\frac{1}{8}$. **9.** $\frac{9}{10}$ by $\frac{2}{3}$. **10.** $37\frac{1}{2}$ ft. by $2\frac{1}{4}$.

Add and also subtract as follows:

11. \$7.25	**12.** \$8.32	**13.** 16.4 mi.	**14.** 26.32 in.
$\underline{6.38}$	$\underline{.76}$	$\underline{7.9\text{ mi.}}$	$\underline{9.76\text{ in.}}$

Multiply and also divide as follows:

15. 172.8 by 1.44. **16.** 473.48 by 6.23. **17.** 47.595 by 5.01.

Find the value in each of the following cases:

18. 45% of \$12. **20.** $37\frac{1}{2}$% of \$72.64. **22.** $6\frac{1}{4}$% of 80.

19. 15% of \$65. **21.** $66\frac{2}{3}$% of \$117.36. **23.** $66\frac{2}{3}$% of $37\frac{1}{2}$.

24. Using a short method, add 125% of 5280 ft. to $\frac{3}{4}$ mi.

Find, to the nearest 0.01, the value of

25. 28.76^2. **26.** 3.14×5.6^2. **27.** $\sqrt{138}$. **28.** $\sqrt{16.75}$.

Find the value of x in each of the following equations:

29. $x - 9 = 6$.

30. $x + 7 = 21$.

31. $2x = 38$.

32. $\frac{1}{4}x = 5$.

33. $\frac{2x-3}{5} = 3$.

34. $2x + 3 = x + 7$.

35. $3.4x = 0.1x + 6.6$.

36. $6x - 40 + 2x = 3x$.

37. $88 + 3.7x = 2.2x + 91$.

38. $\frac{4x-7}{5} = \frac{3x+1}{4}$.

PROBLEMS WITHOUT NUMBERS

All work oral

1. If you know the number of feet in a roll of carpeting, how do you find the number of yards?

2. If you know the number of pints of milk in a can, how do you find the number of quarts?

3. If you know the number of gallons of oil in a tank, how do you find the number of quarts?

4. If you know the number of pupils in each of two schools, how do you find the number in both schools?

5. If you know the number of oranges in one box and the number in another box, how do you find how many more oranges there are in one box than in the other?

6. When you buy something at a store and give the clerk more than the amount of the sale, how do you find how much change is due you?

7. If you are told the number of trees in each row in an orchard and the number of rows, how do you find the number of trees in the orchard?

8. If you know the number of passengers in a trolley car and the total amount collected in fares, how do you find the fare paid by each passenger?

9. If a man, who is going to buy a set of dining-room furniture, finds that by waiting a week he can buy it at a sale at a certain per cent off the regular price, how do you find the amount he can save by waiting?

10. If you know how much a man paid for rent last year and how much more he pays this year, how do you find the per cent of increase? How do you find the amount of the monthly increase in rent?

11. If you know the amount that a real-estate agent receives for selling a house at a certain price, how do you find the per cent of commission he receives?

12. If you know the face of a fire-insurance policy and the rate per hundred, how do you find the premium?

13. If you know a man's bank balance on the first of the month and the amount of his deposits and checks drawn during the month, how do you find his balance on the last day of the month?

14. If you know the market quotation on some railroad stock, how do you find the amount that you must pay your broker if you buy a certain number of shares?

15. If you know the quarterly rate of dividend on a stock, how do you find the amount received annually by the owner of a certain number of shares?

16. If you know the prices at which you can buy a first-class stock and a first-class bond, and the rate of dividend or of interest paid, how do you find which is the more desirable investment for you to make?

17. How do you find the square of a number?

18. If you know the side of a square, how do you find the area? Illustrate by an example.

19. If you know the area of a square, how do you find the side? Illustrate by an example.

20. How do you find the area of a parallelogram? the area of a circle? the volume of a cube?

21. How do you find the hypotenuse of a right triangle when you know the other two sides?

22. If you know the height of a cylinder and the diameter of the base, what other measurements can you find relating to the cylinder and how do you find them?

PROBLEMS FOR COMPLETION

1. Our school baseball team lost $16\frac{2}{3}\%$ of the games played last season, and they won 10 games. Complete the problem in any reasonable way and then solve it.

Complete and solve the following problems:

2. A boy has a newspaper route on which he delivers 75 papers each morning and night, 6 da. a week. He is paid $\frac{1}{2}$¢ for each paper delivered.

3. Between June 30 and Sept. 1 a boy earns $6 a week by working in a grocery store. He saves $33\frac{1}{3}\%$ of his wages.

4. A steel girder 18 ft. 10 in. long weighs 48 lb. to the running foot.

5. In making a scale drawing of a rectangular garden that is 18 ft. long and 12 ft. wide, a girl wishes to make the drawing 6 in. long.

6. Harry, who is 5 ft. tall, finds that his shadow is 4 ft. long at the same time that the telephone pole in front of his house casts a shadow 30 ft. long.

7. A square has an area of 22 sq. in.

8. Another square has a diagonal of 10 in.

9. The edge of a cube is 16 in.

10. The area of one face of a cube is 289 sq. in.

11. The area of a circle is 44 sq. in.

12. A square courtyard with a perimeter of 144 ft. is to be cemented at a cost of $2.90 per square yard.

13. A ladder 28 ft. long leans against a building. The foot of the ladder is 8 ft. from the building.

14. A cylindric water tank is 28 ft. in diameter and 32 ft. high. There are $7\frac{1}{2}$ gal. in 1 cu. ft.

15. A traveling salesman, who covers New York State, is paid a commission of $6\frac{1}{2}\%$ on his sales, and pays his own expenses. Last year his sales amounted to $78,650 and his expenses to $2350.75.

16. A man, who lives in a city where the property-tax rate is $28.70 per $1000, owns a house that is assessed for $8800 and an automobile that is assessed for $1250.

17. A department store borrows $18,500 on a 60-day note which the bank discounts at $5\frac{1}{2}\%$.

18. A manufacturing company wishes to insure its factory, which is valued at $185,000, for 80% of its value for a period of 5 yr. The 5-year rate is four times the annual rate, and on this type of property the annual rate is $1.10 per $100.

19. A man takes out a 20-payment life-insurance policy for $15,000. The annual premium at his age for this type of policy is $26.40 per $1000.

20. Mr. Huntington invests $12,500 in a mortgage that pays interest at $5\frac{3}{4}\%$ a year.

21. Mr. Driscoll buys a house for $9800 and rents it at $75 a month. In a certain year he pays $156 for taxes, $12.50 for insurance, and $75 for repairs.

22. A man buys 50 shares of stock when it is quoted at $108\frac{3}{4}$. The brokerage is $15 per 100 shares.

23. A man owns 25 shares of stock on which a dividend of $2.75 a share is declared.

24. A man buys 30 shares of Western Union Telegraph stock at $127\frac{1}{2}$ and sells it at 130, paying the usual brokerage of $20 per 100 shares.

25. Mr. Rowley has $3000 to invest and finds that with it he can buy, including brokerage, bonds which have par value of $3200 and pay $4\frac{1}{4}\%$.

Testing Your Alertness. In this work, which will show how alert you are, you are to follow directions. For example, for the figures on page 393 the teacher may read directions something like these:

1. In Ex. 11 write 4 in a space that is in the triangle and one circle but not in both circles.

2. In Ex. 12 write 6 in a space that is in one and only one of the two large triangles and also in the circle.

3. In Ex. 13 write 7 in a space that is in the triangle and rectangle but not in the circle, and write 3 in a space that is in the triangle and the circle but not in the rectangle.

4. In Ex. 14 write the second letter of the word "number" in the first circle, the fourth letter in the third circle, and the fourth letter from the last in the second circle.

5. In Ex. 15 write 1 in the third square, 3 in the first square, and half their sum in the second.

6. In Ex. 16 write 7 in the triangle, 6 in the circle, and twice their product in the square.

7. In Ex. 17 write 8 in each circle that comes after a square and write 7 above each square that comes before a circle.

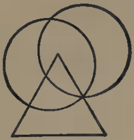
ALERTNESS TEST

Copy the following, and then, allowing 15 sec., insert in the parentheses the number announced by the teacher (it may be 15, 16, or 17) and place a circle (○) before each correct statement and a cross (×) before each incorrect one:

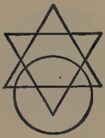
1. $3 + (\quad) = 20.$
2. $(\quad) - 4 = 12.$
3. $2 \times (\quad) = 34.$
4. $(\quad) \div 5 = 3.$
5. $10 \times (\quad) \div 2 = 80.$
6. $5 + (\quad) = 4 \times 5.$
7. $9 \times (\quad) = 153.$
8. $(\quad) + (\quad) = 40.$
9. $10 \times (\quad) = 120.$
10. $(\quad) - 5 = 12.$

Copy the following figures, and, at a signal given by the teacher, do as you are directed, allowing 5 sec. for each example:

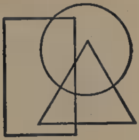
11.



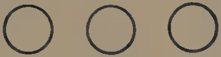
12.



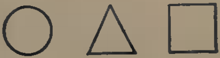
13.



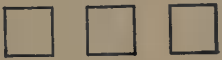
14.



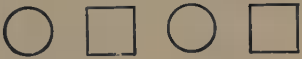
16.



15.



17.



Copy the following numbers as rapidly as possible, and then cancel in each case the numbers (if any) that should be omitted so that the numbers shall run along in a regular order according to some kind of rule or law (for example, Ex. 18 should appear as 9 8 7 6 5 4 ~~2~~*), allowing 5 min. for the work:*

18. 9 8 7 6 5 4 2.

19. 1 7 2 3 4 5 6.

20. 3 4 6 8 10 12 14.

21. 4 7 10 12 13 16 19.

22. 1 2 4 8 10 16 32.

23. 64 32 24 16 8 4 2.

24. 7 11 15 19 23 27 30.

25. 8 4 3 2 1 $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{16}$.

26. 25 20 15 12 10 5 0.

27. 3 9 27 81 92 243 729.

28. 42 40 38 36 34 32 30.

29. 29 30 32 31 32 33 34.

30. 40 34 28 22 16 14 10.

31. 2 6 18 54 162 486 1024.

32. 93 83 73 68 63 53 43.

33. 40 20 10 8 5 $2\frac{1}{2}$ $1\frac{1}{4}$.

34. 81 27 9 3 2 1 $\frac{1}{3}$.

35. 7 7 6 7 7 7 7 7 7 7.

36. 2 8 32 128 512 729 2048.

37. 100 92 84 76 68 62 60.

38. $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{16}$ $\frac{1}{24}$ $\frac{1}{32}$ $\frac{1}{64}$.

39. 0.3 0.9 0.27 0.54 0.81.

40. 1 0.1 0.01 0.001 0.0010.

41. 2.6 3.5 4.4 5.3 5.9 6.2.

SIMPLE PROBLEMS OF EVERYDAY LIFE

1. If 1 pk. of apples makes 3 qt. of sweet cider, 1 bu. of apples will make how many quarts? how many gallons?

2. If 2 bu. of apples make 12 qt. of sweet cider, how many bushels will it take to make 60 gal.?

3. If we put the 60 gal. of cider into quart bottles, how many bottles will it take to hold all the cider? How many pint bottles would it take?

4. A clerk in a grocery store put up 200 lb. of sugar in bags holding 5 lb. each. How many bags did he fill?

5. A bushel of potatoes weighs 60 lb. How many pounds shall we get if we buy 3 pk.? 6 pk.? 75 bu.?

6. On Saturday Helen and Ruth picked 32 qt. of berries in our strawberry patch. Mother canned the berries and found that it took 1 qt. of fresh berries to fill a pint can. How many pint cans did she fill? This was how many quarts?

7. Mother canned 32 qt. of cherries and used 8 lb. of sugar. How many quarts of cherries should she use with 24 lb. of sugar? How many pounds of sugar should she use with 24 qt. of cherries?

8. Mr. Bartley runs a hotel and finds that each guest uses, on an average, 1 oz. of butter at a meal. If he has 10 lb. of butter on hand, how many persons will this amount serve for three meals?

9. On rainy days Mr. Brown and his boys make potato crates in the barn. It takes 2 oz. of nails to make each crate. How many crates can they make with 1 lb. of nails? with 15 lb.? How many pounds of nails will they need for 72 crates?

10. When Ben had only 2 black rabbits, he fed them 2 lb. of cabbage a day. If he now has 16 rabbits, how many pounds of cabbage does he need for the day's feed?

11. Ben had a vegetable garden. On April 14 he planted some early potatoes, which he estimated would be ready for digging in 79 da. If he went on a camping trip in June, on what date did he have to be back to dig the potatoes?

12. Ben raised 42 bu. of potatoes which he sold at an average price of $1.40 a bushel. How much were his receipts?

13. In preparing a spray for his orchard Mr. Brown used 10 lb. of sulphur, 9 lb. of lime, and 1 lb. of arsenate of lead to 50 gal. of water. What part of a pound of each ingredient did he use for each gallon of water? How many pounds of each ingredient should he use for 400 gal. of water?

14. In making out a bill for 56 doz. eggs Ben made the mistake of multiplying the price per dozen by 65 instead of by 56. The result that Ben obtained was $24.70. What result should he have obtained?

15. Ben's uncle works in the chair factory in town. He is paid 70¢ an hour for an 8-hour day, with "time and a half" for overtime. How much does he receive if he works 8 hr. a day for 5 da.? if he works 9 hr. a day for 3 da.? if he works 8 hr. a day for 2 da. and 10 hr. a day for 3 da.?

16. The factory where Ben's uncle works has 12 men on its pay roll. As weekly wages, two of them receive $32.50 each, five receive $34.20 each, three receive $27 each, one receives $33.88, and one receives $34.10. How much money does the factory need for the week's pay roll? How many of each kind of bill or coin should it provide so as to give each man the right change in the most convenient form?

17. During one week that Ben's uncle worked he made $3.15 for overtime. The next week he made $4.20 for overtime, and during the next 3 wk. he made $3.68 a week in the same way. What was the average amount of overtime pay that he received per week during the 5 wk.?

EVERYDAY PROBLEMS IN PERCENTAGE

1. Mr. Brown's cherry crop has averaged 1950 lb. per year for several years. Last year it amounted to 2340 lb. What per cent was this above the average crop?

2. The average amount received annually from the cherry crop in Ex. 1 was $117. Last year Mr. Brown received 4¢ a pound for his crop of 2340 lb. What per cent was this above or below the average yearly receipts?

3. For the annual meeting of the New York State Teachers' Association the railroads sell round-trip tickets for $1\frac{1}{2}$ times the regular one-way fare. What per cent of reduction is this on the regular round-trip cost?

4. If a gasoline-saving device installed on an automobile makes it possible for the car to go 15 mi. instead of 12 mi. on 1 gal. of gasoline, what is the per cent of increase in mileage?

5. With the increased mileage per gallon described in Ex. 4, how much gasoline will be saved on a trip that required 56 gal. of gasoline before the device was installed?

6. If an automobile travels 30 mi. in 50 min., what per cent of a mile will it go in 1 min.?

7. At 30 mi. per hour what per cent of a mile will be traveled in $\frac{1}{4}$ min.?

8. If a produce dealer's expenses are 30% of sales and he wishes to make a profit of 10% on the selling price, how much must he charge for potatoes that cost him 60¢ a bushel in order to cover his expenses and make this profit?

9. If the enrollment in a school was 920 last year and has increased 35% this year, what is the present enrollment?

10. Ben weighed 110 lb. in June and 120 lb. in September. What was the per cent of increase?

11. If our rent is $840 a year and the landlord raises it 15%, what will be the new monthly rental?

12. If the rent in Ex. 11 should be decreased 15% after being raised 15%, how much would it be then per year?

13. If we pay $13.50 a ton for coal and the price is increased 10%, how much shall we have to pay then for 14 T. of coal?

14. If the present rate for natural gas in Dunkirk is 60¢ per 1000 cu. ft., and the rate is increased 25%, what will a family's gas bill be if they burn 15,000 cu. ft. of gas under the new rate?

15. An agent's sales in one day amounted to $144. If he received 15% on his sales, how much did he make?

16. If Mr. Brown borrows $100 at 6% in order to buy seed potatoes and repays the loan at the end of 6 mo., how much interest will he have to pay?

17. Mr. Brown wishes to make a profit of 25% on the total cost of his potato crop. His expense for labor is $75, the interest on his loan at the bank is $3, and his other expenses are $162. How much must he receive for his crop in order to pay all expenses and make the desired profit? If he has 500 bu. to sell, how much must he charge per bushel?

18. If James bought 1 doz. melons for $3 and the express was 60¢, what will he have to charge apiece for the melons in order to make a profit of 20% on the selling price?

19. On an invoice for a shipment of farm tools amounting to $720, Mr. Brown takes advantage of a discount of 2% for paying cash. What is the amount that he actually pays?

20. One gas company requires each customer to deposit $10 when the meter is installed in the house. If the company pays interest at 6% on these deposits, how much will a customer, who moves out at the end of 18 mo., receive from the company, including the returned deposit?

EVERYDAY HOUSEHOLD PROBLEMS

1. A recipe for cream candy calls for $\frac{3}{4}$ lb. of sugar, $\frac{1}{4}$ cup of vinegar, and $\frac{1}{2}$ cup of water. If Mollie wishes to make five times what the recipe allows, how much of each of these ingredients should she take?

2. A recipe for 2 lb. of a certain kind of candy calls for 4 cups of brown sugar, $\frac{1}{2}$ cup of milk, 3 teaspoonfuls of butter, and $\frac{3}{4}$ of a cup of English walnuts. What changes should you make in the recipe if you wished to make 1 lb.? to make 3 lb.? to make 5 lb.?

3. Rachel is making a cover for the library table. The cover is $51\frac{1}{2}$ in. long and $16\frac{3}{4}$ in. wide, and she wishes to put a fringe around it. Estimating the amount needed to the next greater $\frac{1}{4}$ yd., find how much Rachel will have to pay for the fringe at 35¢ a yard.

4. To make a fancy bag Mrs. Jewett needs a piece of homespun 20 in. long and 8 in. wide. She has a strip of homespun 2 yd. long and 32 in. wide. Draw a plan that shows how she should cut the cloth to make as many bags as possible.

5. Marie's mother has 7 yd. of cloth of the right width for towels. Each towel is to be 30 in. long when finished, and to have a hem $\frac{3}{4}$ in. wide at the top and bottom. How many towels can she make from the cloth?

6. If a living room is 24 ft. long and 15 ft. wide, how many strips of wall paper 18 in. wide will be needed to paper the four walls, no allowance being made for windows, doors, fireplaces, or other openings?

7. Make inquiries as to the probable cost of furnishing a room for yourself. Assume that you have $50 to spend and write a statement telling how much each item will cost, but do not let the total cost exceed $50.

8. A housekeeper buys a pound each of mustard seed, allspice, and cayenne pepper, paying 90¢ for the three. The mustard seed costs 18¢ a pound, and the allspice costs the same as the pepper. How much does the allspice cost?

9. A housekeeper bought a can each of cinnamon, nutmeg, and pepper, paying 39¢ for the three. The pepper cost 9¢, and the nutmeg cost twice as much as the cinnamon. How much did the cinnamon cost?

10. If a housekeeper can buy canned corn at 18¢ a can or at $1.90 per dozen cans, how much is saved on 24 cans in buying by the dozen?

11. If a housekeeper can buy canned soup at 24¢ a can or at $2.50 per dozen cans, how much is saved on 36 cans in buying by the dozen?

12. A dealer sells coffee in half-pound packages at 24¢ a package or in 5-pound cans at $2.15 a can. If a housekeeper wishes 5 lb., how much does she save in purchasing a can?

13. A housekeeper can buy Dutch cocoa in $\frac{1}{4}$-pound boxes at 28¢ a box or in 5-pound cans at $4 a can. How much does she save in purchasing by the can? What per cent of the $4 does she save by buying in this way?

14. A housekeeper can buy maple sirup at 85¢ a quart or in gallon cans at $2.85 a can. How much does she save in purchasing by the gallon? What per cent does she save?

15. When flour costs $12.25 a barrel (196 lb.) or 7¢ a pound, what per cent is saved in purchasing by the barrel?

16. A housekeeper's ice bill averages $3.60 a month. A better refrigerator, which will use only 75% as much ice, can be bought for $60. Not counting the saving due to the food being kept in better condition, in how many months will the saving in ice pay for the new refrigerator?

17. If each of 200,000 homes in a certain state wastes $\frac{1}{8}$ cup of milk a day, how many quarts (4 cups = 1 qt.) are wasted per day? How many quarts are wasted per year (365 da.)?

18. In Ex. 17, what is the cost of this waste in a year when the average price of milk is 11.6¢ a quart?

19. What would be the cost in Ex. 18 if, by taking more care, the waste should be reduced 35%?

20. A careless housekeeper wasted a cup of cereal each week. If the cereal costs 12¢ a pound and $2\frac{2}{3}$ cups weigh a pound, how much money is wasted in this way each year?

21. If each of 250,000 housekeepers in a certain state should waste as much as the one in Ex. 20, how much money would be wasted in this way in a year?

22. In making apple sauce and pies a woman pares the apples so carelessly that she loses 15% of the usable part of the fruit. If she buys apples costing 4¢ each and uses on an average $\frac{1}{2}$ doz. apples every fifth day in the year, how much money does she waste in this way every year?

23. A careless housekeeper threw away the water used in boiling meats and vegetables, and thereby lost material which could be used in making soup. If, as a result, she has to buy during the year 28 cans of soup at 15¢ a can, 2 doz. cans of soup stock at $1.25 a dozen, and soup meat costing $6.80, how much money is wasted in this way during the year?

24. In making a cake, a cook uses 3 eggs at 68¢ a dozen, $\frac{1}{2}$ cup of butter at 48¢ a pound, $\frac{1}{2}$ cup of milk at 14¢ a quart, 1 cup of sugar at 8¢ a pound, 2 cups of flour at 7¢ a pound, and 6¢ worth of other materials. Allowing 2 cups of butter or of sugar to the pound, 4 cups of flour to the pound, and 4 cups of milk to the quart, if she burns the cake so that it has to be thrown away, how much money is wasted?

DAIRY PROBLEMS

1. How many pounds of butter fat are credited to a farmer who sells to a creamery 28,000 lb. of milk which tests 4.2% butter fat?

2. If 6 lb. of butter fat are needed in making 7 lb. of butter, what is the value, at 44¢ a pound, of the butter produced from 2136 lb. of butter fat?

3. A farmer shipped to a creamery a quantity of milk which averaged 3.75% butter fat. If the butter fat weighs 890 lb., how many pounds of milk were there in the shipment?

4. How much is the butter fat referred to in Ex. 3 worth at 44¢ a pound?

5. Last month one of two cows belonging to a farmer gave 986 lb. of milk testing 3.1% butter fat, and the other 812 lb. testing 4.2%. Which cow gave the larger receipts for butter fat, and how much more did the farmer receive if the creamery paid 36¢ a pound for butter fat that month?

6. A creamery used 8500 lb. of milk in a certain week. The skim milk, containing 0.6% butter fat, amounted to 80% of the whole milk. During that week, how much butter fat was lost in the skim milk?

PROBLEMS ON SPRAYING

1. Bordeaux mixture, for spraying, contains $1\frac{1}{4}$% by weight of copper sulphate, $1\frac{1}{4}$% of unslaked lime, and the rest of the mixture is water. How many pounds of each substance are there in a barrel containing 250 lb. of Bordeaux mixture?

2. A cubic foot of Bordeaux mixture weighs 63 lb. From the facts given in Ex. 1 find the number of pounds of copper sulphate and unslaked lime in a tank 8 ft. long and 5 ft. wide, filled with Bordeaux mixture to a depth of 18 in.

3. Half of a peach tree was sprayed with Bordeaux mixture and the other half was not. From the sprayed half 340 lb. of fruit were taken, and from the other half 95% less. How many pounds of peaches were taken from the tree?

4. In preparing a spray for his trees, a farmer uses 3 lb. of arsenate of lead and 4 lb. each of lime and copper sulphate to 50 gal. of water. If the arsenate of lead costs 42¢ a pound, the lime 1¢ a pound, and the copper sulphate 16¢ a pound, and the man uses $2\frac{1}{2}$ gal. of the spray on each tree, what will be the cost of the material to spray 450 trees?

5. For spraying potato plants a man prepares a solution of Paris green, using $\frac{1}{2}$ lb. of Paris green costing 60¢ a pound to 100 gal. of water. He uses 300 gal. to an acre and sprays a field containing $4\frac{1}{2}$ A. If the labor costs $18.50, what is the total cost? If the crop brings $74 more than if he had not sprayed the field, how much is his gain?

6. In order to lessen the damage from potato scab a farmer soaks his seed potatoes in a solution made by adding $\frac{1}{2}$ pt. of formalin to 15 gal. of water. What is the per cent of formalin in the mixture?

7. In Ex. 6, what would be the per cent of formalin in the mixture if he had used $\frac{1}{8}$ pt. more to 15 gal. of water?

CAREFUL MANAGEMENT ON THE FARM

1. Frank's father owns two farms of about the same size and value. He runs one farm himself and lets the other on shares. In showing Frank the value of careful management on the farm, he made up a table showing the receipts last year from the home farm and from the tenant's farm. Here is the table from which he asked Frank to find the total receipts on each farm. How much did Frank find that they were?

Items	Home Farm	Tenant's Farm
Crops	$1075.30	$607.22
Dairy products	348.60	193.75
Domestic animals	695.75	388.17
Eggs and poultry	332.60	180.04
Wool	21.75	10.34

2. Frank then found for each item the per cent by which the home-farm receipts exceeded those of the tenant's farm. What results did Frank find?

3. The expenses of the two farms also showed poor management by the tenant. Here are the different items of expenses:

Items	Home Farm	Tenant's Farm
Feed	$52.60	$47.13
Fertilizers	86.75	10.00
Labor	575.45	502.43
Repairs	41.35	89.50
Taxes and miscellaneous	85.70	83.03

Find the difference between each pair of items, and between the total expenses of each farm.

4. From Exs. 1 and 3 find by how much the receipts exceeded the expenses on each farm, and then find the difference between these results.

PROBLEMS ON TRANSPORTATION

1. If a freight car is 38 ft. long, 8 ft. 4 in. wide, and 7 ft. 8 in. high, inside measurements, how many cubic feet does it contain?

2. If the width of the car in Ex. 1 were increased 1 in., the height $1\frac{1}{2}$ in., and the length 1 ft. 6 in., what would be the increase in capacity?

3. If a wooden coal car can carry 80,000 lb., and a steel car 140,000 lb., the steel car can carry what per cent more than the wooden car? The wooden car can carry what per cent less than the steel car?

4. A car 36 ft. long and 8 ft. 4 in. wide is loaded with wheat to a depth of 4 ft. 6 in. Taking a bushel as equivalent to $1\frac{1}{4}$ cu. ft. and the weight of a bushel of wheat as 60 lb., what is the weight of the wheat in the car?

5. If the car mentioned in Ex. 4 has a carrying capacity of 80,000 lb., to what depth must it be filled with wheat so as to load the car to capacity?

6. If a locomotive that weighs $115\frac{1}{2}$ T. can exert a pull of $22\frac{1}{4}\%$ of its own weight in starting a long train, how many pounds of pull does it exert?

7. A locomotive and coal tender together weigh 95 T., and there are seven cars to the train, averaging 21 T. each. If a horizontal pull equal to 0.5% of the total weight of the train is required to maintain a certain speed on a level track, how large a pull in pounds is required?

8. A 20,000-ton modern ship carries 12 times as much freight as the old-style ocean steamers, makes 25% better time, and the expenses per year are only 4 times as much. In a year a modern ship of this type will do how many times the work of the old kind? At the same cost, it will carry how many times as much freight?

PROBLEMS IN MANUFACTURING

1. Last year the sales of a manufacturing company with a capital of $350,000 were $3\frac{1}{4}$ times its capital, and the profit was $7\frac{1}{2}$% of the sales. How much was the profit?

2. A dealer sells a lamp for $21, which is 40% more than the cost to him. If the manufacturer who sold it to the dealer made a profit of 25% on the manufacturing cost, what was the cost of manufacture?

3. If the manufacturer allows a discount of 40%, how much does a dealer pay for 8 rockers listed at $14.50, 9 dressers listed at $29, and 16 bedsteads listed at $37.50?

4. A manufacturer sells 8 doz. plates @ $2.84; 9 doz. cups @ $4.70; $\frac{3}{4}$ doz. covered dishes @ $19.90; 16 doz. plates @ $3.30; $\frac{5}{6}$ doz. pitchers @ $13.50. If the manufacturer's selling price represents a profit of 20% on the manufacturing cost, find the cost of manufacturing the lot.

5. On which will a factory make the greater per cent of profit on the cost, an article which costs $2.90 to make and sells for $3.50 less 2% or one which costs $8.70 to make and sells for $16.50 less 40%? How much greater is the per cent?

6. A manufacturer makes a bid on a contract to turn out certain articles. He wishes to make a profit of 25% on the cost, and finds that the cost of labor and materials is $3782.40. What bid, to the nearest $1, should he make?

7. A company manufactures circular water tanks 20 ft. in diameter and 28 ft. high. Taking $7\frac{1}{2}$ gal. to the cubic foot, find the capacity in gallons that the company would use in describing the size of the tanks.

8. A factory pays its salesmen a commission of 12% on their sales. If a salesman sells $66,725 worth of goods in a certain year, how much is his income that year?

PROBLEMS OF THE MERCHANT

1. How much is paid for an article marked $82.50, when bought at a sale where a discount of 20 % is given?

2. A dealer received $31.50 for a suit of clothes which he sold at 10 % off the marked price. What was the marked price?

3. A merchant, whose expenses of doing business average 20 % of sales, sold for $308 goods which cost him $215.60. What is the per cent of profit on the cost? on the selling price?

4. A furniture dealer buys tables at $16.50 each and sells them at $25 each. His expenses of doing business are 24 % of the cost. What per cent does he make on the cost?

5. If the dealer in Ex. 4 had calculated his cost of doing business as 24 % of the selling price, what would have been his per cent of profit on the selling price?

6. A hardware dealer buys stoves at $28.50 each. His cost of doing business is 28 % of sales, and he wishes to make a profit of 15 % on the selling price. Find the selling price which the dealer should place on the stoves.

7. A merchant buys a line of carpeting at $1.84 a yard. His cost of doing business is 14 % of sales, and his profit is to be 10 % on the selling price. Find, to the next larger multiple of 10¢, the price per yard at which he should sell the carpeting.

8. A dealer buys some kitchen chairs at $2.24 each, with a discount of 25 %, 10 %. If his cost of doing business is 16 % of sales, and the profit is to be 12 % on the selling price, find, to the next larger multiple of 25¢, the price at which he should sell the chairs.

9. A dealer pays $972.40 for a lot of goods. His cost of doing business is 20 % of sales, and the profit is 12 % on the selling price. Find the profit on the sale of the entire lot.

PROBLEMS OF THE BUILDER

1. The cellar of a building is to be 9 ft. deep, 92 ft. long, and 60 ft. wide, outside measurements. The excavation is to be extended 18 in. beyond each of the four walls, so as to allow space for working. How many cubic feet of earth must be removed? How many cubic yards?

2. In figuring the material needed for a brick wall 68 ft. long, 6 ft. high, and 2 bricks thick, a builder allows 14 bricks per square foot of outside surface for a 2-brick wall. How many bricks does he need? At $17 per M, how much will the bricks cost if in buying them he gives his order to the next larger 500 to allow for breakage?

3. The walls of a brick building, which is to be 78 ft. long, 35 ft. wide, and 28 ft. high, are to be 18 in. thick. Deducting 750 cu. ft. for openings, and allowing 22 bricks to a cubic foot, find how much the bricks will cost at $18.50 per M if the builder orders to the next higher M to allow for waste.

4. If a builder makes a profit of $1080 on a house that he sells for $9400, what per cent does he make on the cost?

5. If a builder makes a profit of 20% on the cost of a house that he sells for $7200, how much profit does he make?

6. A builder plans to put up a house costing \$5800 on a lot costing \$1250. If he estimates that he will spend \$250 worth of his time in planning the house and supervising its erection, for what price should he sell it in order to make a profit of 10% on the total cost?

7. A semicircular window over a door has a diameter of 4 ft. 1 in. Find the area and the perimeter of the window.

8. A room is heated by 168 ft. of steam pipe with an outside diameter of 2 in. Find the area of the radiating surface.

9. The distance from the eaves to the ridge of a roof is 13 ft., and the span, the horizontal distance between the eaves, is 24 ft. Find the rise of the roof; that is, the height of the peak above the level of the eaves.

10. The span of an A-shaped roof is 26 ft., and the rise of the roof is 6 ft. What is the distance, to the nearest 0.1 ft., from the eaves to the ridge?

11. A paper hanger measures the rooms of a small house and finds them to be 14 ft. × 16 ft., 8 ft. × 12 ft., 15 ft. × 18 ft., 10 ft. × 12 ft., 16 ft. × 16 ft., and 12 ft. × 12 ft. For openings he deducts 6 ft. from the perimeter of each room and allows one roll of paper for every $3\frac{1}{2}$ ft. of perimeter. How many rolls does he need in all? If the paper for the first two rooms costs 35¢ a roll, for the third and fourth 40¢, and for the last two 55¢, what is the total cost of the paper for the house?

12. In building a concrete wall 66 ft. long, 8 ft. high, and 18 in. thick, the concrete which the builder plans to use is to be $\frac{1}{6}$ cement, which weighs 150 lb. per cubic foot and costs \$3.80 a barrel of 400 lb., and the rest is to be gravel costing \$1.75 a cubic yard delivered. In ordering the material the builder cannot order a fraction of a barrel or of a cubic yard. What will be the cost of the material for the wall?

13. A cubic yard of "1 : 2 : 3" concrete contains approximately $1\frac{1}{2}$ bbl. of cement, $\frac{1}{2}$ cu. yd. of sand, and $\frac{3}{4}$ cu. yd. of crushed rock, the expression "1 : 2 : 3" indicating the proportions of cement, sand, and crushed rock. The sum of the volumes of the different materials is more than 1 cu. yd., but since the smaller particles of the sand and cement fill in around the pieces of rock, thus adding very little to the volume of the mixture, they make only 1 cu. yd. of concrete. How much of each of these materials is needed for the foundation of a building requiring 650 cu. yd. of concrete?

14. A cubic yard of $1 : 2\frac{1}{2} : 3$ concrete contains approximately $1\frac{1}{2}$ bbl. of cement, 0.6 cu. yd. of sand, and 0.7 cu. yd. of crushed rock. How much of each of these materials is required for 240 sq. yd. of concrete pavement 9 in. thick?

15. How many cubic yards of 1 : 2 : 3 concrete will 1 bbl. of cement make? This is equal to how many cubic feet? How many square feet of 1 : 2 : 3 concrete sidewalk can be made for each barrel of cement used if the concrete is 6 in. thick?

16. How many cubic yards of $1 : 2\frac{1}{2} : 3$ concrete will 25 bbl. of cement make? How much sand and gravel (see Ex. 14) is needed to mix with this amount of cement?

17. A farmer plans to build three concrete watering troughs of 1 : 2 : 3 concrete. The inside of each trough is to be 10 ft. long, 3 ft. wide, and 20 in. deep, the walls are to be 6 in. thick, and the bottom is to be 10 in. thick. First computing to the next larger whole cubic yard the amount of concrete required for the three troughs, find how much of each of the materials named in Ex. 13 is needed.

18. In Ex. 17 find the total cost of the material needed if the cement costs $3 per barrel, the sand $1.50 per cubic yard, and the crushed rock $1.75 per cubic yard.

PROBLEMS ABOUT INVESTMENTS

1. A man has $25,000 invested at various rates of income as here shown. Copy the table and fill in the blank column:

AMOUNT	RATE	ANNUAL INCOME
$3000	6 %	
2000	$4\frac{1}{2}$ %	
2000	5 %	
5000	$5\frac{1}{2}$ %	
6000	4 %	
3000	8 %	
2500	$4\frac{1}{4}$ %	
1500	$7\frac{1}{2}$ %	
$25000		

Given the amount invested and the rate of income paid on the investment as follows, find the amount of income received for the stated period:

2. $100, 6 %, 1 yr.
3. $200, 5 %, 2 yr.
4. $150, 4 %, 1 yr.
5. $500, 6 %, 1 yr.
6. $800, 5 %, 6 mo.
7. $960, 4 %, 3 mo.
8. $9000, 6 %, 2 yr.
9. $7500, 5 %, 6 mo.
10. $1500, $5\frac{1}{2}$ %, 6 mo.
11. $2500, 6 %, 2 yr.
12. $7500, 5 %, $2\frac{1}{2}$ yr.
13. $8000, $4\frac{1}{2}$ %, $1\frac{1}{2}$ yr.

14. Robert saved his money and bought two 100-dollar bonds, which pay interest at $4\frac{1}{4}$ % annually, and one 50-dollar bond, which pays interest at $4\frac{1}{2}$ % annually. How much interest does he receive every 6 mo.?

15. Which gives the better rate of income, a bond paying 5 % that can be bought at 113, including brokerage, or a mortgage that pays $4\frac{1}{2}$ %?

16. A man has a choice of three investments which his banker tells him are equally sound. The first is a stock that pays $1\frac{3}{4}\%$ quarterly and can be bought at $129\frac{3}{4}$, the second is a stock that pays 5% and can be bought at $92\frac{1}{2}$, and the third is a bond paying $5\frac{1}{2}\%$ that can be bought at 100.40, including brokerage. Remembering to add the brokerage in the case of the stock, find the rate of income on each investment. In which one should you advise the man to invest?

17. If the rate of dividend on a first-class stock is higher than the average rate of income on good investments, will the stock be selling above or below par? Give the reasons on which you base your answer.

18. If you receive a circular advertising at below par a stock that promises to pay a very high dividend rate, and another circular advertising a new bond issue which will yield the current market rate of interest, in which investment should you prefer to put your money? Give the reasons on which you base your answer.

19. A young man received a legacy of $1000 and had the opportunity of making the following investments: (1) Southern Pacific stock at 105, paying 6%; (2) Pacific Gas stock at 90, paying 6%; (3) a Florida orange grove of 10 A., with an average income per acre of $25; (4) a first mortgage, paying 8%, for two thirds of the value of a farm; (5) Gold B Oil at $10 per share, 50% to be used for drilling, stock to pay 500% when oil is struck. If you were the young man, in which should you invest? Give reasons for your answer.

20. A man bought a house for $6000 as an investment. He finds that the average yearly expenses of maintaining the property are as follows: insurance, $5; taxes, $125; water rent, $15; upkeep, $150. What monthly rent, to the next higher $1, must he charge in order to receive 10% on his investment?

21. A man who bought a house as an investment leases it at a rental of $50 a month. He pays $122.40 a year for taxes, $5.60 for insurance, and an average of $65 for repairs. How much does the owner have left from the rental each year?

22. The owner of the house in Ex. 21 sells it for $6400 and pays $150 to an agent for making the sale. If he then invests the proceeds in a mortgage which pays 6%, is his income increased or is it decreased, and how much?

23. The new owner of the house spends $500 in improvements. What is now his total investment in the house? What is the annual interest on this investment at 6%?

24. Owing to the improvements made in Ex. 23 the taxes are increased to $142.30 a year, and the insurance to $6.40. Since he has just fixed over the house, he has no repair bill the first year. What is the monthly cost of owning the house the first year, including the interest on his investment which he loses by not being able to invest his money in other ways? How does this compare with the rent that the first owner charged for the house as mentioned in Ex. 21?

25. The second year that he owns the house he pays $75 for repairs as well as the other expenses given in Ex. 24. At the end of the year his business takes him to a distant city, and so he sells the house for $7500. In figuring out whether he has lost or gained, he adds the expenses, including the interest on his investment, and compares them with the amount received from the sale. What is the result?

26. In studying the result in Ex. 25 the man realizes that he has made no allowance for the fact that he has lived in the house for 2 yr. and has paid no rent. Owing to the improvements made he could have rented the house for $60 a month. Taking this figure as a fair rental, at what amount should the man in Ex. 25 calculate his gain or loss?

Recreations. Most of the following problems have a long and interesting history. They are part of the inheritance of the school, and show some of the amusing features of arithmetic. They have no practical value, and should be omitted unless taken in the spirit intended; that is, as recreations which show the lighter side of arithmetic.

RECREATIONS AND CURIOSITIES

1. Multiply 142,857 by 7 and notice the result.

2. Multiply 37 by 3; by 6; by 9; by 12; by 15; by 18; by 21. Notice the results.

3. How can you write the number 100 with Arabic figures without using zeros?

4. Write an even number, using only the odd digits.

5. If a goose weighs 10 lb. and half of its own weight, what is the weight of the goose?

6. If a goose weighs 14 lb. when standing on both feet, how much does it weigh when standing on one foot?

7. How many days would it take to cut a piece of cloth 50 yd. long into 1-yard lengths, one length being cut off daily?

8. If a post in a pond stands $\frac{1}{3}$ in the ground, $\frac{1}{4}$ in the water, and 10 ft. above the water, how long is it?

9. A bottle and its stopper cost $1.10, and the bottle cost $1 more than the stopper. How much did the stopper cost?

10. A new steel coal car and its contents weigh 139,700 lb., and the contents weigh 60,300 lb. more than the car. How much does the car weigh?

11. Two men plan to cross the Rocky Mountains. X can walk 13 mi. a day and Y only 10 mi. a day, and so Y starts 9 da. ahead. How long will it take X to overtake Y?

X. LITTLE EXAMINATIONS

I. 1. Find the interest on \$750 for 63 da. at 5%.

2. At $3\frac{1}{2}$%, find the commission on \$14,500.

3. If the tax on \$8600 is \$215, what is the rate?

4. Find the yearly income from \$2250 invested at $5\frac{3}{4}$%.

5. Find the rate of income when \$1500 yields \$67.50 a year.

II. 1. Find the area of a circle with a diameter of 6.3 in.

2. Find the radius when the circumference is 48.4 in.

3. Find the number of gallons in a tank holding 160 cu. ft.

4. Show how to bisect a given angle.

5. Show how to construct one triangle equal to another.

III. *Solve the following equations and check each result:*

1. $x + 10 = 18$.
2. $19x - 7 = 31$.
3. $5x + 3 = 3x + 9$.
4. $7x - 5 = 3x + 19$.
5. $4x + 3 = 2x + 7$.
6. $8x - 37 = 2x - 7$.
7. $8x - 8 = 5x - 2$.
8. $2(3 + x) = 9 + x$.

IV. *Using the formula* $A = \frac{1}{2}bh$, *find the value of* A *when*

1. $b = 6$ and $h = 8$.
2. $b = 7$ and $h = 9$.
3. $b = 12.2$ and $h = 18.6$.
4. $b = 22.4$ and $h = 18.7$.
5. $b = 16\frac{1}{2}$ and $h = 12\frac{3}{4}$.
6. $b = 165\frac{3}{4}$ and $h = 126\frac{1}{2}$.
7. $b = 26.57$ and $h = 18.24$.
8. $b = 117.58$ and $h = 109.67$.

V. *From the following formulas, find formulas as stated:*

1. $A = bh$; for h.
2. $V = lwh$; for l.
3. $C = 2\pi r$; for r.
4. $A = \frac{1}{2}bh$; for b.
5. $V = \pi r^2 h$; for h.
6. $S = 4\pi r^2$; for r.
7. $i = prt$; for p.
8. $V = \frac{1}{3}\pi r^2 h$; for r.

MATERIAL FOR FREQUENT DRILL

How to Use the Exercises. This material is arranged to give practice in the fundamental operations. By going through each exercise, first using the number designated by (a), then the number designated by (b), and so on, nearly seven hundred different examples will be found.

EXERCISE 1

Taking (a) *7.1,* (b) *8.3,* (c) *15.4,* (d) *21.7,* (e) *24.9,* (f) *26.8,* (g) *35.7, or* (h) *28.2, as directed,*

1. Add it to $48.3 + 27.9 + 3.6 + 0.9 + 27.0$.
2. Subtract it from $23.4 + 76.5 + 62.8$.
3. Multiply it by itself.
4. Divide it by 0.65, finding the result to the nearest 0.01.
5. Find $33\frac{1}{3}\%$ of it, using the shortest method.

EXERCISE 2

Taking (a) *26.4,* (b) *31.4,* (c) *28.9,* (d) *34.8,* (e) *37.8,* (f) *39.1,* (g) *36.8, or* (h) *37.4, as directed,*

1. Add it to $27.9 + 36.7 + 4.2 + 29.9 + 0.8$.
2. Subtract it from $48.3 + 51.6 + 4.8$.
3. Multiply it by twice itself.
4. Divide it by 0.071.
5. Find 25% of it; 125% of it; $133\frac{1}{3}\%$ of it.

EXERCISE 3

Taking (a) *34.1,* (b) *35.7,* (c) *32.8,* (d) *38.7,* (e) *36.7,* (f) *39.8,* (g) *38.4, or* (h) *37.6, as directed,*

1. Add it to itself plus 9 times itself.

2. Subtract it from $429.3 + 570.6 + 0.1 + 4.7$.

3. Multiply it by 3.5.

4. Divide it by half itself; by twice itself; by 0.71.

5. Add it to the result of $2\frac{1}{4} \times 96$.

EXERCISE 4

Taking (a) *41.2,* (b) *42.3,* (c) *43.4,* (d) *44.5,* (e) *45.6,* (f) *46.7,* (g) *45.4, or* (h) *47.3, as directed,*

1. Add it to 99 times itself.

2. Subtract it from 1001 times itself.

3. Multiply it by the result of $625 \div 5$.

4. Divide it by the result of dividing it by 47.6.

5. Find 200% of it; 50% of it; 0.1% of it.

EXERCISE 5

Taking (a) *51.8,* (b) *52.7,* (c) *53.6,* (d) *54.5,* (e) *55.4,* (f) *56.3,* (g) *53.2, or* (h) *54.9, as directed,*

1. Add it to 94.3 less itself.

2. Subtract it from 26 times itself.

3. Multiply it by the result of $\frac{2}{3} \times \frac{1}{8}$.

4. Divide it by the result of $\frac{3}{4} \div \frac{7}{8}$.

5. Divide it by 125%; find 125% of it.

EXERCISE [illegible]

Taking (a) *62.7,* (b) *63.6,* (c) *64.5,* (d) *65.4,* (e) *66.3,* (f) *67.2,* (g) *64.7, or* (h) *65.9, as directed,*

1. Add it to $4.6 + 39.8 + 0.9 + 53.7$.
2. Subtract it from $8.2 + 19.6 + 82.6 + 0.8$.
3. Multiply it by $2\frac{1}{2} + 3\frac{2}{5}$.
4. Divide it by $3\frac{7}{8} + 4\frac{3}{8}$.
5. Find $12\frac{1}{2}\%$ of it; $1\frac{1}{4}\%$ of it; 125% of it.

EXERCISE 7

Taking (a) *71.9,* (b) *72.8,* (c) *73.7,* (d) *74.6,* (e) *75.5,* (f) *76.4,* (g) *73.9, or* (h) *74.8, as directed,*

1. Add it to $48.7 + 6.9 + 39.4 + 95.0 + 0.2$.
2. Subtract it from $87.4 + 0.9 + 3.4 + 9.8 + 2.7$.
3. Multiply it by 0.625; by 62.5.
4. Divide it by $62\frac{1}{2}\%$; by 6.25; by $6\frac{1}{4}\%$.
5. Find 20% of it; 40% of it; 60% of it.

EXERCISE 8

Taking (a) *82.7,* (b) *83.6,* (c) *84.5,* (d) *85.4,* (e) *86.3,* (f) *87.2,* (g) *86.9, or* (h) *87.8, as directed,*

1. Add it to 999 times itself.
2. Subtract it from $6\frac{1}{2}$ times itself.
3. Multiply it by $2\frac{1}{2}\%$; by 25%; by 250%.
4. Divide it by $2\frac{1}{2}\%$; by 25%; by 250%.
5. Add it to the result of $42\frac{3}{4} \div \frac{3}{8}$.

EXERCISE 9

Taking (a) *2 ft. 8 in.,* (b) *3 ft. 6 in.,* (c) *5 ft. 4 in.,* (d) *6 ft. 2 in.,* (e) *7 ft. 10 in.,* (f) *8 ft. 4 in.,* (g) *8 ft. 7 in., or* (h) *9 ft. 6 in., as directed,*

1. Add it to 19 times itself.
2. Subtract it from 10 ft. 1 in.
3. Multiply it by 2; by $2\frac{1}{8}$; by $21\frac{1}{4}$.
4. Divide it by 2; by 4; by 6.
5. Divide it by 3 in; by 5 in.; by 2 ft.

EXERCISE 10

Taking (a) *92.7,* (b) *97.2,* (c) *93.4,* (d) *94.3,* (e) *98.6,* (f) *96.8,* (g) *97.7, or* (h) *98.4, as directed,*

1. Add it to $137\frac{7}{8}$ less itself.
2. Subtract it from 126 times itself.
3. Multiply it by the result of $196.4 \div 7.856$.
4. Divide it by 7.264; by $\frac{1}{2}$ of 7.264.
5. Multiply 34 ft. 2 in. by it.

EXERCISE 11

Taking (a) *1 yd. 15 in.,* (b) *58 in.,* (c) *2 yd. 8 in.,* (d) *$2\frac{1}{2}$ yd.,* (e) *$3\frac{1}{2}$ yd.,* (f) *43 in.,* (g) *62 in., or* (h) *$1\frac{1}{2}$ ft., as directed,*

1. Add it to 5 ft. 8 in. + 3 ft. 8 in.
2. Subtract it from 6 yd. 1 ft. + 2 yd. 11 in.
3. Multiply it by $2\frac{1}{2}$; by 25; by 25%.
4. Divide it by 3; by 5; by 7; by 12.
5. Divide it by 1 in.; by 0.1 in.; by $\frac{1}{4}$ in.

TABLES FOR REFERENCE

Length

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet = 1 rod (rd.)
320 rods, or 5280 feet = 1 mile (mi.)

Square Measure

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards = 1 square rod (sq. rd.)
160 square rods = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)

Cubic Measure

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)

Weight

16 ounces (oz.) = 1 pound (lb.)
2000 pounds = 1 ton (T.)

Liquid Measure

2 pints (pt.) = 1 quart (qt.)
4 quarts = 1 gallon (gal.)

Dry Measure

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

TIME

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
12 months (mo.) = 1 year (yr.)

A common year has 365 da.; a leap year, 366 da.; for figuring interest, the commercial year is usually taken as 360 da., and the month as 30 da.

MONEY

10 mills = 1 cent (c., ¢, or ct.)
100 cents = 1 dollar ($)

ANGLES AND ARCS

60 seconds (″) = 1 minute (′)
60 minutes = 1 degree (°)

COUNTING

12 units = 1 dozen (doz.)
12 dozen, or 144 units = 1 gross (gr.)

PAPER

24 or 25 sheets = 1 quire
20 quires = 1 ream

For folded paper a quire is usually 24 sheets; for unfolded paper it is 25 sheets. Paper is sold also by weight.

WEIGHT PER BUSHEL

Barley	48 lb.	Corn, shelled	56 lb.	Peas	60 lb.
Beans	60 lb.	Flaxseed	55 lb.	Potatoes	60 lb.
Buckwheat	48 lb.	Oats	32 lb.	Rye	56 lb.
Clover seed	60 lb.	Onions	57 lb.	Wheat	60 lb.

These weights are the ones used in New York State.

Practical Equivalents

1 bu. contains $1\frac{1}{4}$ cu. ft., or 2150 cu. in.
1 bbl. contains 4.2 cu. ft., or $31\frac{1}{2}$ gal.
1 gal. contains 231 cu. in.
1 cu. ft. contains $7\frac{1}{2}$ gal.
1 cu. ft. of water weighs $62\frac{1}{2}$ lb.
1 gal. of water weighs 8.3 lb.
1 T. of hay occupies 500 cu. ft.
1 cd. (cord) of 4-foot wood occupies 128 cu. ft.
1 T. of hard coal occupies 35 cu. ft.
1 T. of soft coal occupies 42 cu. ft.
1 cu. ft. of brickwork contains 22 bricks (and mortar)

These are generally only approximations, but they give results sufficiently close for ordinary computations.

Convenient Rules for Farmers

Bushels in a Bin. *Take* $\frac{4}{5}$ *of the product of the three dimensions.*

This gives the number of bushels for a rectangular bin when the dimensions are expressed in feet.

Bushels in a Circular Bin. *Multiply* $\frac{5}{8}$ *of the square of the diameter by the depth.*

This gives the number of bushels in a circular bin when the dimensions are expressed in feet.

Bushels of Shelled Corn. *Multiply the product of the three dimensions by 0.45 for old, dry corn; by 0.4 for new, fresh corn.*

This gives the number of bushels of shelled corn for a rectangular bin when the dimensions are expressed in feet.

Feet of Lumber in a Log. *Subtract 4 from the number of inches in the smallest diameter of the log, square the result, and multiply by* $\frac{1}{16}$ *of the number of feet in the length.*

Tons of Hay in a Stack. 1. *Find the approximate height by subtracting the width from the overthrow and dividing by 2. Then divide the product of the three dimensions by 500.*

The overthrow is measured, over the top, from the ground at one side to the ground at the other side. The dimensions are to be expressed in feet. For wild hay divide by 412; for timothy, by 450; for clover and alfalfa, by 512. The rule gives only an approximate result.

2. *Divide the product of the three dimensions in yards by 30.*

The divisor varies from 30 to 36, according to the kind of hay. This does not give such close results as the first rule and assumes that there is considerable slope to the stack.

Tons of Hay in a Mow. *Divide the product of the three dimensions in yards by 15 for well-packed hay.*

If the hay is not well packed, the divisor may vary from 15 to 18, according to conditions.

Gallons in a Tank or Cistern. *Multiply the product of the three dimensions by 7.5 for rectangular tanks; multiply the product of the depth and the square of the diameter by 5.9 for round (cylindric) cisterns.*

The dimensions in both cases are to be expressed in feet.

Approximate Tonnage of a Silo. The following table is useful:

HEIGHT IN FEET	INSIDE DIAMETER IN FEET					
	10	**12**	**14**	**16**	**18**	**20**
30	47 T.	67 T.	91 T.			
35	59 T.	84 T.	114 T.	137 T.		
40	70 T.	101 T.	138 T.	180 T.	229 T.	
45				215 T.	269 T.	330 T.
50					310 T.	382 T.

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